

# Considerations in short-wavelength [extreme UV and X-ray] Free-Electron Laser using Classical and Quantum Interference

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## Abstract

Ordinary Free-Electron Lasers [FELs] can be found in successful operation in the spectral range from millimeters to ultraviolet wavelengths. However the operation of the common FELs in the extreme ultraviolet and X-ray wavelength regimes faces certain adverse effects. Some of the main obstacles in the way of the realization of X-ray FEL are electron momentum spread and angular divergence. Another point to keep in mind is that ordinary FELs work on the principle of “momentum population inversion.” By this one means that electrons with momenta larger than the resonant value contribute to the gain whereas electrons with momenta smaller than the resonant value contribute to the loss. Thus to ensure a net gain we need more electrons with momenta lying in the upper momentum domain than in the lower one i.e. a “momentum population inversion.” Keeping these points in mind Scully and co-workers have proposed using the ideas of Lasing Without Inversion [LWI] to achieve the successful operation of short-wavelength [extreme UV and X-ray] FELs. The purpose of this work is to, as a first step, critically analyze the theoretical and the practical feasibility of the proposals by Scully and co-workers.

## 1 INTRODUCTION

The first proposal of FEL by Madey [1, 2, 3, 4, 5] was made on the basis of Quantum Electrodynamics [QED]. It was later pointed out by Scully and co-workers [6] that for FELs in the visible regime it is sufficient to use classical theory to explain their operation. Quantum approach is necessary in the short-wavelength regimes [ultraviolet [uv] and X-rays] since then the effects of photon recoil and electron position-momentum uncertainty must be taken into account.

In the quest for extreme uv and X-rays lasers several proposals exist. Some of the proposals include the stimulated Cherenkov radiation by quasi-free electrons in a refracting medium and Cherenkov Transition Radiation in a periodic dielectric structure [7, 8]. The quantum theory of Cherenkov lasers along with a discussion of photon statistics is presented in [9]. Other proposals to obtain lasing by FELs in the short wavelength regimes include replacing in the FEL the magnetostatic wiggler by an intense electromagnetic wave [10, 11, 12]. In all these schemes the main obstacles in obtaining X-ray FEL are electron momentum spread and angular divergence.

In order to tackle these obstacles Scully and co-workers have made the ingenious suggestions by proposing FELs

based on the principle of classical and quantum interference [16, 17, 18]. The ideas of lasing without inversion [LWI] and electromagnetically induced transparency [EIT] in atomic [13, 14, 19] and semiconductor systems [15, 19] are based on the concept of quantum interference and coherence [13, 14, 19]. For example in atomic LWI the role of population inversion is replaced by coherence between two electronic states. It is thus tempting to apply the idea of classical and quantum interference and coherence to FELs in order to obtain an efficient laser in the short wavelength regime. FEL's based on quantum interference were proposed in [16], ones based on universal classical mechanism of free-electron lasing without inversion in [17] and lasing without inversion in Cherenkov free-electron lasers in [18].

In order to test the feasibility of the theoretical ideas proposed in [16, 17, 18] for FELs based on interference we need to critically examine and analyze the theoretical and the practical issues pertaining to the proposals by Scully and co-workers. It is also worthwhile to suggest other related schemes to obtain FELs working on the principle of interference and coherence. The purpose of this note is to raise the relevant questions which naturally arise as result of the mentioned proposals and to suggest a two-stage two-section [i.e four sections and two drift regions] Cherenkov TR FEL.

## 2 BASIC REASONS FOR THE LIMITATIONS OF ORDINARY FEL'S IN THE SHORT WAVELENGTH REGIMES AND CHERENKOV TR FEL

The principal concepts of the FEL's may be found in [1, 2, 3, 4, 5]. In this section we summarize some relevant details. We restrict ourselves to the one-dimensional approximation in this note.

