SIMULATION STUDIES ON TRANSVERSE SPACE CHARGE EFFECT IN A HIGH CURRENT COMPACT CYCLOTRON

A. Goswami, P. Sing Babu, and V. S. Pandit*,
Variable Energy Cyclotron Centre, 1-AF, Bidhannagar, Kolkata - 700 064, India

Abstract

In this article we present the results of simulations carried out to evaluate the behavior of the space charge dominated beam envelopes more accurately during the injection in a compact cyclotron. We have studied the change in the beam envelope by changing the current of the injected beam and estimated the limit on the beam current that can be injected. The dependence of the limiting current on various machine and beam related parameters have also been studied.

INTRODUCTION

The Variable Energy Cyclotron Centre at Calcutta is developing a 2.45 GHz microwave ion source, which will deliver 30mA of proton at 100keV. This beam will be transported by a low energy beam injection line and will be injected axially into a 10 MeV, 5-10mA compact radial sector cyclotron. The basic aim of this project is to study the various physics and technological problems associated with the production and handling of high intensity beams. The motivation of our present study is to improve and estimate the transverse space charge limiting current and to make projections as how to achieve this in a low energy compact cyclotron.

In a low energy compact cyclotron the vertical focusing is very weak at the center. The space charge force further reduces vertical focusing and sets an upper limit on the beam current. This limit on the beam current depends on various factors such as injection energy, beam emittance, available vertical aperture, vertical tune, phase acceptance in the central region etc. To improve the limiting current, apart from using a high value of injection energy, the general trend is to use the value of the betatron tune as high as possible. In our recent work [1] we have shown that this procedure generally does not yield the desired result. In this work we first briefly mention the analytical results to get maximum space charge limiting current by optimizing the parameters of the compact cyclotron. Here we assume cyclotron as a uniform focusing channel. Then we describe the more accurate numerical simulations carried out for studying the behavior of beam envelope during the first turn. We have studied the change in the beam envelope by changing the current of the injected beam as well as initial width and divergence of the beam and estimated the limit on the beam current that can be injected. The dependence of the limiting current as well as the behavior of the beam envelope have been studied under various conditions by changing beam as well as machine related parameters.

LIMITING CURRENT

Assuming cyclotron to be a uniform focusing channel, an analytical expression for the limit on the injected beam current I can be obtained by solving the K-V beam envelope equation under the matched condition. We can obtain following expression for the beam current [1,2],

$$I = \frac{I_0}{2}\beta\gamma\,\frac{\Delta\phi}{2\pi}\left[\frac{q^2\overline{B}^2 a^2\,\upsilon_z^2}{m^2c^2} - \frac{\varepsilon_n^2}{a^2}\right] \qquad (1)$$

where a is the matched (allowed) beam radius, R is the orbit radius at the injection, υ_z is the axial betatron tune, q/m charge to mass ratio and $\overline{B}$ is average magnetic field. β and γ are the usual relativistic terms. ε_n is the normalized emittance, $\Delta\phi$ is the accepted beam phase width. I_0 is the well-known characteristic current and for proton, I_0 = 31 MA. For an N sector compact cyclotron with hill field B_H and valley field B_V we have the following relations for average magnetic filed $\overline{B}$ and axial betatron tune υ_z at a particular radius as,

$$\overline{B} = \frac{N\theta_H}{2\pi}(B_H - B_V) + B_V \qquad (2)$$

$$\upsilon_z^2 = -\beta^2\gamma^2 + \frac{N^2}{N^2-1}\left[\frac{\left(B_H-\overline{B}\right)\cdot\left(\overline{B}-B_V\right)}{\overline{B}^2}\right]\left(1+2\tan^2\xi\right) \qquad (3)$$

Here θ_H is the hill angle and ξ is the spiral angle at the injection radius R. It is to be noted here that these formulas are appropriate for sufficiently large injection radius and high injection energy. Putting equation (2) and (3) in equation (1) one can easily find out that maximum of the limiting current I will occur at a particular hill angle where $\upsilon_z\overline{B}$ will be maximum. This occurs at a hill angle $\theta_H \approx \pi/N$.

