REFINED MODELS OF INTRABEAM SCATTERING

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Abstract

Fundamental limitations and two specific extensions of intrabeam-scattering (IBS) theory are discussed. First, starting from the Bjorken-Mtingwa recipe, generalized formulae are derived for the three electro-magnetic intrabeam scattering growth rates, including non-ultrarelativistic terms and vertical dispersion, still maintaining a Gaussian beam approximation. A few applications to LHC and CLIC demonstrate the importance of the vertical dispersion. Other limitations of IBS theory are briefly discussed. Second, one of the these limitations is studied in more detail, namely the importance of nuclear scattering. Again estimates are presented for the LHC.

INTRODUCTION

CERN experiments with IBS at low or moderate energy were reported to strongly disagree with MAD predictions [1]. Discrepancies between standard theory and observations were also observed at RHIC [2]. IBS is an important effect for the LHC, where despite of the high beam energy it will affect the luminosity lifetime, and, even more, for the CLIC damping ring, where IBs leads to a quadrupling of the vertical equilibrium emittance [3].

Classical treatments of IBS are those of A. Piwinski initially using a smooth-focusing approximation [4], the general formalism of Bjorken and Mtingwa who also derived explicit formulae for the ultrarelavistic limit [5], non-ultrarelativistic corrections derived by Conte and Martini [6], the formulae programmed by Zisman et al in ZAP [7], plus the general formulae of Kubo and Oide, who also introduced a factor $\sqrt{2}$ correction for bunched beams [8]. There further exists a "modified Piwinski theory" due to K. Bane [9] which yields predictions in good agreement with Bjorken-Mtingwa. We decided to start from the Bjorken-Mtingwa approach.

This paper is organized as follows. We describe in some detail the recent extension of the IBS module in MAD-X, next highlight other limitations inherent in most treatments of IBS, and then, as an example, investigate the possible importance of nuclear scattering for hadron beams. Finally, some conclusions are drawn.

MAD-X IBS EXTENSION

Input and goals of the 2005 MAD-X IBS developments [10] were a recommendation by M. Martini to implement the "Conte-Martini formulae" [6], the desire to verify the algorithm originally implemented in MAD, and the wish to extend this algorithm so as to include vertical dispersion. The latter is crucially important for both damping rings and for the LHC, as neglecting vertical dispersion often predicts a shrinkage of the vertical emittance which is not observed at operating machines above transition. The general IBS formulae were first re-derived including vertical dispersion; in the limit of zero vertical dispersion these formulae were then compared with the equivalent expression from Conte and Martini; lastly IBS growth rates were computed for the LHC, for LHC upgrade scenarios, and for the CLIC damping ring using both the old and the new version of MAD-X.

The derivation starts with expression (3.4) in [5] for the emittance growth rate $1/\tau_d \equiv (d\epsilon/dt)/\epsilon_d$ in direction d:

$$\frac{1}{\tau_d} = \frac{\pi^2 r_0^2 v_c m^3 N(\log)}{\gamma\Gamma} \left\langle \int_0^\infty \frac{d\lambda\ \lambda^{1/2}}{\left[\det\left(L+\lambda I\right)\right]^{1/2}} \right. \left. \left\{ \mathrm{Tr} L^d\ \mathrm{Tr}\left(\frac{1}{L+\lambda I}\right) - 3\ \mathrm{Tr} L^d \left(\frac{1}{L+\lambda I}\right)\right\}\right\rangle , \quad (1)$$

where $d = x$, y, or l, r_0 is the classical particle radius, v_c the speed of light, m the particle mass, N the number of particles per bunch, $(\log) \equiv \ln(r_{\max}/r_{\min})$ a Coulomb logarithm, with $r_{\max}$ denoting the smaller of σ_x and the Debye length and $r_{\min}$ the larger of the classical distance of closest approach and the quantum diffraction limit from the nuclear radius, typically assuming values of $(\log) \approx 15-20$, γ the Lorentz factor, and, for a bunched beam, $\Gamma = (2\pi)^3(\beta\gamma)^3 m^3 \epsilon_x \epsilon_y \sigma_\delta \sigma_z$ the 6-dimensional invariant phase space volume (corrected by a factor of $\sqrt{2}$ [8]) [1],

$$L = L^{(h)} + L^{(l)} + L^{(v)} , \quad (2)$$

with

$$L^{(h)} = \frac{\beta_x}{\epsilon_x}\begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2 H_x/\beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad (3)$$

$$L^{(l)} = \frac{\gamma^2}{\sigma_\delta^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad (4)$$

and, generalizing the Bjorken-Mtingwa theory to the case of nonzero vertical dispersion,

$$L^{(v)} = \frac{\beta_y}{\epsilon_y}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2 H_y/\beta_y & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix} . \quad (5)$$

