FAST BETATRON TUNE CONTROLLER FOR CIRCULATING BEAM IN A SYNCHROTRON

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Abstract

When rf quadrupole(RFQ) electric field is applied to the circulating beam in a synchrotron, an equation of transverse motion is reduced to *Mathieu's Equation.* A new analytical method to obtain an approximate solution has been developed, while a numerical computation is usually applied. Since the solution suggests that a fast tune control can be achieved by rapid tuning of both amplitude and frequency of rf voltage, this process could be applied to compensate a tune shift caused by a space charge effect. An RFQ fast tune controller has been installed at HIMAC synchrotron in cooperation of RCNP and NIRS, and the performance test is being carried out to control a vertical tune value at injection energy where the space charge effect is largest. The observed tune shift was consistent with that estimated by the approximate solution.

1 INTRODUCTION

For any particle accelerators, a high-intensity beam is always desirable but it is known that a high-intensity beam is largely lost at and near after injection due to a space charge effect. The feature of heavy ion beam at injection has been investigated at HIMAC synchrotron [1] constructed at the National Institute of Radiological Sciences(NIRS). From this result, the beam loss at injection occurs with traversing the resonant lines due to a tune shift caused by a space charge effect, and these processes continue in the time scales from μs to ms order. Although rapid control of betatron tune is strongly expected for efficient beam operation, the ordinary tune shift by quadrupole magnets is not applicable because of its slow response.

In the last paper [2], we have obtained a new analytical expression of an equation of motion reduced to Mathieu's Equation which was obtained by means of RFQ electric field applied to the beam circulating in a synchrotron. Translating the behavior of approximate solution into terms of an RFQ electric field and betatron oscillation, a fast tune control can be achieved by rapid tuning of both amplitude and frequency of rf voltage.

In this paper, we review a principle of fast betatron tune controller and report a result of its beam test at HIMAC synchrotron.

2 PRINCIPLE OF FAST BETATRON TUNE CONTROLLER

When RFQ electric field is applied to the beam in a synchrotron, an equation of motion is reduced to Mathieu's Equation as given by:

$$\frac{d^2y}{dz^2} + a\{1 - \frac{2q}{a}\cos(2z)\}y = 0, \tag{1}$$

where

$$\begin{cases} y = \dfrac{x}{\sqrt{\beta_0}} & , 2z = K\phi + \Phi_k, \\ a = \dfrac{4}{K^2}\nu_0^2, & ; \phi = \dfrac{1}{\nu_0}\displaystyle\int_0^s \frac{ds}{\beta_0} \\ \dfrac{2q}{a} = \dfrac{q_e}{m_0\gamma v^2}G_0\beta_0^2\dfrac{l}{L} & \end{cases}$$

;G_0 is a field gradient;l is an RFQ electrode length; L is a circumference of the ring; and ν_0 is a betatron tune which depends on beta function β_0.

Taking conjugate momentum $p = \frac{1}{\sqrt{a}}y'$, we transform y and p into r and θ in Mathieu's equation as given by:

$$\begin{cases} y = r\cos(Qz - \theta) \\ p = -r\sin(Qz - \theta), \end{cases} \tag{2}$$

where Q is a resultant frequency of oscillation.

Neglecting rapidly oscillating components which are averaged to zero, an approximate solution which gives a relation between parameters q,a and Q in stable region is obtained as given by:

$$-\frac{2q}{a} = 4\frac{\sqrt{(\sqrt{a}-1)^2 - (Q-1)^2}}{\sqrt{a}}. \tag{3}$$

An expression in unstable region is also obtained. Fig.1 shows comparison between numerical solution [3] and analytical one. In stable region, the analytical expression corresponds to the dotted lines of constant value of ν, where $Q = 2 - \nu$. In unstable region, it corresponds to the lines of $S = e^{i\nu\pi}$ =constant.