In Fig.1 we have shown the variation of the limiting current in an N=4 straight sector cyclotron as a function of hill angle for various combinations of the fields in the hill and valley. Typical beam parameters used in the simulation are as follows: 100 keV for the injection energy which corresponds to $\beta\gamma \approx 0.015$, 5mm for the beam radius a, 30 degree for beam phase width $\Delta\phi$ and 0.8π mmmrad for the normalized emittance ε_n.

It is easy to see from Figure 1 that limiting current has a broad maximum in the vicinity of the sector angle $\theta_H \approx 45^0$. The peak value of the limiting current as well as the sharpness of the peak both are seen to increase as the different between the hill and valley fields (B_H -B_V) is increased.

---

*Email: pandit@veccal.ernet.in

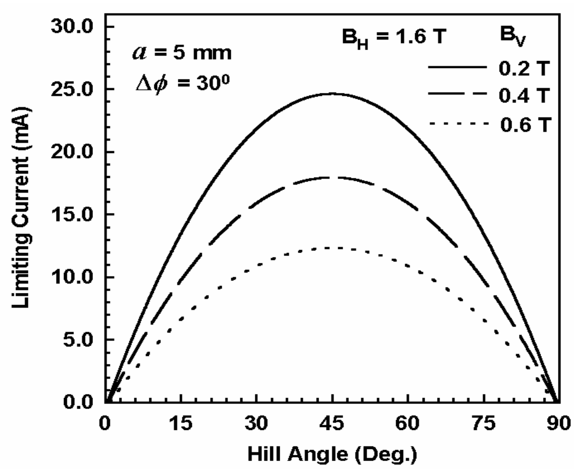

Figure 1: Variation of limiting current I as a function of hill angle in a 4 sector compact cyclotron.

Thus the current that can be injected into a compact cyclotron can be increased substantially by increasing the difference between the hill and the valley fields. Further, limiting beam current increases with the beam energy and reduces if the emittance of the injected beam is large. Clearly a beam with low emittance and high injection energy is desired. Results presented here are valid when the injection energy of the beam is high ~ 100 keV to 200 keV and the average magnetic field $\bar{B}$ is low (< 1.0 tesla).

BEAM ENVELOPE

Results presented in the previous section are based on the assumption that cyclotron is a uniform focusing channel and the available aperture is completely filled. Though these results are very useful for initial estimation, a more accurate study is necessary which considers the envelope oscillations due to the coupling of the axial and horizontal motion. In this section we present numerical simulation method to study the behaviour of the beam envelope during the injection in our proposed compact cyclotron. Important parameters [3] of our proposed 10MeV four sectors compact cyclotron (preliminary design) used for studying the behaviour of beam envelope during the injection are listed in table 1.

Table 1: Parameters of the cyclotron

Injection Energy	100keV	Final Energy	10MeV
Hill Field B_H	1.5T	Valley Field B_V	0.1T
Hill gap	3cm	Valley gap	46cm
Min. hill angle	35.6^0	Max. hill angle	36.0^0
No. of Dee	2	Dee voltage	100kV
Injection radius	6.6cm	Phase width	30^0

We have written a computer code in which hills and valleys are treated as bending magnets with suitable radius and angle. The flaring and edge effect are introduced by using thin lenses at each hill valley boundary. The two accelerating dees in two opposite valleys have been approximated by four step-function accelerating gaps. The following coupled KV beam envelope equations [2] were solved numerically,

$$X'' + k_x^2 X - \frac{4I}{(X+Y)I_0\beta^3\gamma^3}\cdot\frac{2\pi}{\Delta\phi} - \frac{\varepsilon_x^2}{X^3} = 0 \qquad (4)$$

$$Y'' - \frac{4I}{(X+Y)I_0\beta^3\gamma^3}\cdot\frac{2\pi}{\Delta\phi} - \frac{\varepsilon_y^2}{Y^3} = 0 \qquad (5)$$

where, k_x is the well-known focusing strength of magnets (hill or valley) in horizontal direction. The term $(2\pi/\Delta\phi)$ is included with the beam current I to account for the phase acceptance in the central region.