FEL's are essentially devices that convert electron kinetic energies into coherent electromagnetic energy by using the interaction of electron and the photon. Unlike conventional lasers, discrete energy levels which fix the emission wavelength are not involved so that FEL's can cover a broad spectrum of wavelengths from microwaves to visible light and beyond to UV and X-rays regimes. The interconversion of electron kinetic energy and electromagnetic field energy is stated in terms of the conservation law [4] [see page 98 of [4]]

$$\frac{d}{d\tau}(2j_e < \mu > + |\epsilon|^2) = 0. \quad (1)$$

The conservation law in Eq. 1 is written in terms of dimensionless variables.  $\tau$  is the dimensionless time,  $j_e$  is dimensionless electron current density and  $|\epsilon|^2$  is the dimension-

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less electromagnetic field energy [4]. Two quantities of interest are the gain  $G$  and index of refraction  $n$ . These are given by

$$\begin{aligned}\frac{dG}{d\tau} &= \frac{2j_e}{|\epsilon|} \langle \sin(\phi + \psi_L) \rangle, \\ n - 1 &= \frac{j_e}{k_R L_W |\epsilon|} \langle \cos(\phi + \psi_L) \rangle.\end{aligned}\quad (2)$$

In the small-signal approximation the gain assumes a simple form and can be written as [4]

$$G_0 - 1 = 2j_e g(x). \quad (3)$$

The expression for the classical field small-signal gain function  $g(x)$  is given by [page 105 of Ref. [4]]

$$\begin{aligned}g(x) &= \frac{1}{x^3} \left[ 1 - \cos(x) - \frac{x}{2} \sin(x) \right], \\ &= -\frac{d}{dx} \left[ \frac{\sin(x/2)}{x} \right]^2.\end{aligned}\quad (4)$$

$g(x)$  is antisymmetric function of  $x$  and vanishes at resonance. This stands in sharp contrast to the gain function of conventional lasers which are symmetric about atomic resonance. Moreover in conventional lasers the gain curves are independent of photon recoil to a good approximation, whereas in FEL's this is not the case.

Small-signal gain is proportional to the product of [16]: (a):The small photon recoil factor  $\hbar c q / E$ .  $q$  and  $E$  are respectively the magnitude of wavevector and the energy of the photon. (b):The emission rate per photon,  $P(q) / \hbar c q$ .  $P(q)$  is the corresponding emission power. (c):The derivative of the electron momentum distribution  $f(k)$  at the mean resonant momentum  $\bar{k} = \hbar(k_e + k_a) / 2$ , viz  $df(k) / dk|_{\bar{k}}$ .  $k_e$  and  $k_a$  are the magnitudes of the electronic resonant wavevectors for emission and absorption respectively.

The essence of the argument of Scully and co-workers [16, 17, 18] is to make the gain function symmetric by destructive interference of absorption probability and constructive interference of emission probability.

### 2.1 Cherenkov TR FEL

It is well-known that in a Cherenkov laser a beam of relativistic electrons interacts with an electromagnetic wave propagating in a refractive medium. Taking the propagation of electron beam along the z-axis. The laser vector potential can be taken to be a plane wave with propagation wavevector  $\mathbf{k}_l = (k_l \sin \theta, 0, k_l \cos \theta)$  and polarization unit vector  $\hat{\mathbf{e}} = (\cos \theta, 0, -\sin \theta)$ . For a plane wave in a medium with index of refraction  $n$  the usual relation  $k_l = \omega n / c$  between the wavevector and frequency of light holds.

The Fourier expansion of the em vector potential in a medium whose refractive index varies periodically may be written as

$$\begin{aligned}\mathbf{A}_l &= \sum_{j=0}^{\infty} \mathbf{A}_{lj} \exp [i\mathbf{q} \cdot \mathbf{r}], \\ \mathbf{q} &= \mathbf{k}_l + j\mathbf{k}_w.\end{aligned}\quad (5)$$

The harmonics are determined by the inverse period of index variation wavevector  $\mathbf{k}_w$  with amplitudes  $\mathbf{A}_{lj}$  and wavevector  $\mathbf{q}$ . Denoting by the subscript  $i$  the initial state quantities we may write the initial electron energy before it enters the interaction region as  $E_i = \gamma_i m c^2 = \sqrt{p_i^2 c^2 + m^2 c^4}$ ,  $m$  is the mass of the electron,  $\gamma_i$  is the initial Lorentz factor and  $\mathbf{p}_i = \hbar \mathbf{k}_i$ . The initial momentum state may be written as  $|\mathbf{k}_i\rangle$ . Similarly after emission the electron momentum is  $\mathbf{p}_e = \hbar \mathbf{k}_e$  and after absorption  $\mathbf{p}_a = \hbar \mathbf{k}_a$ . The interaction is quasi-free, in other words stationary in time so that conservation of energy holds,  $E_{a,e} = E_i + \hbar \omega$  but it happens in a finite region in space so that there is uncertainty in the momentum, viz  $\hbar \Delta_L = \hbar / L$ . Thus to obtain approximate satisfaction of both the conservation laws, the dispersion relation between  $\omega$  and  $q$  must not be the same as it is in vacuum, which is the case in the periodic medium.

