# SIMULATION STUDY OF HIGH INTENSITY BEAM BUNCHING

P. Sing Babu, A. Goswami and V. S. Pandit\* Variable Energy Cyclotron Centre, 1- AF, Bidhannagar, Kolkata, India

#### Abstract

We present the results of numerical simulations carried out to optimise the bunching performance of sinusoidal, two harmonic and double drift bunchers in the presence of space charge. We have found out the optimum values of buncher parameters. The effect of buncher voltage and drift distance on the efficiency and density distribution at the time focus have been studied for various values of beam current and compared with those obtained in the case of low beam current.

### **INTRODUCTION**

The axial injection system of 10 MeV, 5mA proposed cyclotron consists of a 2.45 GHz microwave ion source to deliver 100keV, 20mA proton beam, two solenoids to transport the beam together with buncher and inflector [1]. In a cyclotron using an external ion source, only a small fraction of the injected continuous beam is accepted in the central region for further acceleration. This is determined by the phase acceptance of the cyclotron. By transforming the dc beam into a suitably bunched beam using a buncher [2,3,4] prior to injection, one can increase the amount of accepted particles considerably. In order to find out a suitable buncher as per our requirement in the limited space, we have carried out studies using a numerical technique to optimise the parameters of a sinusoidal, two harmonics and double drift bunchers suitable for handling high beam current. In the case of a sinusoidal buncher with low beam current and for a given drift distance, one can achieve optimum bunching efficiency by varying only the voltage on the buncher electrode. Same procedure is not true for high beam intensity. In this case one has to optimise both buncher voltage and drift distance to get optimum performance. In the case of double drift buncher, the distance between the two bunchers is also an important parameter. We have calculated the maximum bunching efficiency and optimised parameters for all three bunching systems.

#### THEORY

We have used the well-known disc model [5,6] to incorporate the effect of space charge. A length of beam corresponding to the bunch spacing  $\beta\lambda$ , is divided into *N* number of disc. In order to improve the accuracy, we have also included  $\beta\lambda/2$  period in both side of the period  $\beta\lambda$ . Since the beam radius in our case is small in comparison to  $\beta\lambda$ , it is assumed that the radius of the beam will remain approximately constant throughout. The average electric field of disc *j* on disc *i* is given by

$$E_{ij} = K \sum_{r=1}^{\infty} \exp(\frac{-k_r |z_{ij}|}{b}) \left[\frac{2J_1(k_r a/b)}{k_r J_1(k_r)}\right]^2 sign(z_{ij})$$
(1)

\*pandit@veccal.ernet.in

where,  $K=Q/(2\pi\varepsilon_0 a^2)$ , Q and a are the charge and radius of the disc respectively and b is the radius of the beam pipe.  $z_i$ ,  $z_j$  are the position of the  $i^{\text{th}}$  and  $j^{\text{th}}$  discs and  $J_0$ and  $J_1$  are the well known Bessel functions,  $k_r$  being the  $r^{\text{th}}$  zero of  $J_0$ . The total force acting on  $i^{\text{th}}$  disc can be obtained by summing over  $j^{\text{th}}$  disc i.e.

$$F_i = Q \sum_{j=1}^{2N} E_{ij} , \quad j \neq i$$
 (2)

The total force on any disc due to all other disc depends only on position of the other discs. Since the position of discs changes along the drift length, it is necessary to divide the total drift distance into discrete steps. For the case of sinusoidal buncher when discs pass through the buncher gap they receive voltage impulse and for  $i^{th}$  disc it is given by,

$$\Delta T_i = -V_1 \sin\left(\frac{4\pi(i-1)}{(2N-1)}\right) \tag{3}$$

where  $V_1$  is the amplitude of the buncher voltage. In the case of two harmonic buncher, where rf of frequencies  $\omega$  and  $2\omega$  are applied at the same gap, the resultant voltage impulse on i<sup>th</sup> disc after passing the buncher gap is,

$$\Delta T_i = -V_1 \sin\left(\frac{4\pi(i-1)}{(2N-1)}\right) + V_2 \sin\left(\frac{8\pi(i-1)}{(2N-1)}\right)$$
(4)

where  $V_1$  and  $V_2$  are the amplitude of the buncher voltage for  $\omega$  and  $2\omega$  respectively. The increase in kinetic energy  $T_0$  of the *i*<sup>th</sup> disc after passing the buncher gap is given by,  $T_i = T_0 + \Delta T_i$  (5)