In the above expressions, $\phi_{x,y}$ and $H_{x,y}$ are defined as

$$\phi_{x,y} \equiv D'_{x,y} - \frac{\beta'_{x,y} D_{x,y}}{2\beta_{x,y}} , \quad H_{x,y} \equiv \frac{D_{x,y}^2 + \beta_{x,y}^2 \phi_{x,y}^2}{\beta_{x,y}} .$$

Bjorken and Mtingwa [5] proceeded by solving (1) with zero vertical dispersion, and neglecting β_x/ϵ_x and β_y/ϵ_y relative to $(\gamma D_x)^2/(\epsilon_x\beta_x)$, $(\beta_x/\epsilon_x)\gamma^2\phi_x^2$, and γ^2/σ_δ^2.

[1] For an unbunched beam, (1) also applies, if one uses $\Gamma = 4\pi^{5/2}(\beta\gamma)^3 m^3 \epsilon_x \epsilon_y \sigma_\delta C$, with C the ring circumference. In this case, Γ is equal to the 6-dimensional invariant phase space volume divided by $\sqrt{2}$.

Conte and Martini [6] kept the terms neglected by Bjorken and Mtingwa, which are important for $\gamma < 10$. We also retain these non-ultrarelativistic terms, and, in addition, we include the vertical dispersion.

For all three cases, namely Bjorken-Mtingwa, Conte-Martini, and the generalized expressions described in this report and now implemented in MAD-X, the three growth rates obtained from (1) can be written in the general form:

$$\begin{aligned}
\frac{1}{\tau_x} &= D\left[\frac{\gamma^2 H_x}{\epsilon_x}\right]\int_0^\infty \frac{d\lambda\,\lambda^{1/2}\,[a_x\lambda+b_x]}{(\lambda^3+a\lambda^2+b\lambda+c)^{3/2}}\ , \\
\frac{1}{\tau_l} &= D\left[\frac{\gamma^2}{\sigma_\delta^2}\right]\int_0^\infty \frac{d\lambda\,\lambda^{1/2}\,[a_l\lambda+b_l]}{(\lambda^3+a\lambda^2+b\lambda+c)^{3/2}}\ , \\
\frac{1}{\tau_y} &= D\left[\frac{\beta_y}{\epsilon_y}\right]\int_0^\infty \frac{d\lambda\,\lambda^{1/2}\,[a_y\lambda+b_y]}{(\lambda^3+a\lambda^2+b\lambda+c)^{3/2}}\ , \qquad (6)
\end{aligned}$$

where $D \equiv \pi^2 r_0^2 v_c m^3 N(\log)/(\gamma\Gamma)$. The coefficients a, b and c of the denominator are the same for the three planes. The nine coefficients a, b, c, a_x, b_x, a_l, b_l, a_y, and b_y depend on the approximation used [10]. For the most general case considered, i.e., including non-ultrarelativistic terms and vertical dispersion, they are listed in Table 1. If we set the vertical dispersion to zero, our vertical and longitudinal growth rates agree with those of Conte-Martini. However, our expressions for the coefficients a_x and b_x in Table 1 still differ from those of Conte and Martini, the latter containing the additional terms $6(\beta_x/\epsilon_x)\gamma^2\phi_x^2$ and $6\beta_x\beta_y/(\epsilon_x\epsilon_y)\gamma^2\phi_x^2$, respectively. For all examples we have considered so far, these terms made a negligible contribution to the growth rate.

