RESONANCE TRAPPING DUE TO SPACE CHARGE AND SYNCHROTRON MOTION, IN THEORY, SIMULATIONS, AND EXPERIMENTS

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Abstract

With the development of high intensity accelerators, the role of space charge effects in a nonlinear lattice has gained special attention, as in the FAIR project at GSI, where long term storage of high intensity beams is required. The simultaneous presence of space charge and a nonlinear lattice creates an unprecedented challenge for ring designers as well as a new area of studies in beam physics. We present our understanding of the effect of space charge and chromaticity on the nonlinear beam dynamics of a bunched beam. We apply our findings also to an earlier CERN-PS experiment.

INTRODUCTION

Beam loss produced during long term storage in synchrotrons has become very important for new projects. In the FAIR project [1, 2] the beam loss in a bunched beam in the SIS100 synchrotron is assumed not to exceed significantly 1% during 1 second storage. In fact the large ionization cross section of the stored ion U^{+28} with residual gas atoms makes the vacuum quality very sensitive to beam loss. A too large beam loss triggers a progressive vacuum degradation which may reduce considerably the beam lifetime. The standard value of 1 W/m beam loss and protection of cold superconducting part of magnets imposes also beam loss at the % level. In the SIS100 [1, 2], bunched beams with $\Delta Q_x = -0.3$ are stored for 1 second, and the role of high intensity in a nonlinear lattice has become a critical subject of study. In order to describe the basic beam degradation mechanism deriving from the space charge in bunches, we consider a high intensity bunch stored in a ring having only one vertical lattice resonance. This assumption is done for the sake of simplicity without losing generality. First we discuss the resonance trapping in absence of the chromaticity. The main features of this dynamical system are:

- The space charge couples transverse and longitudinal planes: the instantaneous transverse Coulomb force depends on where in the longitudinal plane a particle is located;
- The self consistent effects in the absence of synchrotron motion do not cause emittance blow-up for non KV-distribution [3]: coherent resonances are not excited by transverse Gaussian distributions;
- The longitudinal motion induces, via space charge, a slow variation of transverse tunes. This condition is common in synchrotrons, for instance in the SIS100, $Q_{x0} \sim 20$ while the longitudinal tune can be of the order of $Q_{z0} = 10^{-3}$ so that the $Q_{z0}/Q_{x0} \sim 10^{-5}$; for the LHC we find $Q_{z0}/Q_{x0} \sim 10^{-4}$;
- The presence of a relatively small tune shift ($\Delta Q_x/Q_{x0} \sim 1.5\%$ for SIS100), does not destroy the standard transverse nonlinear dynamics, but rather induces a slow modulation of transverse tunes according to the synchrotron frequency;
- The transverse-longitudinal space charge coupling influences, via the depression of tunes, the transverse position where the resonance condition is met.

Resonances in phase space

The main consequence is that the position of instantaneous transverse islands (resonances) in phase space is depending on the longitudinal position of particles within the bunch. According to the position of the bare tune Q_{x0} with respect to the resonance $Q_{x,res}$ and the maximum tunes shift ΔQ_x, we can distinguish the following cases:

1. $Q_{x0} - Q_{x,res} > |\Delta Q_x|$. With this condition, the tune-spread never intercepts the resonance and no island can appear in the transverse phase space;
2. $Q_{x0} - Q_{x,res} = |\Delta Q_x|$. Here the particles with maximum tune depression touch the resonance, but no islands are formed as the space charge detuning prevents the resonance condition to be met;
3. $|\Delta Q_x| > Q_{x0} - Q_{x,res} > 0$. When this condition occurs, particles with maximum tune depression (at $z = x = y = 0$) fall below the resonance (bare tune above it), whereas particles at large transverse amplitudes are always above the resonance: the islands are formed at an intermediate amplitude, which depends on the particle longitudinal position. The outer position of the fixed points occurs at $z = 0$, whereas the smaller is at $x = 0$, and it happens only in two symmetric longitudinal bunch positions. Note that the outer position of the fixed points depends on $Q_{x0} - Q_{x,res}$: for $Q_{x0} \to Q_{x,res}$ the maximum position of the fixed points is (virtually) infinite;
4. $Q_{x0} - Q_{x,res} \leq 0$. In this case the tune-spread cannot cross the resonance. When $Q_{x0} = Q_{x,res}$ only particles at very large amplitude can be resonant.

