CODE BENCHMARKING ON SPACE CHARGE INDUCED PARTICLE TRAPPING

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Abstract

Trapping of particles in a high intensity bunch has been studied by using the MICROMAP library. The numerical studies were used to interpret the CERN-PS experiments and explore the underlying beam loss/emittance growth mechanisms. We present here the first attempt of code benchmarking in modeling the long term storage of a high intensity bunch. The code benchmarking is initiated between MICROMAP and SIMPSONS.

INTRODUCTION

The single particle dynamics in a high intensity bunch stored for long term is challenging especially when the chromaticity is taken into account. The interest in this special operating regime comes from the new generation of high intensity synchrotrons such as the SIS100 for the FAIR project [1]. Several studies of this regime [2, 3] lead to the interpretation that space charge may induce particle trapping into lattice induced resonances via synchrotron motion. The latest results [4, 5] have shown that the combined contribution of space charge and chromaticity enhances the beam loss prediction; for the CERN-PS experiment the prediction of beam loss reaches 16% versus the 32% observed experimentally. Until now all numerical predictions have been made using the MICROMAP library [6], but so far no other code with a frozen space charge model has been applied to particle trapping phenomena. It is therefore necessary to confirm the proposed mechanism by benchmarking different codes on this particular high intensity operating regime.

We present here a comparison between results obtained with MICROMAP and SIMPSONS [7]. The benchmarking is made for the SIS18 synchrotron of GSI. An intensity upgrade for the SIS18 is foreseen which aims at the delivery of 7.5×10^{10} U^{28+} in bunches with emittances of $\epsilon_{x,2\sigma} = 34, \epsilon_{y,2\sigma} = 14$ mm mrad with $\Delta Q_x \approx -0.3$. As SIS18 has several significant nonlinear resonances [8], the understanding of beam degradation is essential for the upgrade the operation. The tolerable beam loss should not exceed 1-5% in order to avoid a progressive vacuum degradation. For these reasons an approved experimental campaign, named S317 and consisting of 24 shifts, will start in the near future at the SIS18 for exploring the effect of space charge on beam loss and emittance growth under well-controlled conditions. Consequently we make the code benchmarking for the SIS18 with realistic parameters for the S317 experiment. The SIS18 lattice is taken with the standard triplet configuration typically used at injection energy. The lattice nonlinearities are created by a sextupole magnets in order to excite 3rd order resonances.

THE BENCHMARKING

In Table 1 we report the parameters used in the benchmarking unless otherwise specified. The bunch consists of a 6D matched Gaussian distribution. The space charge is modeled in both codes by an analytic force which is locally matched with the lattice for the Gaussian ellipsoid with rms properties following the exact local beta functions.

Table 1: Settings for the benchmarking

Parameter	value	units
Sextupole strength K_2	0.2	m^{-2}
Maximum tuneshift ΔQ_x	0.1	
Horiz. size X_{rms}	5	mm
Vert. size Y_{rms}	5	mm
Longitudinal size Z_{rms}	40.35	m
Horiz. emittance (2σ) ϵ_x	12.57	mm mrad
Vert. emittance (2σ) ϵ_y	9.30	mm mrad
Turns for 1 synch. osc. N_{synch}	15000	
Bunch length $(4\sigma_z)$ τ	3472.7	ns
Kinetic energy E_k	11.4	MeV/u
Gamma transition γ_t	5	
$\Delta p/p$ at $3\sigma_z$	2.5×10^{-4}	

Step 1. Transverse phase space. The first step of the benchmarking has the purpose of assuring that the transverse Poincare' sections are identical in the two codes. Nonlinearities are excited using the sextupole strength quoted in Table 1. The space charge is absent for the time being. In order to control the phase space topology we take a working point close to the 3rd order resonance at $Q_{x0} = 4.338, Q_{y0} = 3.2$. In Fig. 1 we show the result of the comparison. The red curve from SIMPSONS is located at the edge of the stability domain: all curves further out are unstable.

Step 2. Transverse tune vs. transverse amplitude without sextupole. In this step we control if the modeling of the frozen space charge has the same impact on the single particle dynamics if the sextupole is deactivated. To this end we compute the nonlinear tune in both transverse planes as a function of the transverse amplitude at $z = 0$. The transverse bare tunes remain as in Step 1 and we take

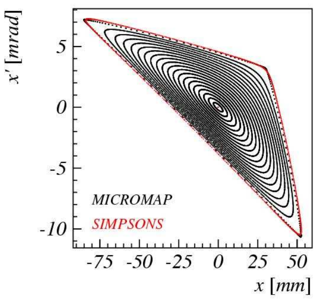

Figure 1: Benchmarking of the phase space without space charge.

infinitesimal longitudinal oscillations. The tunes are computed with an FFT method in 1024 turns. The results are shown in Fig. 2.