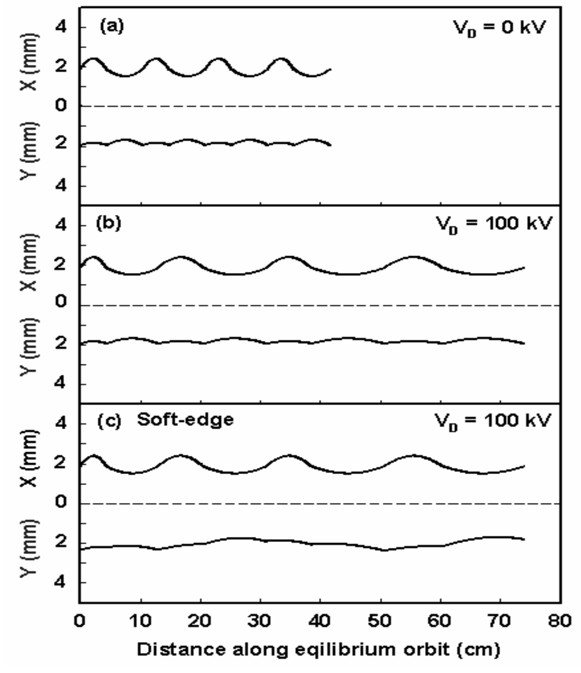

Figure 2: Beam envelopes in radial and axial directions during the first turn.

In our calculations we have used same value for the normalised emittance ε_n ($=\beta\gamma\cdot\varepsilon_x=\beta\gamma\cdot\varepsilon_y$) in both the planes equal to 0.8π mmmrad. Equations (4) and (5) were solved numerically in the hill magnet and the valley magnet to obtain X, X' and Y, Y'. We have used step size of 0.1mm. At each hill valley boundary X, X' and Y, Y', obtained from the solutions are converted to the well known twiss parameters α, β and γ using the following relations,

$$\beta_x = \frac{X^2}{\varepsilon_x}, \quad \alpha_x = -\frac{XX'}{\varepsilon_x}, \quad \gamma_x = \frac{1+\alpha_x^2}{\beta_x} \qquad (6)$$

Similar expressions are used for Y direction also. These twiss parameters are then transformed to new values by the thin lens matrices **R,** which include the edge effect, flaring and soft edge [4,5] as,

$$\mathbf{J_2} = \mathbf{R}\cdot\mathbf{J_1}\cdot\mathbf{R}^{-1}, \qquad \mathbf{J} = \begin{bmatrix}\alpha & \beta \\ -\alpha & -\gamma\end{bmatrix} \qquad (7)$$

where, $\mathbf{J_1}$ and $\mathbf{J_2}$ stand for initial and final matrix containing twiss parameters respectively. The final twiss parameters obtained from $\mathbf{J_2}$ are then reverted back to X,

X' and Y, Y' for further calculations using relations of equation (6).

RESULTS AND DISCUSSIONS

Figure 2(a) shows the beam envelope for the first turn at injection radius of 6.6 cm without acceleration and with zero beam current whereas figure 2(b) shows the envelope with acceleration using four gaps with voltage 100kV in two opposite valleys. We have chosen the starting point at the entry of a hill and the first acceleration gap is at the beginning of the first valley. We can see that the periodicity in the envelope oscillation is still maintained except for the path length, which is increased due to the acceleration. Figure 2(c) shows the beam envelope with acceleration and soft edge. The introduction of soft edge has increased the envelope amplitude as well as added an extra oscillation in the axial envelope.

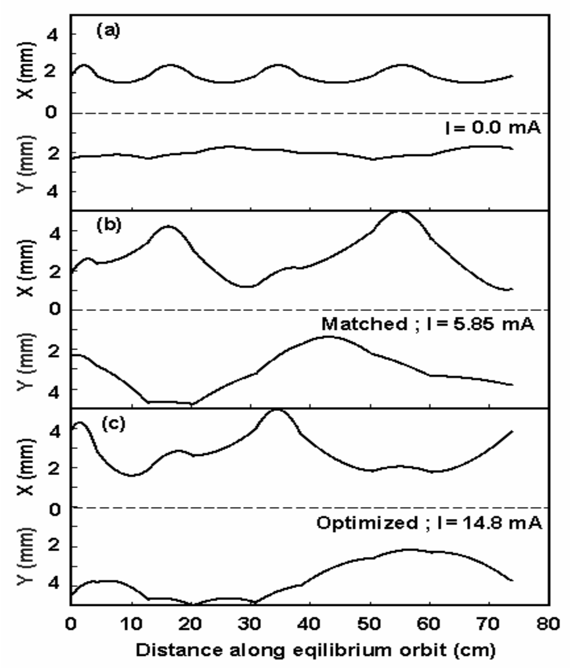

Figure 3: Beam envelopes in the radial and axial directions under matched and optimized conditions.

