Analysis of the magnetized friction force *

A.V. Fedotov †, BNL, Upton, NY 11973, USA
D.L. Bruhwiler, Tech-X, Boulder, CO 80303, USA
A.O. Sidorin, JINR, Dubna, Russia

Abstract

A comprehensive examination of theoretical models for the friction force, in use by the electron cooling community, was performed. Here, we present our insights about the models gained as a result of comparison between the friction force formulas and direct numerical simulations, as well as studies of the cooling process as a whole.

INTRODUCTION

A theoretical calculation of the energy loss by an ion passing through a cloud of electrons in an external magnetic field has been extensively studied by the plasma community (see, for example, recent Refs. [1, 2] and references therein). A treatment is typically done via two complementary approaches: binary collision model and dielectric linear response treatment.

In the presence of a finite-strength magnetic field, both analytic approaches have complications. The binary collision treatment does not provide a closed form solution anymore, because the relative motion and the center of mass motion are now coupled. With some approximations, the closed form expressions for the friction force can be obtained [3, 4]. For arbitrary magnetic field strength, numerical simulations are required. In the dielectric treatment, there exists a closed form expression for the friction force, but it requires numerical evaluation of multi-dimensional integrals with strongly oscillatory integrands [5, 6]. A practical expression, in the form of a one-dimensional integral, is possible in the limit of a very strong magnetic field [7, 8].

A variety of theoretical models for the friction force have been developed [3]-[10]. Unfortunately, the available expressions make various approximations, and the discrepancy between theory and experiments can be large.

In recent years, numerical simulations have been used to explore in detail the collisions between ions and magnetized electrons for arbitrary magnetic field strengths [2]. However, to the best of our knowledge, a systematic comparison with the friction force formulas used by the electron cooling community has not been reported.

Recently, we reported numerical studies [11] with the VORPAL code [12], which includes an algorithm to explicitly resolve close binary collisions [13]. Validation of VORPAL results at least for some limiting cases with numerical integration within the BETACOOL code [14] was also presented [11].

* Work supported by the US Department of Energy
† fedotov@bnl.gov

Following a previous report, we explored various aspects of the magnetized cooling. Some insights are summarized in this paper.

MODELS AND LIMITATIONS

For the parameters of the cooler discussed in this paper, the dominant contribution comes from adiabatic collisions of the magnetized type. In the limiting case of a very strong magnetic field a practical expression for such adiabatic collisions, in the form of a one-dimensional integral, was obtained by Derbenev and Skrinsky [5, 7]:

$$\vec{F}_A = -\frac{2\pi n_e e^4 Z^2}{m} \frac{\partial}{\partial \vec{V}} \int \left[\frac{V_\perp^2}{U_A^3} L_M + \frac{2}{U_A} \right] f(v_e) dv_e, \tag{1}$$

where $\vec{V} = (V_\perp, V_\parallel)$ is the ion velocity, and $U_A = \sqrt{V_\perp^2 + (V_\parallel - v_e)^2}$ is the relative velocity of the ion and an electron "Larmor circle", with the transverse electron velocities assumed to be completely suppressed ("adiabatic" or "magnetized" collisions). The actual values of the magnetic field and transverse rms electron velocity spread enter only via the cutoff parameters under the Coulomb logarithm, which is defined as $L_M = \ln(\rho_{max}/\rho_{min}^A)$. Here, a minimum impact parameter of adiabatic collisions ρ_{min}^A is defined as $\rho_{min}^A = max(\rho_L, Ze^2/mU_A^2)$, where $\rho_L = mc\Delta_{e,\perp}/(eB)$ is the radius of Larmor rotation.

The function in Eq. (1) has asymptotes in the region of small ($V \ll \Delta_{e,\parallel}$) and large ($V \gg \Delta_{e,\parallel}$) ion velocities, where $\Delta_{e,\parallel}$ is the rms longitudinal spread of the electrons. Such asymptotic expressions [7, 9] are widely used for the estimates of the magnetized cooling force in various scenarios.

The asymptotic limits of Eq. (1), while useful qualitative guides, are not sufficient for the design of the electron cooling system, where an accurate description of the friction force is needed for a large range of relative velocities between the ions and electrons. Our computational results [11] showed that the use of such asymptotic limits to construct a friction force expression to cover a full range of relative velocities [9] leads to a significant overestimate of the force. This is the case even when the values of the magnetized logarithm are significant. For example, VORPAL simulations, presented in Fig. 1, are done for the following parameters: B=5T, time of interaction in the beam frame τ =0.4 ns, the rms velocity spreads of the electron beam $\Delta_{e,\perp} = 4.2 \cdot 10^5$ m/s, $\Delta_{e,\parallel} = 1.0 \cdot 10^5$ m/s, Z=79 and the density of electrons in the beam frame $n_e = 2 \cdot 10^{15}\ m^{-3}$.