The Gain is proportional to the difference between the squares of the amplitudes of emission  $T_e$  and absorption  $T_a$ . Following [3] the amplitudes for the  $j$ th harmonic emission and absorption can be written as

$$\begin{aligned}T_e &= \langle \mathbf{k}_e | e \mathbf{A}_{lj}^* \cdot \mathbf{p}_i / m \gamma_i | \mathbf{k}_i \rangle, \\ T_a &= \langle \mathbf{k}_a | e \mathbf{A}_{lj} \cdot \mathbf{p}_i / m \gamma_i | \mathbf{k}_i \rangle.\end{aligned}\quad (6)$$

The probabilities of emission and absorption are given by

$$\begin{aligned}M_e &= |C|^2 \text{sinc}^2 [\Delta_e L / 2], \\ M_a &= |C|^2 \text{sinc}^2 [\Delta_a L / 2], \\ C &= \frac{e A_{lj} \hbar k_i \sin \theta}{m \gamma_i}, \\ \Delta_e &= k_{iz} - k_{ez} - q_{jz}, \\ \Delta_a &= k_{az} - k_{iz} - q_{jz}.\end{aligned}\quad (7)$$

We may introduce the calculation of detunings and gain systematically by decomposing the variation of energy  $\hbar \omega$  with momentum difference in terms of Taylor's series in terms of  $\hbar(k_{az} - k_{iz})$  for absorption and  $\hbar(k_{ez} - k_{iz})$  for emission. In the first-order approximation the dispersion curve is given in terms of a straight line with slope equal to the initial z-component of electron velocity  $v$ , viz

$$\Delta_e = \Delta_a = \Delta_0 = \omega / v - q_{jz}. \quad (8)$$

In this approximation, as is clear from Eq. 8 [i.e. the emission and absorption detunings coincide], there is no net gain since the absorption and emission cancel each other. Moreover we note that zero detuning corresponds to resonance, where the momentum is conserved precisely. The usual Cherenkov radiation condition, viz,  $c/v = n \cos \theta$  is obtained for the fundamental harmonic  $j = 0$  for zero detuning. It is at the second-order that one obtains a non-zero contribution to the gain. Keeping in mind that the laser light propagates at an angle  $\theta$  with respect to the z-axis, so that we must take into account of the transverse variation of momentum one obtains unequal detunings for emission and absorption upon combining the longitudinal and transverse momentum variation contributing to the second-order

curvature of the dispersion curve,

$$\begin{aligned}
\Delta_a &= \Delta_0 - \Delta_R, \\
\Delta_e &= \Delta_0 + \Delta_R, \\
\Delta_R &= \frac{\hbar\omega^2}{2m_{\parallel}v^3} + \frac{n^2 \sin^2 \theta \hbar\omega^2}{2m_{\perp}vc^2}, \\
m_{\parallel} &= m\gamma^3, \\
m_{\perp} &= m\gamma, \\
\frac{\Delta_R}{\Delta_L} &= \varepsilon \frac{2\pi^2 c^3}{v^3} \left[ 1 + \frac{n^2 \sin^2 \theta \gamma^2 v^2}{c^2} \right], \\
\hbar\Delta_L &= \hbar/L, \\
\varepsilon &= \frac{\lambda_c L}{\lambda^2 \gamma^3}, \\
\lambda_c &= \frac{\hbar}{mc} \sim 4 \times 10^{-13} m. \tag{9}
\end{aligned}$$

The unequal detunings for emission and absorption is the result of recoil of the electrons due to photons. Due to smallness of the Compton wavelength of the electron  $\lambda_c$ , the quantum regime of FEL's which corresponds to  $\varepsilon \sim 1$  is reached when the wavelength is on the order of several nanometers, i.e. for X-rays lasers. We note that  $\varepsilon$  results on taking the ratio of the shift between the centers of emission and absorption curves  $\Delta_R$  and the homogeneous width  $\Delta_L$  of emission and absorption profiles and is thus a measure of the regime of the operation of the FEL, the regime being classical if  $\varepsilon \ll 1$  and quantum if  $\varepsilon \sim 1$ .