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When synchrotron motion is present, a periodic crossing of the resonance takes place. All particles, which periodically cross the resonance, will slowly diffuse out to form a halo. Its density and extension depend on the number of particles that cross the resonance, and on the outer position of the islands (see [4]). If the outer position of islands intercepts the beam pipe or reaches the dynamic aperture beam loss occurs according to a rate which is function of the distance from the resonance. This process is related to halo formation in mismatched linac beams, where a second order resonance appears [5].

ADIABATICITY

In normal condition, without slow change of parameters (tune modulation), if a particle is inside an island, it always remains there. However, when parameters change, islands migrate through the particle orbit, and a process known as crossing of the separatrix takes place [6, 7, 8]. When the islands move and the particle is initially inside of them, then the particle may follow it or not: if the motion of the island is slow enough with respect to the frequency of revolution Q_{xf} around the fixed points, then we expect that the particle remains trapped. This situation can be formulated in terms of an adiabaticity parameter

$$T \equiv \frac{\partial x_f(n)}{\partial n} \frac{1}{Q_{xf}(n)\Delta x(n)}, \qquad (1)$$

where $x_f(n)$ is the transverse position of the fixed point and $\Delta x(n)$ is the island size. All these quantities depend on the number of turns n and on the longitudinal dynamics, i.e. on the type of longitudinal motion (1 RF, 2 RF, or a barrier bucket system, see [9]). The term $(\partial x_f(n)/\partial n)/Q_{xf}(n)$ in Eq. 1 represents the transverse shift of the fixed point x_f due to the combined effect of space charge and synchrotron motion during the time needed for one revolution around the fixed points. When this shift is small compared with the island size ($T < 1$) the particle will follow the island motion (trapping). If the shift is too large ($T > 1$), the particle will not be able to follow the motion of the fixed points (scattering).

TRAPPING AND SCATTERING REGIME

If the condition 3) is satisfied, particles in the bunch may undergo a periodic crossing of a lattice induced resonance and have a finite probability of being trapped into transverse islands. When trapping does not occur, the particle orbit is subjected to a jump (scattering of the invariant). We show these effects for the SIS18 synchrotron. We take $Q_{x0} = 4.35$, $Q_{y0} = 3.2$, and add a sextupole to a linear constant focusing lattice to excite the 3rd order resonance $3Q_x = 13$. The space charge is chosen such as to create a maximum tune-spread of $\Delta Q_x = -0.1$. In Fig. 1a is shown an example of how the adiabaticity parameter T varies along the bunch for a longitudinal tune of $Q_{z0} = 10^{-3}$. Note that for $|z|/\sigma_z > 0.7$ the fixed points are in a non-adiabatic regime as $T > 1$. The consequence of this regime is shown in Fig. 1b: when the islands crosses the particle orbit, the particle is not able to follow the fixed point and the invariant (single particle emittance) is subjected to a small jump (scattering of the invariant). The test particle used here has initial coordinates $x = 1.5\sigma_x, p_x = y = p_y = p_z = 0$, and $z = 3\sigma_z$.

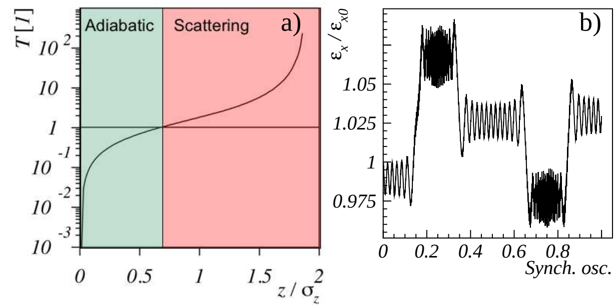

Figure 1: Scattering regime: a) adiabaticity parameter, and b) scattering of the single particle invariant.