From the eq.(3), a variation of $\frac{2q}{a}$ for a fixed a corresponds to a variation of Q, where the parameter $\frac{2q}{a}$ is proportional to the amplitude of RFQ electric field: and the parameter a is related with frequency of RFQ and that of revolution as given by:

$$f_{rfq} = f_{rev}(\frac{2\nu_0}{\sqrt{a}} - m) \qquad m = 0, \pm 1, \pm 2, \cdots). \tag{4}$$

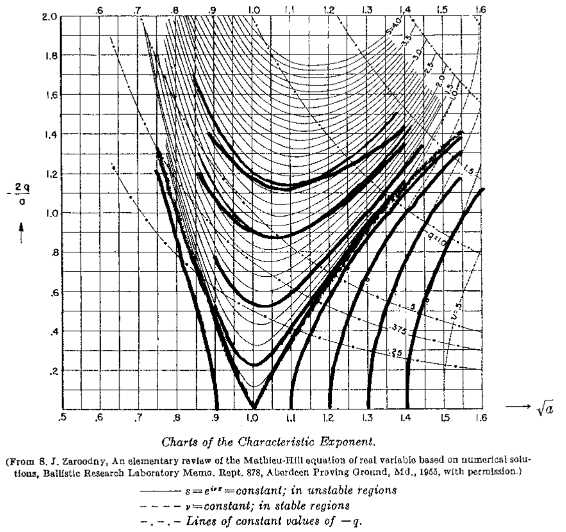

Figure 1: Comparison between numerical solution and analytical one. Bold lines correspond to the lines of constant values of ν.

The eq.(4) is derived from the relation between parameters in the eq.(1). As a result, the original tune ν_0 is shifted to the resultant tune $\frac{Q}{\sqrt{a}}\nu_0$ by tuning of both amplitude and frequency of rf voltage.

3 BEAM TEST AT HIMAC SYNCHROTRON

3.1 RFQ fast tune controller

The fast betatron tune controller was constructed in cooperation of RCNP and NIRS, and it was designed to manipulate a vertical tune shift caused by RFQ electric field because a space charge effect is larger at vertical direction rather than horizontal one. Table 1 shows an example of tune shift expected for vertical tune, where the experiment is assumed to be performed at HIMAC synchrotron($L = 130[m]$). The parameters of the fast tune controller is also shown in Table 1.

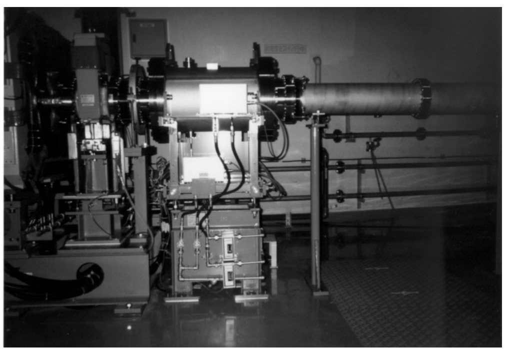

Figure 2: Fast tune controller installed at HIMAC synchrotron

Fig. 2 shows the phtograph of the fast betatron tune controller installed at HIMAC.

Table 1: An example of tune shift for vertical tune at HIMAC synchrotron

original tune	$\nu_0 = 3.15$
resultant tune	$\frac{Q}{\sqrt{a}}\nu_0 = 3.141$
value of tune shift	$\Delta\nu = -0.009$
ratio between f_{rfq} and f_{rev}	$\frac{f_{rfq}}{f_{rev}} = 2.265$
energy par nucleon	$6[MeV/u]$
β function	$\beta_0 = 1\ 54[m]$
bore radius	$65[mm]$
length of electrode	$0.6[m]$
field gradient	$426[kV/m^2]$
RF Power	$1.0[kW]$

3.2 Method of experiment

In this experiment, we utilize transverse Schottky signals to measure a betatron tune value [4]. Assuming a single particle with a constant revolution frequency f_{rev} which performs a betatron oscillation $x_k = a_k \cos(Q_x \omega t + \phi_k)$ around the ideal orbit $x = 0$, a position sensitive pick-up records a short pulse modulated in amplitude by the oscillation at each traversal. The signals from a position monitor are changed to the frequency spectrum by spectrum analyzer, which contains spectral lines at the two sideband frequencies $(n \pm Q_x)f_{rev}$ of each revolution harmonic nf_{rev}. The difference between the revolution harmonic frequency and its sideband frequencies gives a betatron tune value. Table 2 shows the experimental condition of HIMAC syn-

Table 2: Experimental condition

beam	$^4He^{2+}$
energy	6.1MeV/u
beam intensity	700μ A (4.4×10^{11}ppp)
flat-base frequency	f_b=1.0456MHz
revolution frequency	f_{rev}=0.2614MHz

chrotron carried out at an injection energy.

