A STUDY OF SMOOTHING ANALYSIS OF ALIGNMENT BASED ON BEPC

Y. Luo and C. Zhang
IHEP, P. O. Box 918-9, 100039, Beijing, P. R. China

Abstract

Smoothing analysis is an effective method to save the manpower and working period occupied by the alignment of magnets in circular accelerators. Now it has been used in a few accelerators [1,2] with different smoothing methods. This paper is aimed at the basic discussion about it and under what condition it is effective. A program named FIT is written by using Fourier series fitting to find the smooth curve and as an example, the results of Beijing Electron-Positron Collider (BEPC) horizontal alignments in 1991 and 1993 are dealt with.

1. INTRODUCTION

In order to save the manpower and the working period occupied in the process of magnet alignment, it is, especially for large accelerators, essential to do some simulations before the adjustments to decide which magnets and how many to move. The final maximum and rms closed orbits in reference to the design orbit should be accepted.

In many magnet alignments, it is often found that the distribution of all the magnets' positional errors has a smooth tendency along the design orbit, they seems to be 'aligned' to a smooth curve which is slightly displaced from the design orbit. It is mainly caused by systematic errors, such as the movements of the ground. In general, if the smooth curve is smooth enough and close enough to the design orbit, all the magnets could be aligned to the smooth curve. The method is called smoothing analysis of magnets' positional errors. It can largely reduce the number of the magnets to be adjusted and the maximum and rms closed orbits.

Different smoothing methods have been used in some accelerators. The major difference among them is what method is used to find the smooth curve. The results of different smoothing methods are slightly different, too. In this paper, a basic discussion of the smoothing analysis is presented, and Fourier series fitting is adopted to find the smooth curve. The results are read by the computer code MAD [3] to simulate the maximum and rms closed orbit.

2. DISCUSSION OF SMOOTHING ANALYSIS

Among the positional errors of magnets, the horizontal and vertical positional errors of quadrupoles and the rotation errors of dipoles are the major sources of the closed orbit distortion. For the convenience of discussion, only the positional errors of quadrupoles are considered here. If a quadrupole has a horizontal or vertical positional error, Δx or Δz, there will be an additional dipole field which will cause the closed orbit distortion. Taking the horizontal orbit as an example, the equation is written as

$$x_{co}''+k(s)x_{co}=-\frac{1}{(B\rho)}\frac{\partial B_z}{\partial x}\Delta x \qquad (1)$$

The solution of eq. (1) is

$$x_{co}(s)=\frac{\sqrt{\beta(s)}}{2\pi\sin(\pi Q)}\int_s^{s+C}\sqrt{\beta(t)}\,f(t)\cos(\phi(t)-\phi(s)-\pi Q)dt \qquad (2)$$

where $f=-\frac{1}{(B\rho)}\frac{\partial B_z}{\partial x}\Delta x$.

If the distribution of positional errors of quadrupoles has a smooth tendency, the position error then can be divided into two parts. The first part Δx_1 is the displacement of a quadrupole from the smooth curve, the second part Δx_2 is the difference between the smooth curve and the design orbit. The whole positional error is

$$\Delta x=\Delta x_1+\Delta x_2 \qquad (3)$$

As a sequence, the closed orbit x_{co} can also be divided into two parts. One part $x_{co,1}$ is the closed orbit of Δx_1 in reference to the smooth curve, the second $x_{co,2}$ is the closed orbit of Δx_2 in reference to the design orbit, that is,

$$x_{co}=x_{co,1}+x_{co,2} \qquad (4)$$

and they satisfy

$$x_{co,1}+k(s)\,x_{co,1}=f_1(s) \qquad (5)$$

$$x_{co,2}+k(s)\,x_{co,2}=f_2(s) \qquad (6)$$

It can be seen from eq. (5) that in order to get small $x_{co,1}$, all the magnets should be close enough to the smooth curve, that is, the rms displacement of the quadrupoles from the smooth curve should be small. The number of quadrupoles to be adjusted is decided by the rms displacement.

On the other hand, in order to get small $x_{co,2}$, the smooth curve should be smooth enough and close enough to the design orbit, as shown in eq. (6). After adjustments, the maximum and rms closed orbit in reference to the design orbit are mainly decided by the smooth curve.