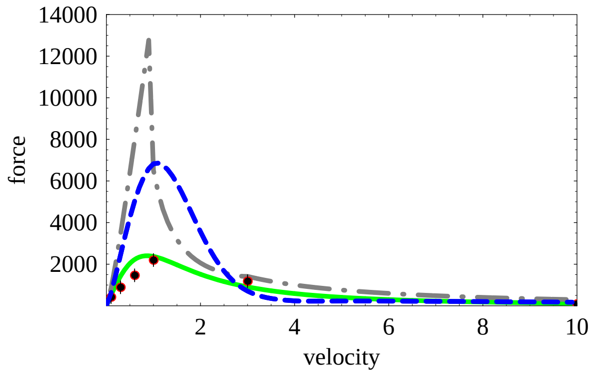

Figure 1: Longitudinal component of the force [eV/m] vs velocity [$\times 10^5$ m/s]. Asymptotic expressions [9] - dot-dash line (gray); Eq. (1) without the non-logarithmic term - dash line (blue); Eq. (4) - solid line (green); VORPAL results - dots with error bars.

The fact that the asymptotic expressions may significantly overestimate the friction force in the vicinity of the force maximum, as shown in Fig. 1, is not surprising since the validity condition for the asymptotic expressions is not satisfied there. The use of Eq. (1) instead of the asymptotic expression helps to avoid strong overestimate of the friction force in the vicinity of the longitudinal spread of the electrons. However, the accuracy of the expression in Eq. (1) is itself of a concern since it was obtained with several approximations, including an approximation of a very strong magnetic field.

One of the features of the integral in Eq. (1) which comes as a result of an assumption of strong magnetic field is that at zero transverse ion velocities, the friction force for $V_{ion} \gg \Delta_{e,\parallel}$ goes to zero in the absence of the non-logarithmic term. Such behavior is, in fact, expected for the asymptotic case of the infinite magnetic field, assuming symmetric and complete collisions.

However, for ion velocities smaller or comparable to the longitudinal velocity spread of the electrons, the integral in Eq. (1) does not go to zero when the transverse velocity of an ion is zero and results in the finite value for the longitudinal component of the force, as was pointed out by Pestrikov [15]. This fact is typically overlooked in most of the literature on electron cooling. In fact, the integral in Eq. (1) and its asymptotic expressions obtained by Derbenev and Skrinsky [7] is typically referred to as the results obtained using the binary-collision approach. This appears to be not accurate, as it was pointed out in Ref. [16]. The expression in Eq. (1) was actually obtained using the dielectric linear plasma response technique [5].

Such a behavior for the zero transverse ion velocity (transverse angle with respect to the magnetic field lines $\theta = 0$) is of special interest. For Gaussian distribution of the electrons

$$f(v_e) = \frac{1}{\sqrt{2\pi}\Delta_{e,\parallel}} \exp\left(-\frac{v_e^2}{2\Delta_{e,\parallel}^2}\right), \qquad (2)$$

the integral in Eq. (1) can be evaluated analytically, and, without the non-logarithmic term, gives the following functional dependence [15]:

$$F_\parallel(0, V_\parallel) = -V_\parallel \frac{4\pi Z^2 e^4 n_e L_M}{m\Delta_{e,\parallel}^3} \exp\left(-\frac{V_\parallel^2}{2\Delta_{e,\parallel}^2}\right), \qquad (3)$$

which is a standard result that can be found in plasma literature, and which appears as a result of the linearized treatment. A functional form of the force in Eq. (3) is very different from a typical functional behavior of the friction force as well as behavior for the non-zero transverse ion velocities. In fact, in some of the literature, it is argued that such a behavior at zero transverse angle with the "enhanced" force values is an artifact due to a failure of the linearized plasma approach to treat hard collisions correctly, and has nothing to do with the collective plasma response [4]. Such arguments are based on numerical simulation of the non-linear plasma response which does not produce the "ehancement" observed in Fig. 1, when Eq. (1) or Eq. (3) is used.