Table 1: Coefficients for IBS growth rates in Eq. (6).

$$\begin{aligned}
a &= \gamma^2\left(\frac{H_x}{\epsilon_x}+\frac{H_y}{\epsilon_y}\right)+\frac{\gamma^2}{\sigma_\delta^2}+\left(\frac{\beta_x}{\epsilon_x}+\frac{\beta_y}{\epsilon_y}\right) \\
b &= \left(\frac{\beta_x}{\epsilon_x}+\frac{\beta_y}{\epsilon_y}\right)\left(\frac{\gamma^2 D_x^2}{\epsilon_x\beta_x}+\frac{\gamma^2 D_y^2}{\epsilon_y\beta_y}+\frac{\gamma^2}{\sigma_\delta^2}\right)+\frac{\beta_x\beta_y\gamma^2}{\epsilon_x\epsilon_y}\left(\phi_x^2+\phi_y^2+\frac{1}{\gamma^2}\right) \\
c &= \frac{\beta_x\beta_y}{\epsilon_x\epsilon_y}\left(\frac{\gamma^2 D_x^2}{\epsilon_x\beta_x}+\frac{\gamma^2 D_y^2}{\epsilon_y\beta_y}+\frac{\gamma^2}{\sigma_\delta^2}\right) \\
a_x &= 2\gamma^2\left(\frac{H_x}{\epsilon_x}+\frac{H_y}{\epsilon_y}+\frac{1}{\sigma_\delta^2}\right)-\frac{\beta_x H_y}{H_x\epsilon_y}+\frac{\beta_x}{H_x\gamma^2}\left(\frac{2\beta_x}{\epsilon_x}-\frac{\beta_y}{\epsilon_y}-\frac{\gamma^2}{\sigma_\delta^2}\right) \\
b_x &= \left(\frac{\beta_x}{\epsilon_x}+\frac{\beta_y}{\epsilon_y}\right)\left(\frac{\gamma^2 H_x}{\epsilon_x}+\frac{\gamma^2 H_y}{\epsilon_x}+\frac{\gamma^2}{\sigma_\delta^2}\right)-\gamma^2\left(\frac{\beta_x^2}{\epsilon_x^2}\phi_x^2+\frac{\beta_y^2}{\epsilon_y^2}\phi_y^2\right) \\
&\quad+\left(\frac{\beta_x}{\epsilon_x}-\frac{4\beta_y}{\epsilon_y}\right)\frac{\beta_x}{\epsilon_x}+\frac{\beta_x}{\gamma^2 H_x}\left(\frac{\gamma^2}{\sigma_\delta^2}\left(\frac{\beta_x}{\epsilon_x}-\frac{2\beta_y}{\epsilon_y}\right)+\frac{\beta_x\beta_y}{\epsilon_x\epsilon_y}\right. \\
&\quad\left.+\gamma^2\left(\frac{2\beta_y^2\phi_y^2}{\epsilon_y^2}-\frac{\beta_x^2\phi_x^2}{\epsilon_x^2}\right)\right)+\frac{\beta_x H_y}{\epsilon_y H_x}\left(\frac{\beta_x}{\epsilon_x}-\frac{2\beta_y}{\epsilon_y}\right) \\
a_l &= 2\gamma^2\left(\frac{H_x}{\epsilon_x}+\frac{H_y}{\epsilon_y}+\frac{1}{\sigma_\delta^2}\right)-\frac{\beta_x}{\epsilon_x}-\frac{\beta_y}{\epsilon_y} \\
b_l &= \left(\frac{\beta_x}{\epsilon_x}+\frac{\beta_y}{\epsilon_y}\right)\gamma^2\left(\frac{H_x}{\epsilon_x}+\frac{H_y}{\epsilon_y}+\frac{1}{\sigma_\delta^2}\right)-2\frac{\beta_x\beta_y}{\epsilon_x\epsilon_y} \\
&\quad-\gamma^2\left(\frac{\beta_x^2\phi_x^2}{\epsilon_x^2}+\frac{\beta_y^2\phi_y^2}{\epsilon_y^2}\right) \\
a_y &= -\gamma^2\left(\frac{H_x}{\epsilon_x}+\frac{2H_y}{\epsilon_y}+\frac{\beta_x}{\beta_y}\frac{H_y}{\epsilon_x}+\frac{1}{\sigma_\delta^2}\right)+2\gamma^4\frac{H_y}{\beta_y}\left(\frac{H_y}{\epsilon_y}+\frac{H_x}{\epsilon_x}\right) \\
&\quad+\frac{2\gamma^4 H_y}{\beta_y\sigma_\delta^2}-\left(\frac{\beta_x}{\epsilon_x}-\frac{2\beta_y}{\epsilon_y}\right) \\
b_y &= \gamma^2\left(\frac{\beta_y}{\epsilon_y}-\frac{2\beta_x}{\epsilon_x}\right)\left(\frac{H_x}{\epsilon_x}+\frac{1}{\sigma_\delta^2}\right)+\left(\frac{\beta_y}{\epsilon_y}-\frac{4\beta_x}{\epsilon_x}\right)\frac{\gamma^2 H_y}{\epsilon_y} \\
&\quad+\frac{\beta_x\beta_y}{\epsilon_x\epsilon_y}+\gamma^2\left(\frac{2\beta_x^2\phi_x^2}{\epsilon_x^2}-\frac{\beta_y^2\phi_y^2}{\epsilon_y^2}\right)+\frac{\gamma^4 H_y}{\beta_y}\left(\frac{\beta_x}{\epsilon_x}+\frac{\beta_y}{\epsilon_y}\right)\left(\frac{H_y}{\epsilon_y}+\frac{1}{\sigma_\delta^2}\right) \\
&\quad+\left(\frac{\beta_y}{\epsilon_y}+\frac{\beta_x}{\epsilon_x}\right)\gamma^4\frac{H_x H_y}{\beta_y\epsilon_x}-\gamma^4\frac{H_y}{\beta_y}\left(\frac{\beta_x^2}{\epsilon_x^2}\phi_x^2+\frac{\beta_y^2}{\epsilon_y^2}\phi_y^2\right)
\end{aligned}$$