3. SMOOTHING ANALYSIS BY USING FOURIER SERIES FITTING

In this paper, Fourier series fitting of points with equal interval is used for smoothing analysis. The method is chosen just because the given curve by using Fourier series fitting is distinctly smooth and periodic and the program is simple.

If $2N$ points are given, the intervals between any two close points are equal, such as,

$$x_p = \frac{p\pi}{N}, p = 0,1,2,\ldots\ldots,(2N-1) \qquad (7)$$

The values on x_p are

$$y_p, p = 0,1,2,\ldots\ldots,(2N-1) \qquad (8)$$

Those data can fitted by *f(x)* as

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{N-1}(a_k \cos x + b_k \sin x) + \frac{a_N}{2} \qquad (9)$$

The coefficients are given by

$$\sum_{P=0}^{2N-1} y_p = N a_0$$

$$\sum_{P=0}^{2N-1} y_p \cos(\pi p) = a_N$$

$$\sum_{p=0}^{2N-1} y_p \cos(\frac{\pi mp}{N}) = N a_m \qquad (10)$$

$$\sum_{p=0}^{2N-1} y_p \sin(\frac{\pi mp}{N}) = N b_m$$

$$1 \le m \le N-1$$

Since the intervals of any two close magnets are usually unequal, so before performing Fourier series fitting, it is essential to interpolate (2*N-1*) points equal in interval. In the process, the periodicity of accelerator ring or the positional errors of quadrupoles should be satisfied. There are 68 quadrupoles along the BEPC ring, so the number of interpolated points is selected to be 68 in order to represent fully the positional errors of magnets and not to induce other errors.

After having the coefficients of all orders, *f*(x) can be replaced by the sum of the former series *f′(x)* as

$$f'(x) = \frac{a_0}{2} + \sum_{k=1}^{K}(a_k \cos x + b_k \sin x) \qquad (11)$$

where *f′(x)* is defined as a smoothing curve, *K* is cutoff order.

Then by comparing the positional errors of all the quadrupoles with the smooth curve, which quadrupoles and how many to be moved are determined. In general, only those magnets whose displacements from the smooth curve are bigger than the tolerance should be adjusted.

4. PROGRAM AND EXAMPLES

The main program to perform the smoothing analysis is FIT. When the cutoff order is given, it will give a report on which magnets and how many to be adjusted. The code MAD can read its results directly to calculate the closed orbit.

As an example, we use these programs to deal with the horizontal positional errors of BEPC in 1991 and 1993.

Figures 1 and Fig. 2 show the smooth analyses of the measurements in 1991 and 1993 with *K*=4. It can be seen from the figures that the distribution of the positional errors in 1993 is indeed smooth along the ring. But that in 1991 is not smooth enough, and its rms displacement of the quadrupoles from the smooth curve is big.

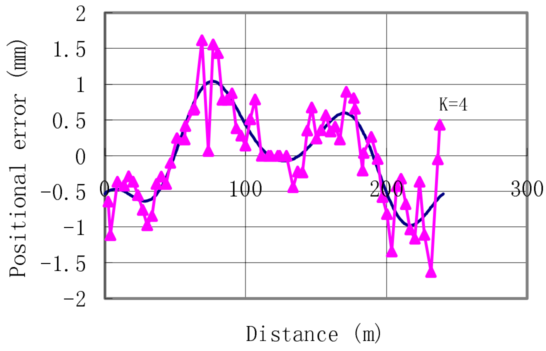

Fig. 1 The results of 1991 measurement

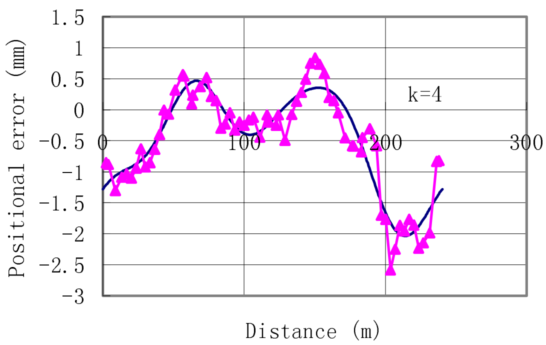

Fig. 2 The results of 1993 measurement

Table 1 and Table 2 give some information about the raw data and their closed orbits calculated by MAD code. The closed orbits are in reference to the design orbit.