Besides the doubts whether the limiting case for the zero transverse ion velocity in Eq. (3) can be used at all, the force value in Eq. (3) vanishes for the ion velocity equal to $4\Delta_{e,\parallel}$ or higher. The presence of the non-logarithmic term in Eq. (1) helps to get finite force values for zero transverse ion velocities even at relative velocities much higher than the longitudinal velocity spread of the electrons. However, as we reported before [11], it does not provide correct scaling with the magnetized logarithm at zero angle. Also, the use of such a non-logarithmic term may be not even justified in many cases.

The origin of this non-logarithmic term in Eq. (1) is due to collective plasma waves [5], [15], [17]. In a typical low-energy cooler, plasma effects may become important (depending on the parameters) so that inclusion of a term resulting from the collective plasma oscillations may be justified. But for the high-energy coolers, the time of flight of an ion through the cooler in the beam frame becomes extremely short due to the large relativistic factor, so that the maximum impact parameter is determined by the finite interaction time rather than by a dynamic Debye screening.

Our numerical studies, presented in this paper, were done for the parameters of the proposed RHIC cooler ($\gamma = 107$) with the time of flight through the interaction region (in the beam frame) smaller than the plasma period, so that in our case such a non-logarithmic term should be omitted from Eq. (1).

To avoid limitations of the models described above an empirical model for the force was introduced by Parkhomchuk [10]:

$$\vec{F} = -\vec{V}\frac{4Z^2 e^4 n_e L_p}{m}\frac{1}{(V^2 + \Delta_{e,eff}^2)^{3/2}}, \qquad (4)$$

where $\Delta_{e,eff}$ is the effective velocity spread of the electrons. The Coulomb logarithm in Eq. (4) is given by

$$L_p = \ln\left(\frac{\rho_{max} + \rho_{min} + \rho_L}{\rho_{min} + \rho_L}\right). \quad (5)$$

For the special case of zero transverse ion velocity, as in Fig. 1, we find that Eq. (4) is in remarkable agreement with our simulation results using the VORPAL code. This is shown in Fig. 2, where the solid curve corresponds to Eq. (4) with $\Delta_{e,eff} = \Delta_{e,\|}$, and the points with error bars are the VORPAL results. An agreement observed is

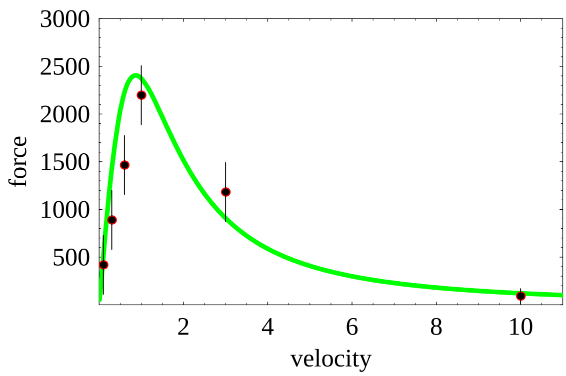

Figure 2: Longitudinal component of the force [eV/m] vs velocity [$\times 10^5$ m/s] for zero transverse angle $\theta = 0$ with respect to the magnetic field lines. VORPAL results: dots with error bars; Eq. (4) - solid line.

not unreasonable, because Eq. (4) was obtained through a systematic parametric fitting of the longitudinal friction force measurements from experiments with ion beams that were already cooled and so had small transverse velocity spread.

However, for the design of future high-energy electron coolers, it is extremely important to have an accurate description of the friction force for the initial state of the ion beam, when the transverse velocities are still large. Thus, it is important to have an accurate description of the longitudinal friction force as a function of the angle between the ion velocity vector and the magnetic field lines.

For the ion motion along the magnetic field lines ($\theta = 0$) Eq. (1) clearly overestimated the force values obtained with the VORPAL simulations, as well as the one predicted by the empirical formula in Eq. (4). For all other angles ($\theta \neq 0$), the functional dependence in Eq. (1) is closer to the expected one and is in reasonable agreement with the dependence in Eq. (4), which is shown in Figs. 3 and 4 for the $\theta = 45$ and $\theta = 60$ degrees angles with respect to the direction of the magnetic field, respectively.

Unfortunately, Eq. (4) does not provide anisotropic behavior of the friction force expected in the presence of the strong magnetic field. For relative velocities larger than the longitudinal spread of the electrons, this model overestimates the friction force for some angles, while underestimating it for others, compared to the VORPAL results [18].