In the present calculation we have not considered the effect of energy spread. We have calculated position and velocity of all the discs with respect to the central disc, which gets no impulse from the buncher. The position and velocity of the  $i^{th}$  disc with respect to the central disc are,

$$z_i = (i - \frac{2N+1}{2})h, \quad \delta\beta_i = (\beta_i - \beta_0) \tag{6}$$

Here  $h = \beta \lambda / N$ , is the width of each disc and  $\beta_0$  and  $\beta_i$  are the velocity parameters of the central disc and the *i*<sup>th</sup> disc. The position  $zI_i$  of the *i*<sup>th</sup> disc at a distance *d* from the buncher is given by

$$z1_i = z_i + \delta\beta_i \frac{d}{\beta_0} \tag{7}$$

*d* is the fraction of the drift length and can be suitably chosen to improve the accuracy. The space charge forces on discs changes with the position of discs. The space charge force then modify the velocity of the discs. The modified velocity of the  $i^{th}$  disc with mass M after first step is given by ,

$$\delta\beta n_i = \delta\beta_i + \frac{F_i}{M} \cdot \frac{d}{\beta_0 \cdot c^2} \tag{8}$$

(11)

The position of the discs changes due to the modified velocity and it is given by,

$$zn_i = z_i + \frac{\delta\beta_i + \delta\beta n_i}{2} \cdot \frac{d}{\beta_0}$$
(9)

We can calculate the position and velocity of the discs at distance 2d, 3d, 4d ... and so on till the central disc completes the total drift length.

In case of double drift buncher first harmonic rf voltage is applied at one gap and second harmonic rf voltage is applied at an another gap at some distance from the first gap. In this case  $i^{th}$  disc get the energy at the first gap as given by equations (3) and (5) and moves ahead. As it reaches the second gap the voltage impulse received by the  $i^{th}$  disc is given by

$$\Delta T2_i = -V_2 \sin\left(\frac{8\pi}{(2N-1)} \cdot \frac{zn_i}{h}\right) \tag{10}$$

and

 $T1n_i = Tn_i + \Delta T2_i$ 

where,  $zn_i$  is the position of the *i*<sup>th</sup> disc at the second buncher gap. We can calculate the position and velocity of the discs at distance (n+1)d, (n+2)d, ... and so on till the central disc completes the total drift length L.

We have written a computer code, which calculates the position and velocity of all the discs including the space charge forces at any specified position along the drift distance.

## **RESULTS AND DISCUSSIONS**

In the present calculation we have taken 360 discs in one bunch spacing  $\beta\lambda$ , but this can be changed as required. Before performing the simulation, the force between discs for 5000 discs separation have been calculated in advance and stored in a file for interpolation. We have observed that one disc per degree gives a good accuracy. We have calculated the efficiency of all the bunchers for 100keV protons at the time focus for a phase widths of 30° of rf. We have assumed the beam radius *a* to remain constant (*a*=5mm) during the drift.



Figure 1: Optimised bunching efficiency as a function of drift length for various values of beam current.

In the case of sinusoidal buncher we have optimised the bunching efficiency by varying the buncher voltage and the drift distance. We have observed that at nominal beam current (<1mA) the bunching efficiency remains almost independent of the drift length. However, the optimum buncher voltage increases with decreasing the drift distance. This behaviour is similar to the bunching process as observed in the absence of space charge. For higher current the bunching efficiency remains almost constant up to a certain drift distance and then decreases rapidly as the drift is increased. This behaviour is independent of the voltage on the buncher and is shown in Fig. 1 for several values of beam current optimised for the beam phase width 30° of rf.