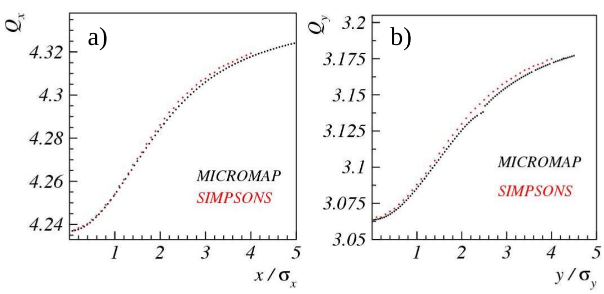

Figure 2: Comparison of a) transverse horizontal and b) vertical tunes.

Step 3. Transverse tune vs. transverse amplitude with sextupole. When the sextupole is activated, transverse islands are created at a position controlled by the space charge tune spread ΔQ_x, by the distance of the bare tunes from the resonance, and by the resonance strength. A preliminary test showed that the working point for the steps 1-2 creates islands so far in the phase space to exceed the domain ($\sim 8\sigma_x$) in which the space charge frozen algorithms are applicable. For this reason we move the tunes to $Q_{x0} = 4.3504, Q_{y0} = 3.2$. The dependence of tunes vs. transverse amplitude is shown in Fig. 3. We find an excellent agreement between SIMPSONS and MICROMAP. This test confirms that in both codes frozen space charge produces the same detuning and the islands are located at the same amplitude (flat between 3.5 and 5.5 σ_x in Q_x).

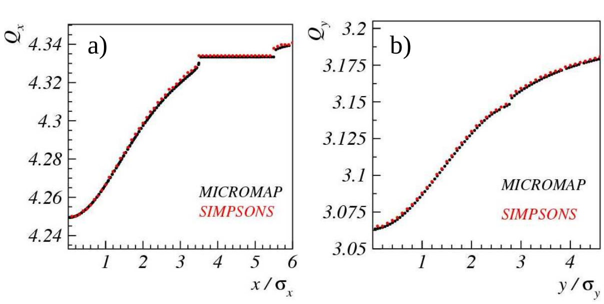

Figure 3: Transverse tunes when the 3rd order resonance is excited.

Step 4. Phase space with space charge. We compare here the phase space topology in the bunch center at $z = 0$ when sextupole and space charge are present. The results are shown in Fig. 4.

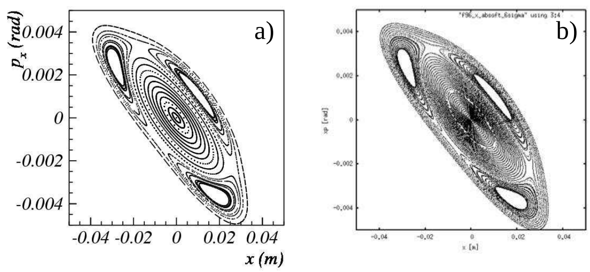

Figure 4: Comparison of phase space when the 3rd order resonance is excited [a) MICROMAP, b) SIMPSONS].

Step 5. Particle trapping. This step benchmarks the full trapping of one test particle during one synchrotron oscillation. The trapping regime is obtained taking a synchrotron tune of $Q_{z0} = 6.6 \times 10^{-5}$. The parameters of the simulation are those used in the steps 3-4. We take a test particle with coordinates: $x = 5$ mm, $x' = y = y' = z' = 0$, and $z = 2.5\sigma_z$ and compute the single particle invariant. Fig. 5 shows the full trapping. In SIMPSONS, the particle leaves the bucket after the first half synchrotron oscillation. This discrepancy might be due to slight differences in the way the optical elements are represented in the two codes.

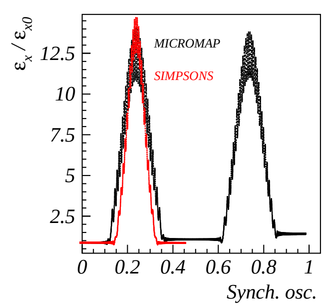

Figure 5: Comparison of single particle invariant in trapping regime.

Step 6. Scattering regime. In this step we compare the effect of the crossing of the 3rd order resonance in 1 synchrotron oscillation for $Q_{z0} = 10^{-3}$. Note that the bunch length is now reduced by a factor of 15 in order to keep the momentum spread as for the steps 1-5. The maxi-

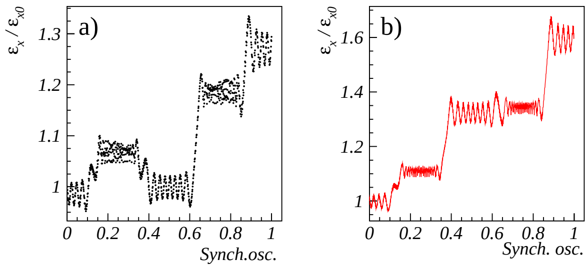

Figure 6: Behavior of the single particle invariant in 1 synchrotron oscillation [a) MICROMAP, b) SIMPSONS].

mum nominal tune shift in Table 1 is kept also by reducing the number of particles by the same factor. The results are shown in Fig. 6. Note the scatter in the invariant is not equal in both codes as the dynamics is now extremely sensitive to the initial conditions.