In Fig. 2a is shown an example of how the adiabaticity parameter T varies along the bunch for $Q_{z0} = 5 \times 10^{-5}$. Here for $|z|/\sigma_z < 1.7$ islands cross the particle orbit very slowly and full trapping may occur (Fig. 2a).

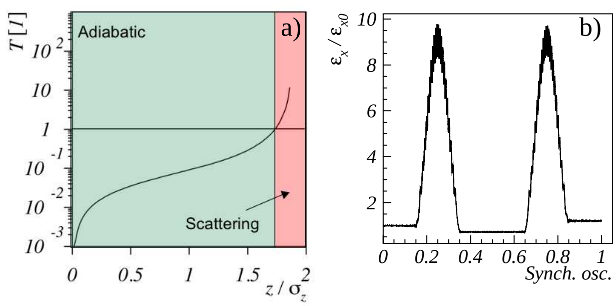

Figure 2: a) Adiabaticity parameter T and b) evolution of the single particle invariant in an adiabatic regime for the same test particle of Fig. 1.

ESTIMATE OF ASYMPTOTIC TRAPPED PARTICLES

We attempt here to estimate the fraction of particles which cross the resonance for a 6D matched Gaussian distribution [4, 10]. For a particle with small transverse amplitude the depressed tune Q_x is

$$Q_x = Q_{x0} - \Delta Q_x \exp\left[-0.5\left(\frac{z}{\sigma_z}\right)^2\right], \qquad (2)$$

where z is the longitudinal amplitude of the particle and σ_z the longitudinal rms size. If Q_{x0} satisfies the condition 3) (see Introduction) we can define a transition longitudinal emittance ϵ_{z_t} such that $Q_{x,res} = Q_{x0}$, see Fig. 3. The red line represents the longitudinal orbit which crosses z_t, the longitudinal position where the islands merge to the longitudinal axis. A particle with small transverse amplitude will cross the resonance 4 times per synchrotron oscillation, if $\epsilon_z > \epsilon_{z_t}$ (the external shell in Fig. 3), and never in case $\epsilon_z < \epsilon_{z_t}$. In addition, particles inside ϵ_{z_t} may also cross the resonance if their transverse amplitude is large enough to intercept the smaller separatrix. Only particles with transverse amplitude larger then the outer island separatrix cannot be trapped. This fraction of particles is, however, small for Q_{x0} close the resonance.

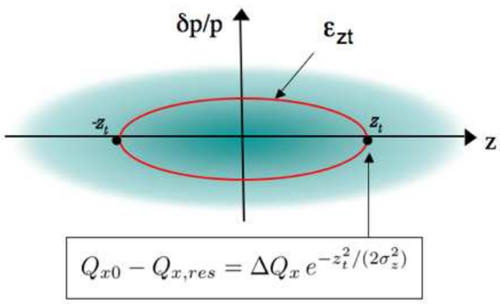

Figure 3: Schematic of the transition emittance ϵ_{z_t} and relation between z_t and ϵ_{z_t}.

The total number of particles crossing the resonance can be written as

$$\frac{\Delta N_t}{N} = \alpha \frac{Q_{x0} - Q_{x,res}}{|\Delta Q_x|}. \tag{3}$$

α depends on the topology of the islands, and its lower limit is obtained by a direct integration in $(z, \delta p/p)$ over the distribution for particles satisfying $\epsilon_z > \epsilon_{z_t}$: we find $\alpha > 1$. Note that Eq. 3 is valid for $Q_{x,res} < Q_{x0} < Q_{x,res} + |\Delta Q_x|/\alpha$. As we do not know the exact value of α, we benchmarked Eq. 3 for $\alpha = 1$ by using the SIS18 including now in the lattice a scraper placed at $3\sigma_x$ of the beam. We also used $Q_{z0} = 10^{-3}$ and $\Delta Q_x = -0.1$. The beam loss is counted after 2.5×10^5 storage turns. In Fig. 4a we show the results of beam loss when the scraper is used and without it.