Figure 3 (b) and (c) show the space charge dominated beam envelope in the matched condition and optimized conditions (using initial up-right ellipses i.e. $\alpha_x = \alpha_y = 0$) together with the envelope (Figure 3(a)) without any beam current for comparison. The maximum envelope amplitude is restricted to remain within 5mm in the both planes in all cases. It is clear that for space charge dominated beams matched condition is not at all suitable for injection because best aperture filling does not occur for a matched beam. We have observed that optimised case is very sensitive to initial beam parameters and machine parameters. A change in dee voltage from 100kV to 50 kV reduces the injected beam current marginally (0.25mA) in the matched condition, whereas, this reduction is substantial in the case of optimised condition (2.2mA). During the optimization we observed that for a particular value of the beam current there is a particular set of initial parameters, which gives the optimum beam envelope. An optimized initial set of X, X' and Y, Y' which confines 14.8 mA within 5mm aperture is not at all suitable for say 10 mA or 11 mA. We believe that this effect is due to the coupling between X and Y motions as a result of the space charge effects which couples the both motions. We have also tried to optimize the beam envelope using tilted initial ellipses and we noticed a marginal increase in the beam current (15.8 mA). Figure 4 shows the beam ellipses during the injection for matched and optimized cases. Results of the simulation indicates that a compact cyclotron of the parameters listed in table 1 can easily handle an injected beam current of 10 mA within the 5mm aperture half width in both the horizontal and vertical planes.

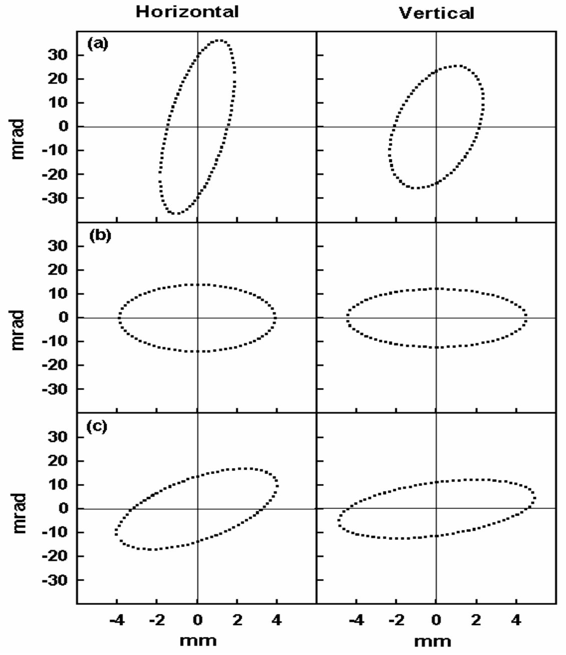

Figure 4: Beam ellipses for horizontal and axial plane (a) matched condition (5.85mA) (b) optimized case with upright ellipses (14.8 mA) (c) optimized case with tilted ellipses (15.8 mA).

REFERENCES

[1] V.S. Pandit and P.S. Babu, “Estimation of the Transverse Space Charge Limiting Current in a Compact Isochronous Cyclotron”, Nucl. Instr. and Meth **A** 523 (2004) 19.

[2] M. Reiser, “Theory and Design of Charged Particle Beams”, John Wiley and Sons, New York (1994).

[3] V.S. Pandit, “ Conceptual Design of a High Current Injector Cyclotron for ADSS,” InPAC’2003, Indore, Feb 2003, p. 69.

[4] J.R. Richardson, “Sector Focusing Cyclotron”, Progress in Nucl. Tech. & Instr, Vol. 1 North Holland, Amsterdam, 1961, p. 21.

[5] H.A. Enge, “Focusing of Charge Particles” Edited by A. Septier, Academic Press, Vol 2, 1967, p. 203.