### 3 FELS BASED ON CLASSICAL AND QUANTUM INTERFERENCE

The proposal to obtain FEL action in UV and X-ray wavelengths using the idea of quantum coherence and interference was first suggested by Scully and his co-workers in [16]. In summary it was suggested in [16] to achieve lasing in free-electron devices without the standard population inversion between the two parts of the electron momentum distribution that contribute to stimulated emission and absorption respectively. By making the absorption states destructively interfere without disturbing the stimulated emission, one obtains a gain profile which is symmetric about the emission resonance with a gain value much larger than achieved by the antisymmetric gain curve of a standard FEL with same parameters.

It was further noticed by Scully and co-workers that the real fundamental limitation of the LWI scheme of ref. [16] is that it depends on the quantum mechanical distinguishability between final momentum states for emission as opposed to the indistinguishability of their counterpart for absorption making it *highly sensitive to the angular spread of the electron beam*. To overcome the limitation due to angular spread a new proposal of a classical mechanism of absorption suppression and emission enhancement by selective interference in two sequential regions was given in ref. [17].

In ref. [18] a scheme for absorption cancellation with enhanced gain is proposed. The main point of the suggestion in [18] is based on the adjustment of the phases of

electrons and light by ‘‘appropriate’’ dispersion between the two sections. An achromatic cancellation of absorption is suggested, which means to cancel absorption for a broad distribution of electron momenta by making the interference term independent of detuning [i.e. achromatic]. The argument may be summarized as follows. A two-section Cherenkov TR FEL, which has two interaction regions separated by a drift region is proposed. In order to make the total *phase difference* between the two absorption amplitudes independent of detuning, an adjustment of the paths of the electrons and light in the drift region is suggested. The paths of the electrons and light in the drift region are adjusted according to their velocities. The probability of absorption may be written as

$$\begin{aligned}
M_a &= |C|^2 \frac{1}{2} [\text{sinc}^2(\Delta_a L/4) \\
&\quad + \text{sinc}^2(\Delta_a L/4) \cos(\Delta_a L/2 - \phi)]. \tag{10}
\end{aligned}$$

If  $\Delta_a L/2 - \phi$  is adjusted to  $\pi$ , Eq. 10 tells us that  $M_a$  will vanish. The total phase difference after the first interaction region of length  $L_l$  and drift region can be written as

$$\begin{aligned}
-\phi &= k_i(s_i + L_l) - k_a(s_a + L_l) \\
&\quad + q(s_1 + L_l) + jk_w L_l, \tag{11}
\end{aligned}$$

where  $s_i$  and  $s_a$  are respectively the paths traversed in drift region by those electrons which have not been affected by the light field and those which have absorbed light.  $s_l$  is the path length of light in the drift region. The idea is to make the phase difference equal  $\pi$  for any initial electron momentum  $k_i$  and any laser wavevector  $q$ . Thus the phase  $k_i(s_i + L_l) - k_a(s_a + L_l)$  in Eq. 11 must be made independent of  $k_i$  or approximately independent of  $k_i$  and  $q(s_1 + L_l)$  adjusted so that the overall phase  $\phi$  is an odd multiple of  $\pi$ , i.e.  $-\phi = (2N + 1)\pi$ .  $k_a$  is determined by recoil, viz  $k_a = k_i + \delta k$ , where  $\delta k \sim qc/v_i$  in the classical limit. We note that  $\delta k$  is much smaller than any other momenta and further  $k_w$  is a constant. By using Taylor series expansion the phase term  $k_i(s_i + L_l) - k_a(s_a + L_l)$ , viz,

$$k_i(s_i + L_l) - k_a(s_a + L_l) \approx -\frac{qc}{v_i} \left( s_i + L_l + \left( \frac{ds}{dk} \right)_i \right) \tag{12}$$

leads to the dispersion relation,

$$k_i \left[ 2 \left( \frac{ds}{dk} \right)_i + k_i \left( \frac{d^2s}{dk^2} \right)_i \right] \approx \frac{s_i + L_l}{\gamma^2}. \tag{13}$$

Eq. 13 can be further simplified to the form

$$k_i \left( \frac{ds}{dk} \right)_i \approx \frac{s_i + L_l}{\gamma^2}. \tag{14}$$

Eq. 14 along with the phase written as <sup>1</sup>

$$\begin{aligned}
-\phi &= jk_w L_l + q \left[ L_l + s_l - \frac{c}{v_i} \left( s_i + L_l + \left( \frac{ds}{dk} \right)_i \right) \right] \\
&= \pi(2N + 1), \tag{15}
\end{aligned}$$

<sup>1</sup>Eq. 34 of Ref. [18] seems to be misprinted

constitute the two main conditions for the interference term in the probability of absorption [namely Eq. 10] to be independent of momenta.