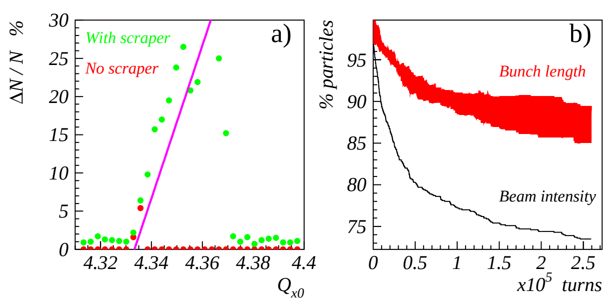

Figure 4: a) Beam loss induced by trapping/scattering of particles (green markers), in red markers beam loss by shrinking of the DA (no scraper); b) Bunch intensity/length evolution vs. storage time for $Q_{x0} = 4.3525$.

When the scraper is removed (red markers), a tiny beam loss occurs because of the shrinking of the dynamic aperture. Contrarily, when the scraper is activated large beam loss is found (green markers): the pink line is given by Eq. 3 with $\alpha = 1$. As expected, Eq. 3 slightly underestimates the beam loss; a fit of Eq. 3 with the simulated beam loss requires $\alpha = 1.5$ as suggested in [11]. Note that the loss in the region $4.352 < Q_{x0} < 4.37$ is below the analytic estimate as the beam loss has not reached saturation. This is due to the small area of phase space, which intercepts the scraper: in fact, slightly below $Q_{x0} = 4.37$, the beam halo barely touches the scraper and therefore the probability that a particle reaches that tiny area in phase space is very small. Consequently the characteristic time for losing all particles, which periodically cross the resonance, is drastically enhanced. Another important consequence of this beam loss mechanism is that the loss of halo particles is accompanied by a bunch shortening (see [10]) as lost particles are characterized by large ϵ_z. This is seen in Fig. 4b where the beam loss (black curve) and bunch length (red curve) are plotted for $Q_{x0} = 4.3525$. The red curve oscillates because of the small number of macro-particles in the bunch (1000). The correlation beam loss vs. bunch shortening is evident.

ESTIMATE OF THE ASYMPTOTIC RMS EMITTANCE

When beam loss does not occur, particle trapping causes an emittance growth [4, 10]. For design and machine operation it is important to develop a fast estimate of the asymptotic emittance growth factor $\tilde{\epsilon}_x/\tilde{\epsilon}_{x0}$, where $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_{x0}$ are the rms asymptotic, respectively initial emittances. As usual the rms emittance of the matched beam at the beginning of the storage is given by $\tilde{\epsilon}_{x0} = \beta_x^{-1}\langle x^2\rangle$. We can distinguish between the $N - \Delta N_t$ particles, which never cross the resonance (labeled here as x_n), and the ΔN_t that will periodically cross it (labeled here as x_c). Due to the trapping/scattering phenomena the particles x_c will diffuse occupying all the phase space spanned by the islands. Therefore the asymptotic rms emittance can be estimated as $\tilde{\epsilon}_x = (N - \Delta N_t)/N\beta_x^{-1}\langle x_n^2\rangle + \Delta N_t/N\beta_x^{-1}\langle x_c^2\rangle$. Note that for $(Q_{x0} - Q_{x,res}) < \Delta Q_x$ the particles x_n do not have a transverse distribution sensibly different from the initial one, therefore it is reasonable to take $\beta_x^{-1}\langle x_n^2\rangle \simeq \tilde{\epsilon}_{x0}$. By assuming that the trapped particles spread uniformly over the area bounded by outer separatrix, of maximum single particle emittance $\epsilon_{x,max}$, we find that $\beta_x^{-1}\langle x_c^2\rangle \simeq \epsilon_{x,max}/4$. The final estimate reads