Step 7. Long term behavior. We compare here the effect of the multiple resonance crossing. The tracking is performed for 200 synchrotron oscillations while all the simulation parameters are as in step 6. The results are shown in Fig. 7. Note the trapping which occurs in a different sequence in a) than in b) due to quasi random process. The maximum value of the invariants do not exceed the outer position of the islands, almost equal in both codes. Note that the results of step 5, $\epsilon_x/\epsilon_{x0} \simeq 12.5$ do not contradict the actual $\epsilon_{x,max}/\epsilon_{x0} = 21$. In step 5 the adiabaticity allows the test particle to remain close to the fixed point, whereas here the particle explores the full area allowed by the islands up to the separatrix because of the scattering regime.

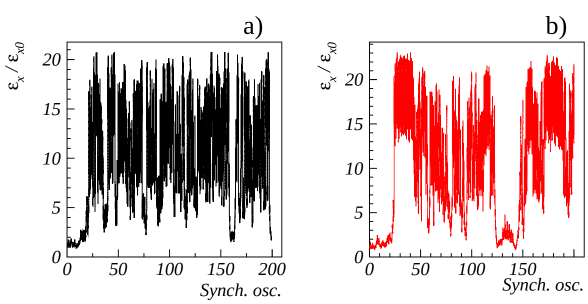

Figure 7: Single particle invariant during long term tracking [a) MICROMAP, b) SIMPSONS].

Step 8. Long term behavior of full bunch. This step benchmarks the transverse emittance evolution of the full bunch of 1000 macro-particles. The number ΔN_t of particles, which can become trapped is given by (see [4])

$$\frac{\Delta N_t}{N} \geq \frac{Q_{x0} - Q_{x,res}}{\Delta Q_x}. \quad (1)$$

We improve the statistics in the halo by changing the horizontal tune to $Q_{x0} = 4.3604$ so as to increase the halo density to $\simeq 27\%$ of the total number of particles. Fig. 8 shows the result of this benchmark. By assuming that all

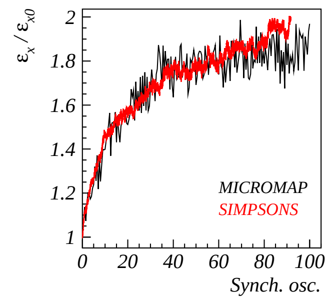

Figure 8: Emittance evolution for the full bunch.

trapped particles are uniformly distributed in the halo we can estimate the asymptotic rms emittance growth as

$$\frac{\tilde{\epsilon}_x}{\tilde{\epsilon}_{x0}} \geq 1 - \frac{\Delta N_t}{N} + \frac{\Delta N_t}{N}\frac{\epsilon_{x,max}}{4\,\tilde{\epsilon}_{x0}}, \quad (2)$$

where $\epsilon_{x,max}$ is the maximum single particle emittance [4]. Repeating step 7 for $Q_{x0} = 4.3604$ we find $\epsilon_{x,max}/\epsilon_{x0} = 10$, which in terms of the rms emittance used here yields $\epsilon_{x,max}/\tilde{\epsilon}_{x0} = 16.5$. By applying Eq. 2 we then find $\tilde{\epsilon}_x/\tilde{\epsilon}_{x0} \geq 1.84$ which is consistent with Fig. 8.

CONCLUSION

The benchmarking between MICROMAP and SIMPSONS has produced excellent agreement. The trapping and scattering regimes have been found identical for a full ensemble of particles. Obviously, we cannot expect identical orbits for single particle in a chaotic regime, but the agreement is excellent as far as ensemble averages are concerned like rms emittances and halo radii. The comparison of the emittance growth has also shown excellent agreement. A benchmarking on loss prediction and on the contribution of the chromaticity as well as on the effect of self-consistency (update of space charge force) is left for future studies.

REFERENCES

[1] P. Spiller, these proceedings.

[2] G. Franchetti, I. Hofmann, M. Giovannozzi, M. Martini, and E. Métral, *Phys. Rev. ST Accel. Beams*, **6**, 124201 (2003).

[3] G. Franchetti, I. Hofmann, Proc. 33rd ICFA workshop, AIP **773**, 122.

[4] G. Franchetti and I. Hofmann, these proceedings.

[5] G. Franchetti, I. Hofmann, Nucl. Instr. and Meth. A 561, (2006), 195.

[6] G. Franchetti, I. Hofmann, and G. Turchetti, *AIP Conference Proceedings* **448**, 233 (1998).

[7] S. Machida, AIP-297,1993.

[8] G. Franchetti *et al.* GSI Scientific report 2005, ISSN 0174-0814.