Inspection of the original MAD-X code revealed that the Conte-Martini formulae [6] were the ones implemented (presumably they had been copied from ZAP [7]), and not the original expressions from Bjorken and Mtingwa [5]. Correcting the formulae for the horizontal plane and adding the terms containing vertical dispersion resulted in the new MAD-X IBS module.

In the LHC vertical dispersion is generated by the vertical crossing angles at interaction points 1 and 2 as well as by the detector fields of ALICE and LHCB. The peak vertical dispersion in the arcs is about 0.2 m, or 10% of the horizontal. Comparing IBS growth rates from the previous MAD-X version and those by the new one, a large difference is seen in the vertical growth rate [10]. Namely, the vertical IBS growth time in LHC changes from -2.9×10^6 h (damping) for the old version of MAD to +436 h for the new MAD-X. As a crosscheck, without crossing angles and detector fields the growth rates from the old and new MAD-X versions agree in the first three digits. The new MAD-X provides as further output the local IBS growth rates around the ring. For LHC by far the highest vertical IBS growth rates are found in the low-beta interaction regions 1 and 5. LHC upgrade scenarios feature higher bunch charge, reduced longitudinal emittance, shorter bunch length, larger crossing angles, or higher rf voltage [10]. The smallest longitudinal growth rate is of order 10 h (nominal 58 h), and the lowest vertical one about 80 h [10].

For the CLIC damping ring the vertical dispersion is zero by design, but spurious dispersion is generated by misalignment errors and quadrupole tilts [3, 11]. Considering random quadrupole roll angles of $\sigma = 200\ \mu$rad rms, with a Gaussian distribution cut at 2.5 σ, the growth rates are $\tau_l \approx 2.2$ ms, $\tau_x \approx 2.1$ ms, and $\tau_y \approx 2.0$ ms, to be compared with $\tau_l \approx 2.2$ ms, $\tau_x \approx 2.2$ ms, and $\tau_y \approx 12.6$ ms, obtained by the previous MAD-X. That is, with the new MAD-X the vertical growth time is a factor 6 shorter when errors generating vertical dispersion are included. The IBS growth rates are large in the arcs.

LIMITATIONS

All existing IBS theories are seriously limited: The rms emittance growth is calculated assuming Gaussian beams, but real beams are not Gaussian, e.g., due to the very effect of IBS. We still lack an accurate description of beam tails arising from IBS. One attempt was made to correctly estimate the "core emittance" growth in e^+ or e^- rings, by discarding the scattering events leading to tails [12]. An alternative approach may be multiparticle Monta-Carlo simulations based on a binary collision model [13, 14], which could eventually lead to a self-consistent prediction. In our treatment above, coupling between the horizontal and vertical motion was not included. We note that IBS computation with arbitrary coupling between the three degrees of freedom is already available in the SAD code [8]. All existing theories consider only binary collisions, ignoring the possible contribution from many-body interactions. Further, they all start from a Coulomb scattering amplitude of the form $M_{\rm em} = 4\pi\alpha/q^2$, with α the fine-structure constant, and q the momentum transfer, which is an approximation for small-angle scattering of non-relativistic spinless point particles [8]. Missing are relativistic corrections (known to be a significant effect for the closely related Touschek scattering [15]), the spin dependence of the cross section in case of polarized beams (which could be important for polarized proton beams and for polarized e^+ or e^- beams in linear-collider damping rings, as the spin dependence of the cross section [16] is large for Touschek scattering and the basis of many resonant-depolarization measurements [17, 18]), and correction terms for scattering of identical particles [8, 19]. Normally, electro-magnetic scattering alone is taken into account, and any interference with nuclear scattering [19] is neglected.