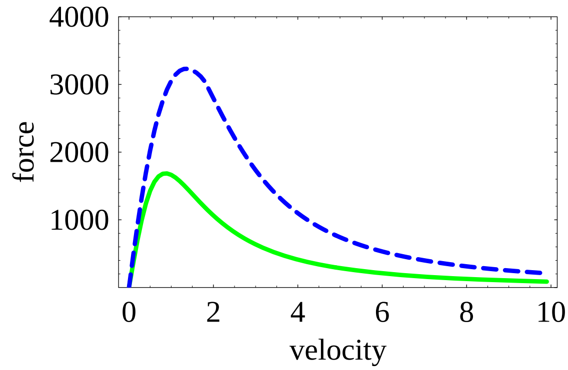

Figure 3: Longitudinal component of the force [eV/m] vs velocity [$\times 10^5$ m/s] for $\theta = 45$ degrees. Eq. (1) without the non-logarithmic term - dash line (blue); Eq. (4) - solid line (green).

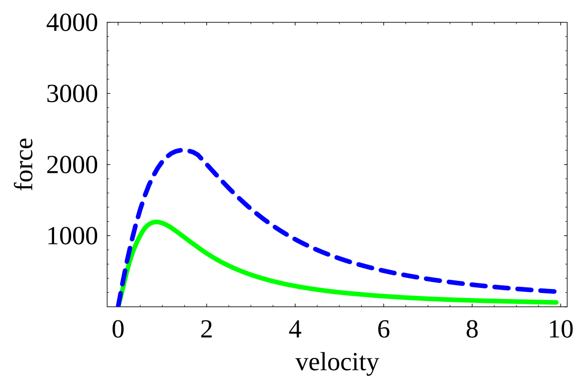

Figure 4: Longitudinal component of the force [eV/m] vs velocity [$\times 10^5$ m/s] for $\theta = 60$ degrees. Eq. (1) without the non-logarithmic term - dash line (blue); Eq. (4) - solid line (green).

However, direct numerical computation of the friction force show weak dependence on angle. This was demonstrated by Parkhomchuk using simulations with the zero-temperature electrons [10]. For finite temperature of the electrons our simulations show even weaker dependence on angle than for the case of the zero-temperature electrons [18]. Such a limitation of Eq. (4) is expected to average out since for the net cooling power one needs to average over all particle amplitudes and phases.

DEPENDENCE ON THE MAGNETIC FIELD VALUES

An important property of the magnetized friction force is its dependence on the strength of the magnetic field in the cooling section. In many experimental measurements, it was reported that no increase of the friction force with the magnetic field increase is actually observed. However, with an accurate set-up of the measurements and with well-aligned ion and electron beams the logarithmic increase with the magnetic field strength is recovered [16], [19],

[20].

Similar ambiguous dependence on the strength of the magnetic field is also reported in cooling simulation [2], [10]. Note, that even in the report where empirical expression in Eq. (4) is advocated, which should provide logarithmic increase in the force values with the magnetic field, the numeric results reported show a reduction in the friction force as the strength of the magnetic field is increased, which is due to the assumption of the zero-temperature electrons. Such a disagreement between Eq. (4) and numerical simulations is confusing unless an explanation of a behavior observed is provided.

In our simulations with the zero-temperature electrons we confirmed such a dependence on the magnetic field, which is shown in Fig. 5. We also find that the reason for such "abnormal" behavior are the fast collisions. The maximum impact parameters in such collisions is limited not by the Larmor radius but by the time of the collision. An interaction time in such a collision increases linearly with the decrease of the magnetic field, which results in such a dependence of the friction force on the magnetic field strength at small transverse angles with respect to the magnetic field lines. To confirm this we plot analytic expression for the force keeping asymptotics for both the magnetized and fast collisions [7, 9] and setting the transverse velocity spread of the electrons to zero. Resulting curves in Fig. 6 clearly show the behavior observed in direct numerical simulations using VORPAL (Fig. 5).

Similar dependence occurs even for the finite temperature electrons when one considers relative velocity much higher than the thermal velocity of the electrons.

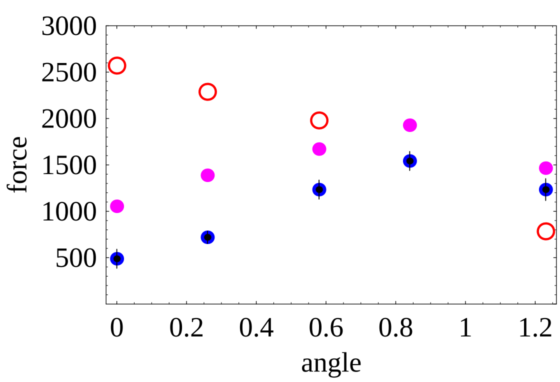

Figure 5: Longitudinal force [eV/m] vs angle [rad] for ion velocity of $V = 3{\cdot}10^5$ m/s, and zero-temperature electrons. VORPAL results: dots with error bars - B=5T, dots without error bars - B=1T, open circles - B=0.1T.