Table 1 Information about the raw data in 1991

The maximum positional error(*mm*)	1.633
The rms positional error(*mm*)	0.674
The maximum closed orbit(*mm*)	16.680
The rms closed orbit (*mm*)	4.535
Number of quadrupoles be adjusted	62 (92%)

Table 2 Information about the raw data in 1993

The maximum positional error (*mm*)	2.590
The rms positional error (*mm*)	0.973
The maximum closed orbit (*mm*)	17.859
The rms closed orbit (*mm)*	6.458
Number of quadrupoles be adjusted	68(100%)

Table 3 and Table 4 give the results of analyses of different cutting orders. Here *N* represents the number of quadrupoles to be adjusted, rms_1 and rms_2 the rms displacement of the quadrupoles from the smooth curve before and after adjustments, respectively.

Table 3 Results of smoothing analysis for 1991

K	*N*	rms_1 (*mm*)	rms_2 (10^{-2} *mm*)
0	56	0.674	2.53
2	42	0.447	7.60
4	34	0.355	8.42
6	35	0.353	8.80
8	31	0.342	9.61
10	32	0.339	8.52

Table 4 Result of smoothing analysis for 1993

K	*N*	rms_1 (*mm*)	rms_2 (10^{-2} *mm*)
0	55	0.830	5.072
2	35	0.366	7.065
4	34	0.292	8.358
6	25	0.241	8.815
8	24	0.220	8.581
10	13	0.148	9.920

From Table 3 and Table 4, it can be seen that the rms_1 of 1991 changes a little with the rise of *K*, so the number of quadrupoles to be adjusted can not be reduced largely. Big rms_1 will affect the number of quadrupoles to be adjusted and the closed orbit $x_{co,1}$ before the adjustments is large. After adjustments, rms_2 is less than 0.1 *mm*.

Table 5 and Table 6 give the closed orbit after adjustments calculated by MAD code, where $x_{co,2}$ and x_{co} are the closed orbits defined in equation (5).

Table 5 Closed orbit calculated with MAD for 1991

K	x_{co2} (*mm*)		x_{co} (*mm*)	
	max	*rms*	*max*	*rms*
0	0.007	0.007	1.607	0.453
2	3.669	1.104	10.855	3.574
4	2.791	1.054	4.206	1.521
6	3.217	1.229	4.933	2.010
8	3.954	1.420	5.710	2.293
10	4.480	1.566	6.065	2.692

Table 6 Closed orbit calculated with MAD for 1993

K	x_{co2} (*mm*)		x_{co} (*mm*)	
	max	*rms*	*max*	*rms*
0	0.501	0.501	5.467	1.521
2	6.082	2.118	8.423	3.021
4	6.022	2.173	7.368	2.4761
6	7.090	2.841	8.801	3.303
8	6.556	2.154	7.813	3.226
10	7.452	2.576	8.703	3.140

From Table 5 and Table 6, it can be seen that the closed orbit $x_{co,2}$ induced by the smooth curve in 1993 is bigger than that in 1991 because the amplitude of the smooth curve in 1993 is bigger than that in 1991. So in order to get small $x_{co,2}$, the smooth curve should be close enough to the design orbit. The closed orbit x_{co} after adjustments is mainly decided by $x_{co,2}$ after adjustments.

As for how to choose the cutoff order, it is a process of compromise. If *K* is chosen larger, the smooth curve is closer to the raw data, the number *N* is smaller, but $x_{co,2}$ increases, so the closed orbit after adjustments is bigger. In contrary, if *K* is chosen smaller, the number *N* will be larger, so it can not save much manpower and working period. It should be noticed that not for all the results of alignments in accelerators, smoothing analysis is effective.

5. CONCLUSION

If the smooth curve is smooth enough and close enough to the design orbit and the rms displacement of the quadrupoles from the smooth curve is small, smoothing analysis will be effective to reduce largely the number of the quadrupoles to be adjusted, while the closed orbit distortion is also reduced. The number of the quadrupoles to be adjusted is decided by the rms displacement of quadrupoles from the smooth orbit. After adjustments, the maximum and rms closed orbit in reference to the design orbit are mainly decided by the smooth curve.

6. REFERENCES

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