$$\frac{\tilde{\epsilon}_x}{\tilde{\epsilon}_{x0}} \simeq 1 - \frac{\Delta N_t}{N} + \frac{\Delta N_t}{N}\frac{\epsilon_{x,max}}{4\,\tilde{\epsilon}_{x0}}, \tag{4}$$

where the number $\Delta N_t/N$ is estimated by Eq. 3. Note that Eq. 4 is valid only when $\epsilon_{x,max}$ is larger then the edge emittance of the beam (i.e. $\sim 9\,\tilde{\epsilon}_{x0}$ for a Gaussian beam). We have benchmarked Eq. 4 for $\alpha = 1$ by computing the rms emittance in the SIS18 after 1000 synchrotron oscillation ($Q_{z0} = 10^{-3}$) for the same working points as in Fig. 4a. The results of these simulations with 2000 macro-particles are shown in Fig. 5 (red curve). In black we plot the result from Eq. 4. Note that as expected, Eq. 4 yields wrongly $\tilde{\epsilon}_x/\tilde{\epsilon}_{x0} < 1$ for $Q_{x0} > 4.39$ because $\epsilon_{x,max} < 4\epsilon_{x0}$.

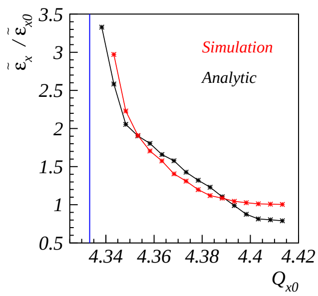

Figure 5: Comparison of the asymptotic rms emittance between simulation (red curve) and analytic estimate (black curve).

SCALING LAW FOR 3RD ORDER RESONANCE TRAPPING

We start here a first discussion on scaling laws for space charge induced trapping effects. The first step is to perform a scaling in order to keep the topology of the phase space orbits at $z = 0$ invariant. We consider the scaling

$$\left\{ \begin{array}{l} K_2 \\ \Delta Q_x \\ Q_{x0} - Q_{x,res} \end{array} \right. \rightarrow \Sigma \times \left\{ \begin{array}{l} K_2 \\ \Delta Q_x \\ Q_{x0} - Q_{x,res} \end{array} \right. \quad (5)$$

This transformation keeps unchanged the relative position of the resonance in the tune-spread. As the contribution of space charge is changed by a factor Σ, then in order to keep the size of the island unchanged we scale the sextupole strength of the same factor. In Fig. 6a is shown the phase space topology for the standard parameters used in this paper: $Q_{x0} = 4.35, \Delta Q_x = -0.1$. In Fig. 6b are shown: in red the orbits obtained with the scaling Eq. 5 for $\Sigma = 1/4$, that is for $Q_{x0} = 4.3375, \Delta Q_x = 0.025$.

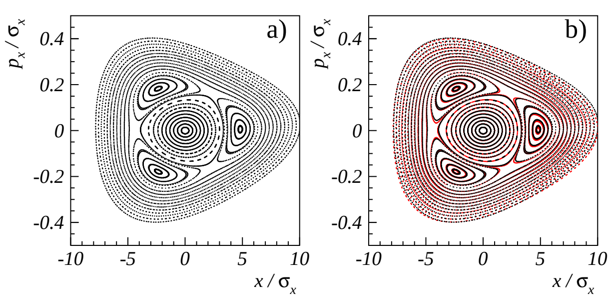

Figure 6: a) Phase space for $\Sigma = 1$ (for standard parameters). In b) phase space for $\Sigma = 1/4$ (red orbits) overlapped with orbits in a) for better comparison.

The orbits practically coincide. An important consequence of the scaling in Eq. 5 results to the adiabaticity parameter T: as K_2 and ΔQ_x are weaker after the scaling is applied, the tune of fixed points Q_{xf} becomes smaller, which increases consequently T (see Eq. 1). This effect is shown in Fig. 7a in the lower curves ($\Sigma = 1/4$ green, $\Sigma = 1/8$ blue): as T increases, the trapping efficiency decreases and the two curves exhibit a small emittance increase.