NUCLEAR SCATTERING

At 7 TeV, the rms transverse momentum spread of protons in an LHC bunch in both beam and laboratory frame is of order 14 MeV/c. For nuclear scattering, we approximate the invariant scattering amplitude as [20] $M_{\rm nucl} = 4\pi g/(q^2 + m_\pi^2 c^2)$, which corresponds to the classical Yukawa theory; see, *e.g.*, [21]. Here, $g \approx 1$ denotes the coupling constant, and $m_\pi \approx 140$ MeV, characterizes the range of the force. The relative importance of the nuclear and electro-magnetic scattering amplitudes depends on the value of q, since $M_{\rm nucl} \approx q^2/(m_\pi^2 c^2)(g/\alpha) M_{\rm em}$. The two are equal for $q \approx 12$ MeV/c, while in the LHC protons are scattered outside of the rf bucket (Touschek effect) already at $q \approx 1.6$ MeV/c. For large-angle scattering the nuclear interaction is dominant. Experimental data indicate a nuclear elastic cross section $\sigma_{\rm tot}$ for proton-proton scattering at low energy of about 0.3 barn [22]. The corresponding beam lifetime $\tau_{\rm nucl}$ is estimated as

$$\frac{1}{\tau_{\rm nucl}} \approx 4 \frac{p_0 \sigma_{\rm tot}}{m\gamma^2} \frac{N_b}{8\pi^{3/2}\beta_x \sigma_y \sigma_z} , \tag{7}$$

where p_0 denotes the average momentum in the laboratory frame, N_b the bunch population, β_x the horizontal beta function, σ_y the vertical beam size, and σ_z the rms bunch length. Inserting LHC parameters at 7 TeV, we find $\tau \approx 3 \times 10^6$ days, which appears enormous, and at 450 GeV, the estimated lifetime still is a considerable 10^6 days.

The effect of intrabunch nuclear scattering on the beam lifetime is much smaller than that of the nuclear interactions with the residual gas and in beam-beam collisions, which are known to limit the LHC beam lifetime to about 100 h or 20 h, respectively. This difference can be understood by comparing the equivalent proton densities in the beam rest frame. Taking into account Lorentz contraction and expansion, the 7-TeV beam proton density in the beam frame is 3×10^{14} m^{-3}, which is almost five orders of magnitude smaller than both the proton density of the residual hydrogen gas for 100 h beam lifetime, which is 1.4×10^{19} m^{-3} and the effective density of the opposite beam, 1.6×10^{19} m^{-3}, assuming collisions at two interaction points. This explains most of the large difference in the associated beam lifetimes. The non-relativistic relative velocities in intrabunch scattering contribute another factor of about 40, which accounts for the remaining difference.

For ions heavier than hydrogen the fusion cross sections are appreciable, with a plateau value of about 1 barn above 10–20 MeV. However, the density of heavy ions in an LHC bunch is about 10^3 times lower than the density of LHC protons. The cross sections of both proton-proton nuclear elastic scattering ($\sim$0.3 barn) and of heavier-ion fusion ($\sim$1 barn) are significantly smaller than that of Touschek scattering ($\sim$1 kbarn) and the total Coulomb cross section ($\sim$36 Mbarn). Clearly, electro-magnetic scattering dominates, though most of it occurs at small angles.

CONCLUSIONS

The updated MAD-X IBS module correctly accounts for vertical dispersion and it gives more realistic vertical growth rates than both the previous MAD-X version and MAD8 [10]. The alternative and previously existing Kubo-Oide treatment of SAD [8] in addition includes arbitrary coupling. In all IBS theories the scattering cross section is approximated. E.g., relativistic corrections and spin-dependent effects are commonly neglected, though they are likely to be important for many applications. The restriction of the theoretical treatments to Gaussian bunches is not self consistent. Here extended Monte-Carlo simulations in MOCAC style [13] could provide a path forward. Nuclear elastic scattering and even fusion can occur inside an LHC proton or ion bunch [23], respectively, but the event rates are found to be so low that these processes will have a negligible effect on the LHC beam lifetime and emittance [20].

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