However, for the ion velocities smaller than the transverse velocity spread of the electrons ($V \ll \Delta_{e,\perp}$) the contribution from the fast collisions is strongly reduced as $F \sim 1/(\Delta^2_{e,\perp})$. Such condition occurs very quickly in a typical low-energy cooler and is, in fact, a typical starting condition for the high-energy cooler, for example RHIC-II [21], where initial rms velocity of ions before cooling is

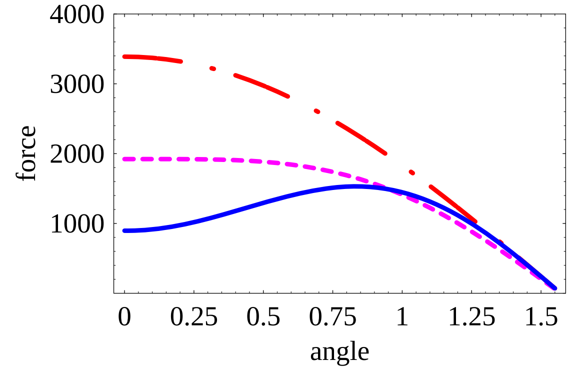

Figure 6: Longitudinal force [eV/m] vs angle [rad] for ion velocity of $V = 3{\cdot}10^5$ m/s, and zero-temperature electrons. Derbenev-Skrinsky-Meshkov asymptotics [9] results: blue (solid) line - B=5T, pink (dash) curve - B=1T, red (dash dot) curve - B=0.1T.

significantly smaller than the transverse velocity spread of the magnetized electron beam.

As a result, for such a condition in the cooler, for the transverse velocity spread of the electrons higher than the ion velocity, our simulations confirm that the friction force increases logarithmically with the increase of the magnetic field strength. Such dependence is expected for the magnetized cooling, and is confirmed by the measurements [19, 16, 20]. This dependence is shown in Fig. 7, where we plot Eq. (4) and results of VORPAL simulations.

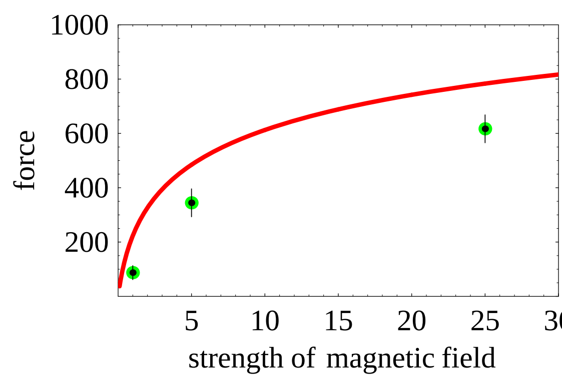

Figure 7: Longitudinal friction force [eV/m] for $(V_{ion,\parallel}, V_{ion,\perp}) = (3 \cdot 10^5, 0)$ m/s vs magnetic field B [T], for the finite temperature electron beam with $(\Delta_{e,\parallel}, \Delta_{e,\perp})$=$(1 \cdot 10^5, 1 \cdot 10^7)$ m/s. Solid curve - Eq. (4); dots with error bars - VORPAL results.

The finite values of the longitudinal friction force at zero transverse angles with respect to the magnetic field lines observed in our simulations (Figs. 1, 2, 7), in the absence of collective plasma effects, are attributed to incomplete electron-ion collisions. We also find that this simulated finite longitudinal force scales with the magnetized logarithm, with maximum impact parameter ρ_{max} determined by the finite interaction time [11, 18].

SUMMARY

The asymptotic limits [7, 9] of Eq. (1) are useful qualitative guides and provide several features typical for the magnetized collisions. However, they are not recommended for the design of the electron cooling system since they can overestimate the cooling power significantly.