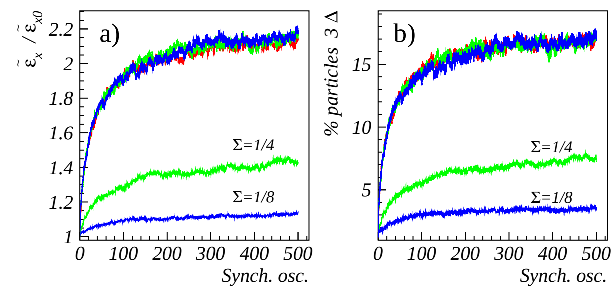

Figure 7: a) Emittance growth for $\Sigma = 1/4$ (green), $\Sigma = 1/8$ (blue), with $Q_{z0} = 10^{-3}$ and for $Q_{z0} \rightarrow Q_{z0}/\Sigma$ (all overlapping). b) % of halo particles. The meaning of the curves is as in a).

In Fig. 7b is plotted the percentage of halo particles (particles beyond $3\sigma_x$ of the beam) showing the same feature as for Fig. 7a. In order to restore the same trapping efficiency, the adiabaticity parameter T should return to its original value after the scaling Eq. 5 is applied. This is readily obtained by scaling the synchrotron tune according to

$$Q_{z0} \rightarrow Q_{z0}/\Sigma \quad (6)$$

so that the term $\partial x_f(n)/\partial n$ in Eq. 1 compensates the reduction of Q_{xf}. By applying this scaling in Q_{z0} jointly to Eq. 5 all the lower curves in Figs. 7a,b rise to overlap with the correspondent original curve obtained for $\Sigma = 1$ (red curves) confirming the interpretation of the scaling. Note that in Figs. 7a,b the evolution of emittance and % of halo particles is plotted as function of the number of synchrotron oscillations. It should be added that for $\Sigma > 1$ the scaling law applies until the shrinking of the DA is too pronounced and the border of stability is too close to the island separatrix.

EFFECT OF THE CHROMATICITY

Including chromaticity complicates the particle dynamics. The key feature of the space charge driven tune modulation stems from the symmetry of the longitudinal distribution: the tune modulation has a periodicity, which is half of the synchrotron one. The tune modulation introduced by the chromaticity, instead, has the same periodicity as the synchrotron motion. When space charge maximum detuning and maximum chromaticity detuning are comparable, the resulting slow modulation of the transverse tunes is the composition of these two effects, which have different frequencies. In Fig. 8a we show the single particle invariant in one synchrotron oscillation as for Fig. 2b, but now including the effect of the chromaticity. For a particle with $\delta p/p = 2.3 \times 10^{-3}$ the natural chromaticity yields a maximum detuning of $\delta Q_c = 0.01$. In Fig. 8b we plot the islands at $z = 0$ for $\delta Q_c = 0.01$, where loss of momentum takes place (red), and for $\delta Q_c = -0.01$, where gain of momentum occurs (blue). The asymmetry of the position of the fixed points with respect to half a synchrotron oscillation is evident. The overall effect is that islands are pushed further out and increase the halo size. For comparison, in

Fig. 8b we plot also the islands in absence of chromaticity (black curve).

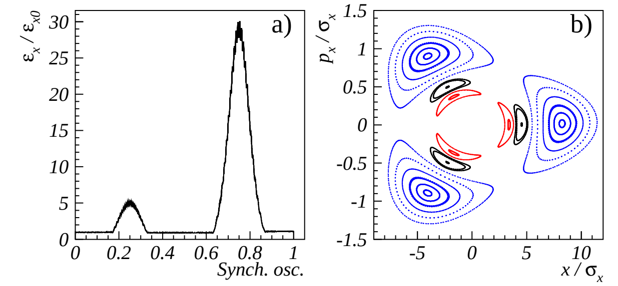

Figure 8: a) Asymmetry of the invariant in one synchrotron oscillation; b) Transverse islands for $\delta Q_c = +0.01$ (red), $\delta Q_c = -0.01$ (blue), corresponding to the loss/increase of particle momentum.