To achieve these delays it is suggested to use a magnetic field  $B$  to deflect the electrons by angle  $\varphi$  at the end of the first interaction region and by angle  $-\varphi$  in the middle of the drift region, in between the electron is assumed to move freely. The dispersion in the experimental situation, in this scenario, is given by

$$k\left(\frac{ds}{dk}\right) \approx 4 \times 10^5 B^2 L_m^2 \frac{L}{\gamma^2}, \quad (16)$$

where  $L_m$  is the length of magnetic field region. To satisfy Eq. 15 a medium with anomalous refraction  $n(q)$  where

$$\frac{dn}{dq} = -\frac{C_2}{q^2 L_d}, \quad (17)$$

is proposed to be inserted in the drift region. It is further noted that for short-wavelength [XUV or X-ray] photons the same effect as in a medium of anomalous refraction can be attained with reflection off a Bragg structure.

#### 4 QUESTIONS

The following questions naturally arise in the context of FEL's based on quantum interference as suggested in [16, 17, 18]:

- Can the LWI FEL's operate efficiently even with a strongly inhomogeneous, broad electron momentum distribution?
- How practically feasible is the two-section Cherenkov Transition Radiation [TR] FEL?
- Does this Cherenkov TR FEL allow a complete absorption cancellation and LWI operation even in the case of a very broad electron momentum distribution compared to the homogeneous width?
- Is the gain of LWI FEL greater than the gain of the usual FEL by a factor of 100 [i.e. by two orders of magnitude]?
- How realistic is the proposed "classical selective interference"?
- Moreover is the "classical selective interference" free of the angular spread limitation?
- How valid is the small-signal gain calculation?
- The saturation would invalidate the small-signal analysis: when precisely does the saturation set in?
- The saturation is expected to affect the phase coherence required for the interference: what is the form of dependence of phase coherence on the saturation?
- What can we say about the noise in the two-section Cherenkov TR FEL based on quantum interference?

#### 5 PRACTICAL AND THEORETICAL CONSIDERATIONS

In realistic conditions the complete vanishing of absorption probability  $M_a$  given in Eq. 10 above is not possible. Thus the actual gain profile will not be the symmetric shape given in [18] but it will have a "skewed" antisymmetric shape, with the positive gain part dominating under favorable conditions. Our initial crude calculations seem to indicate at least a 10% reduction in the ratio gain LWI/gain usual advertised in [18]. We note that the ratio gain LWI/gain usual given in [18] is

$$\frac{\text{gain LWI}}{\text{gain usual}} \sim \frac{\lambda \Delta \gamma}{\lambda_c} \sim 100, \quad (18)$$

for typical parameters  $\Delta \gamma / \gamma \sim 10^{-4}$ ,  $\gamma \sim 100$ ,  $\lambda \sim 3$  nm.

Yet another practical issue which needs to be addressed in the context of FELs based on interference is that of feedback. As a start we suggest the three-mirror Bragg reflecting cavity [11] to achieve the necessary feedback to bring the system above threshold. An additional advantage of Bragg resonators is that they determine very sensitively the operating wavelength of the system.

As mentioned above that in order to satisfy the conditions Eqs. 14 and 15 that are needed to make the interference term independent of momenta, deflection in magnetic field was suggested in [18] as a practical way to achieve the necessary delays. From practical viewpoint, in this context we need to take into account the electromagnetic edge radiation. It is known that electromagnetic edge radiation is generated by a relativistic charged particle when it passes through a region of a rapid change in magnetic field [20]. In [18] it was suggested to apply deflection in magnetic field  $B$  to rotate the velocity of electrons by angle  $\varphi$  at the end of the first interaction region and by angle  $-\varphi$  in the middle of the drift region, between these deflecting fields the electrons are assumed to move freely. It is clear that a practical consideration would be to understand the role of electromagnetic edge radiation caused due to the magnetic field configuration proposed in [18].