3.3 Experimental results and discussion

The variation of vertical tune value was measured by using a realtime spectrum analyzer at several frequencies in the range of $f_{rfq}/f_{rev} = 2.0 \sim 2.5$. The dependence on rf voltage was also measured at each frequency by varying an rf power up to 1kW.

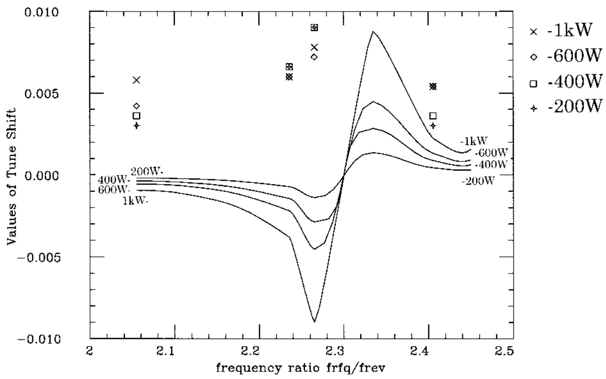

Figure 3: Comparison of tune shift between analytical expression(solid lines) and experimental raw data(plots)

Fig. 3 shows the comparison of values of tune shift between analytical expression and experimental raw data. All the experimental raw data have the same tendency that vertical tune value increases from the original tune. We have found immediately that the beam intensity varies at every measuring time depending on frequency and voltage of RFQ. When the RFQ is turned-on, the unexpected beam loss occurs at all the measurements. Thus we must correct the experimental raw data by the dependence of tune value on the beam intensity for accurate investigation of performance of RFQ. Fig. 4 shows the comparison of tune shift between analytical expression and experimental data corrected by the beam intensity-dependence of tune. The observed tune shift is comparable to that of the analytical solution, although the data at the point where the beam intensity is rather small contain some errors. The unstable region where the beam is lost in the instant that the RFQ is turned-on was also observed as expected.

4 CONCLUSION

An analytical solution of Mathieu's equation, which describes a motion of circulating beam perturbed by RFQ, was obtained. From this solution, a fast tune control is expected by rapid tuning of both amplitude and frequency of rf voltage. This process could be applied to compensate a tune shift caused by a space charge effect of a high-intensity beam. The performance in unstable region would be applied to the slow beam extraction. An RFQ fast tune

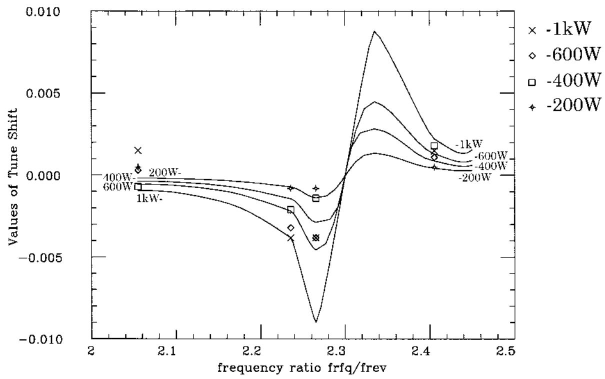

Figure 4: Comparison of tune shift between analytical expression(solid lines) and experimental data(plots) corrected by the dependence of tune value on the beam intensity

controller has been installed at HIMAC synchrotron, and the performance test is being carried out to control a vertical tune value at injection energy. Since the observation shows a tune shift which is comparable to that of the analytical solution, RFQ is applicable to suppress a tune shift caused by a space charge effect and it will contribute the advanced study of beam physics and any other sciences. A further investigation, however, is required because the unexpected beam loss still occurs in most cases when the RFQ is turned-on. In near future, authors plan to apply this system to slow beam extraction which utilizes the behavior of unstable region.

5 REFERENCES

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