Space charge - chromaticity induced beam loss

An intuitive approach to qualitatively understand the behavior of beams in presence of the chromaticity is to consider the chromaticity induced tune shift. If $Q_{x0} + \delta Q_c$ gets close to the resonance from above, the trapped particles will be (virtually) brought to infinity. If ΔQ_c is the tune-spread induced by the chromaticity, setting the bare tune in the region $Q_{x,res} < Q_{x0} < Q_{x,res} + \Delta Q_c$ will always allow some particles in the bunch to hit the pipe in our model of SIS18. A more detailed description of this beam loss is found in [13]. In Fig. 9a we plot beam loss when chromaticity is included and the beam pipe is shifted to $100\sigma_x$. As the beam used has a chromaticity induced tune-spread of $\Delta Q_c = 0.01$, we see a beam loss regime for $4.333 < Q_{x0} < 4.343$. Note also a beam loss region for $4.32 < Q_{x0} < 4.333$.

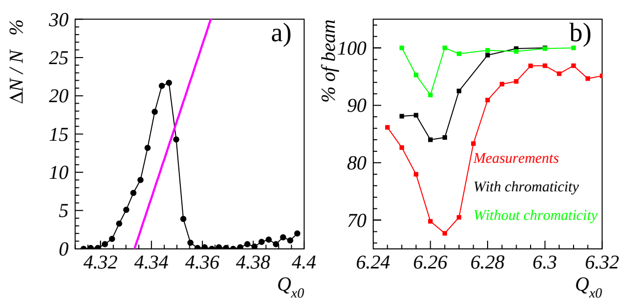

Figure 9: a) beam loss induced by chromaticity; b) New beam loss prediction for the PS-experiment: the black curve gives the simulated beam loss in presence of chromaticity.

EXPERIMENTAL FINDINGS

In the CERN-PS experiment a bunched beam with $\Delta Q_x = 0.075$ was stored for 5×10^5 turns and $Q_{y0} = 6.12$. The experimental finding is that the beam undergoes an emittance growth regime for $6.28 < Q_{x0} < 6.32$ with a maximum emittance growth of 42% at $Q_{x0} = 6.265$. For tunes $6.25 < Q_{x0} < 6.28$ a beam loss regime was found with a maximum beam loss of 32% at $Q_{x0} = 6.265$. The results and simulations on this experiment are documented in Ref. [4, 10, 12]. The maximum beam loss obtained in the simulation is 8% at $Q_{x0} = 6.26$, ignoring chromaticity. However, considering the effect of the natural chromaticity we find that an rms momentum spread of $\Delta p/p = 1.5 \times 10^{-3}$ induces the maximum tune spread of $\Delta Q_c = 0.028$, which corresponds to the width of the observed beam loss regime. We have then repeated the simulation made in [10] including the chromaticity (Fig. 9b). In red we show the measured beam loss, in green the beam loss computed in [10], where the effect of chromaticity was absent, and in black the simulation where the natural chromaticity of the PS synchrotron is included. The beam loss increases up to 16%, which is about 50% of the total measured beam loss.

OUTLOOK

Trapping phenomena are an important subject in high intensity machines as well as in rings with electron clouds [14]. We presented here the status of the present understanding: simple formulae for asymptotic beam loss and rms emittance growth have been found. Scaling laws for trapping induced rms emittance growth are possible and will be studied in details in the near future. The chromaticity also plays an important role: the CERN-PS experiment modeling has been considerably improved by including the chromaticity bringing the beam loss prediction to 50% of that found in the experiment. The remaining discrepancy will be the subject of future studies, which should include fully self-consistent simulations.

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