Figure 2: Variation of optimum drift length and voltage on buncher as a function of beam current.

We have observed that for a given beam current and fixed phase width there is an optimum drift length below which the bunching efficiency remains almost constant. However the buncher voltage increases as we decrease the drift distance from the optimum distance. It is obvious that one should use buncher voltage as minimum as possible to save the rf power. We define the optimum drift length as one which requires minimum buncher voltage for bunching efficiency ~ 90% of the maximum. The optimum drift length decreases as we increase the beam current. Fig.2 shows the variation of the optimum drift length and optimised buncher voltage for various values of the beam current and beam radius.

In the case of two harmonics buncher the rf voltage of frequency  $\omega$  and  $2\omega$  are applied at the same gap. We have optimised the bunching efficiency by varying the voltages of the two harmonics. For the double drift buncher system, we have optimised the bunching efficiency by varying the voltages of the two harmonics and the separation between  $\omega$  and  $2\omega$  bunchers. It has been observed that for a particular value of the beam current the optimum drift length *L* remains same for sinusoidal, two harmonics and double drift bunchers. Thus the drift length *L* between the buncher and the time focus is a crucial parameter to obtain a good bunching efficiency for a given beam current.

The variations of buncher voltage  $V_1$  (fundamental  $\omega$ ) and the ratio of voltages  $V_2$  (second harmonic  $2\omega$ ) and  $V_1$ for two harmonics buncher are shown in Fig. 3 as a function of beam current. In all the cases bunching efficiency is ~73% for phase width  $30^0$  of rf.



Figure 3: Variation of parameters of a two harmonics buncher as a function of beam current.

The variation of bunching efficiency of double drift buncher with l/L (the ratio of distance between two buncher gaps l and total drift length L) is shown in Fig. 4 for three values of beam current and optimum drift length.



Figure 4: Variation of bunching efficiency for a double drift buncher as a function of l/L for various current.

#### **LEBT AND BUNCHER**

The length of Low Energy Beam Transport (LEBT) line is  $\sim 2.8$  meter. It consists of two solenoid magnets (S1 and S2) each of physical length 45 cm. In between the two solenoids there is a slit to remove undesired part of the beam and a faraday cup to measure the beam current. The optimised envelop for 10 mA of beam current (cw), obtained solving K-V beam envelope equations is shown in Fig. 5 (dotted lines). The maximum drift distance available in our transport line for the buncher is ~100 cm with one solenoid magnet S2 in between. We have carried out the optimisation of a sinusoidal buncher for this particular case using the above mentioned disc model in the longitudinal direction and K-V beam envelop equations in the transverse direction. This way we have taken care of the change in beam radius and hence the change of intensity in discs in the given beam phase width as the beam travels in the drift space. In this optimisation we have varied the position and strength of the solenoid S2 to get the beam waist at the same position. The dashed curve in Fig. 5 shows the beam envelop with buncher on. As it appears the waist of the beam is formed earlier and size is also increased. It was not possible to form the waist at the earlier location with a reasonable beam size. In order to get the beam waist at the same location as in the case of cw beam, we have to shift the position of the solenoid S2 by 15cm. In the optimisation we have taken care that beam waist and time focus both are formed at the same location. As seen in Fig. 5 (solid line) there is an increase in the beam size in the solenoid and at the waist. Fig. 6 shows the phase space plot at the time focus in the present case and bunching efficiency is ~ 47% for phase width of  $\pm 15^{0}$  rf and energy width of  $\pm 4.5$ keV.



Figure 5: Beam envelopes for 10 mA proton beam in the injection with and without buncher.



Figure 6: Phase space plot at the beam waist.

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