The free-electron lasers are three-dimensional devices. The one-dimensional treatment is an important tool. However for more complete and realistic treatments we need to consider the full three dimensional case. In the one-dimensional approximation it is assumed that the laser extends without variation to infinity in dimensions transverse to the wiggler axis. In reality however the diameters of electron and light beams are on the order of few millimeters and we must take into account the variation of properties of electron beam, optical beam and the wiggler field along the transverse direction.

We must also consider the contributions of collective effects, since when the electron density in electron beam is sufficiently high, the electron-electron interactions cannot be ignored and collective motions which result from such interactions must be taken into account. When the space-charge waves are important, the first-order effect of space-charge fields is to resist the bunching of electrons, thereby

decreasing the performance of ordinary FELs [4]. It remains to be seen, how space-charge fields effect the performance of FELs based on classical and quantum interference.

## 6 PROPOSAL FOR COMPOUND CONFIGURATIONS BASED ON THE TWO-SECTION CHERENKOV FEL

A simple extension of the two-section FEL of [18] is a two-stage two-section FEL. A three-ring Bragg resonator is proposed to provide the feedback to bring the system above threshold. For a slightly better performance as far as total reflectivity is concerned a five-mirror cavity can be used [11]. However as is clear the three-ring cavity is more easily handled experimentally. Moreover the three-ring resonator allows for a longer wavelength. We note that multilayer mirrors, operating on Bragg diffraction allow for the extension to longer wavelengths.

## 7 CONCLUSIONS

We have presented, as a first step, some relevant issues pertaining to the practical feasibility of quantum interference based FEL's. The discussion of noise in such FEL's will be discussed elsewhere.

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## 9 REFERENCES

- [1] J. M. Madey, J. Appl. Phys. 42 (1991) 1906.
- [2] Thomas. C. Marshall, FREE-ELECTRON LASERS (Macmillan Publishing Company, New York, 1985).
- [3] A. Friedman, A. Gover, G. Kurizki, S. Ruschin and A. Yariv, Rev. Mod. Phys. 60 (1988) 471.
- [4] C. A. Brau, Free-electron Lasers (Academic Press, New York, 1990).
- [5] H. P. Freund and T. M. Antonsen, Principles of Free-electron Lasers (Chapman & Hall, London, 1992).
- [6] F. A. Hopf, P. Meystre, M. O. Scully and W. H. Louisell, Phys. Rev. Lett. 37 (1976) 1342; Optics Comm. 18 (1976) 413.
- [7] A. Gover and P. Sprangle, IEEE J. Quantum Electron. QE-17 (1981) 1196.
- [8] S. Datta and A. E. Kaplan, Phys. Rev. A 31 (1985) 790.
- [9] W. Becker and J. K. McIver, Phys. Rev. A 25 (1982) 956.
- [10] L. R. Elias, Phys. Rev. Lett. 42 (1979) 977.
- [11] P. Dobiash, P. Meystre and M. O. Scully, IEEE J. Quantum Electron. QE-19 (1983) 1812.
- [12] J. Gea-Banacloche, G. T. Moore, R. R. Schlicher, M. O. Scully and H. Walther IEEE J. Quantum Electron. QE-23 (1987) 1558.
- [13] M. O. Scully et al., Phys. Rev. Lett. 62 (1989) 2813. E. Fry et al., Phys. Rev. Lett. 70 (1993) 3235. O. Kocharovskaya and Y. Khanin, JETP Lett. 48, (1988) 630.
- [14] S. E. Harris et al., Phys. Rev. Lett. 62 (1989) 1033. K. J. Boller et al., Phys. Rev. Lett. 66 (1991) 2593.
- [15] Gershon Kurizki and M. Shapiro Phys. Rev. B 39 (1989) 3435.
- [16] Gershon Kurizki, M. O. Scully and C. Keitel Phys. Rev. Lett. 70 (1993) 1433.
- [17] B. Sherman, Gershon Kurizki, D. E. Nikonov and M. O. Scully, Phys. Rev. Lett. 75 (1995) 4602.
- [18] D. E. Nikonov, B. Sherman, Gershon Kurizki and M. O. Scully, Optics Comm. 121 (1995) 101.
- [19] S. Alam, LASERS WITHOUT INVERSION and ELECTROMAGNETICALLY INDUCED TRANSPARENCY, 1997, to be published by SPIE.
- [20] Nikolay Smolyakov, Atsunari Hiraya and Hiroaki Yoshida, APAC98 conference March 1998, Tsukuba, Japan.