The use of Eq. (1) directly by means of a numerical evaluation of the integral avoids significant overestimate of the friction force compared to the asymptotic expressions. However, it requires the use of the non-logarithmic term (not necessarily justified in some case) to prevent unphysical behavior at high relative velocities. Also, its functional behavior at zero transverse ion velocity with the enhanced values for the force may be attributed to the limitation of the linearized dielectric approach to treat accurately close collisions. Thus, this expression should be used with caution.

For a simple estimate of the net cooling power and for finding basic parameters needed for the cooler, the use of empirical expression in Eq. (4) seems sufficient.

For an accurate description of the friction force in a magnetic field of arbitrary strength, with accuracy better than factor of two, direct numerical simulations with a code like VORPAL are required. Numerically generated table for the force values can be used within the BETACOOL code for an accurate prediction of the cooling power.

ACKNOWLEDGMENTS

We are grateful for Ilan Ben-Zvi and Vladimir Litvinenko for many useful discussions and suggestions during these studies. We also thank the Accelerator Physics Group of Electron Cooling Project of RHIC for constant support and stimulating discussions. The support of the VORPAL team, and of the Dubna Cooling group with BETACOOL, is greatly appreciated. This work is supported by the U.S. DOE Office of Science, Office of Nuclear Physics.

REFERENCES

[1] O. Boine-Frankenheim, J. D'Avanzo, Phys. Plasmas, 3, p. 792 (1996), and references therein.

[2] H.B. Nersisyan, G. Zwicknagel, C. Toepffer, Phys. Rev. E 67, 026411 (2003), and references therein.

[3] V.V. Parkhomchuk, Dissertation (BINP, Novosibirsk), 1984.

[4] C. Toepffer, Phys. Rev. A 66, 022714 (2002).

[5] Ya. Derbenev, Dissertation (BINP, Novosibirsk), 1978.

[6] A. Sorensen, E. Bonderup, Nuc. Inst. Meth. 215, p. 27 (1983).

[7] Ya. Derbenev, A. Skrinsky, Part. Acc. 8 (1978) , p. 235.

[8] Ya. Derbenev, A. Skrinsky, Sov. Phys. Rev 1 (1981), p. 165.

[9] I.N. Meshkov, Nuclear. Inst. Meth. **A 391**, 1 (1997).

[10] V.V. Parkhomchuk, Nucl. Inst. Meth. A 441, p. 9 (2000).

[11] A.V. Fedotov, D.L. Bruhwiler, D.T. Abell, A.O. Sidorin, AIP Conf. Proc. 821, p. 319 (2005).

[12] C. Nieter, J. Cary, J. Comp. Phys. 196 (2004) , p. 448.

[13] D.L. Bruhwiler, R. Busby, A.V. Fedotov, I. Ben-Zvi, J.R. Cary, P. Stoltz, A. Burov, V.N. Litvinenko, P. Messmer, D. Abell, C. Nieter, AIP Conf. Proceed. 773 (Bensheim, Germany, 2004), p. 394.

[14] A.O. Sidorin, I.N. Meshkov, I.A. Seleznev, A.V. Smirnov, E.M. Syresin, G.V. Trubnikov, Nucl. Instr. Meth. A 558, p. 325 (2006); http://lepta.jinr.ru.

[15] D.V. Pestrikov, BINP Preprint 2002-57 (Novosibirsk).

[16] N.S. Dikansky, V.I. Kudelainen, V.A. Lebedev, I.N. Meshkov, V.V. Parkhomchuk, A.A. Sery, A.N. Skrinsky, B.N. Sukhina, "Ultimate possibilities of electron cooling", BINP Pripriint 1988-61 (Novosibirsk).

[17] Ya. Derbenev, A. Skrinsky, Fizika plazmy (Sov. J. Plasma Physics), V. 4, p. 492 (1978).

[18] A.V. Fedotov, D.L. Bruhwiler, A.O. Sidorin, D.T. Abell, I. Ben-Zvi, R. Busby, J.R. Cary, V.N. Litvinenko, "Numerical Study of the Magnetized Friction Force", submitted to Phys. Rev. ST AB (2006).

[19] V.V. Parkhomchuk, A.N. Skrinsky, Rep. Prog. Phys. 54, p. 919 (1991).

[20] A.V. Fedotov, B. Gålnander, V.N. Litvinenko, T. Lofnes, A. Sidorin, A. Smirnov, V. Ziemann, "Experimental studies of the magnetized friction force" , submitted to PRE (2006).

[21] A.V. Fedotov, I. Ben-Zvi, V.N. Litvinenko, A. Sidorin, A. Smirnov, G. Trubnikov, Proceedings of PAC05, p. 4236 (2005).