Intuitive Description of Emittance Compensation and Its Application to Beam Transport Lines

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Abstract

A solid and intuitive picture of emittance compensation process is presented. The emittance compensation technique is originally applied in low emittance and high intensity RF guns. The key component is a solenoid which confines the beam to forming a Brillouin flow and transfers angular momentum to a focusing force at its exit. With this picture in mind, we successfully extend the technique to the beam transport line of the TESLA Test Facility (TTF) FEL [1]. Some 30% reduction of the transverse emittance compared to that obtained with common triplets is achieved.

1 INTRODUCTION

Low emittance and high intensity beams are needed by modern single pass free electron lasers (FELs) at short wavelengths like the TTF FEL which adopts the self amplified spontaneous emission (SASE) lasing scheme. It requires a normalized transverse emittance of 2 π mm-mrad. A photo RF gun has been optimized to be able to deliver a beam of 1 π mm-mrad. Preserving the beam emittance at a relatively low energy (20 MeV) along a beam transport line becomes an unavoidable issue. Common focusing elements like triplets or doublets are found not adequate for this purpose.

The emittance compensation technique originally applied to RF guns [2] is an effective method to preserve the beam emittance. The key component is a solenoid which confines the beam to forming a Brillouin flow and transfers particles' angular momentum to a focusing force at its exit. When the focusing force and the space charge repulsion reach their balance, a subsequent acceleration of the beam to infinity would result in a zero emittance beam. With this in mind, we successfully applied the gun space charge compensation to the beam transport line of TTF FEL. Some 30% reduction of emittance compared to that obtained with triplets is achieved.

In the following, a real space picture of the space charge compensation process is presented. A comparison between different focusing schemes is given. For a more theoretical description of the emittance compensation technique, see e.g. [3]

2 EMITTANCE COMPENSATION

The essence of the so-called emittance compensation is the balance between external focusing forces and the space charge repulsion. Linear forces do not alter rms emittances, which can be directly proved from the definition of normalized rms emittance

$$\epsilon_x = \sqrt{\overline{x^2}\ \overline{p_x^2} - \overline{xp_x}^2}.$$

As first order approximation, the space charge forces are

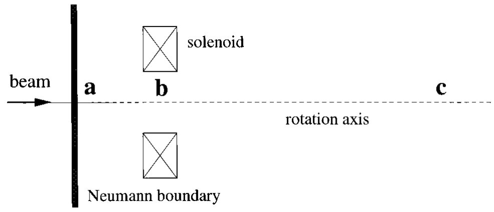

Figure 1: A schematic setup for emittance compensation. At **a** a beam enters the solenoid field where $\mathbf{B} = 0$. In the drift space from **a** to **c**, all the particles in the beam rotate across the axis, forming the so-called Brillouin flow. At **c** where the fringe solenoid field becomes very weak, the rotating momentum of the particles turns to a focusing one. How well the balance is achieved largely depends on the distance **a**-**b** and the solenoid field strength.

linear for a round beam of uniform charge distribution. There should be no space charge induced emittance growth. Coming from thermal interactions, particles are not well correlated. The r-r' correlation goes worse with increase of radius. Outer particles contribute the most to the emittance growth. Other non-linear effects are RF and wake fields.

According to Busch's theorem [4]

$$rP_\phi + \frac{q}{2\pi}\psi = \text{const},$$

where r is radius, P_ϕ angular momentum, q particle's charge, and $\psi = \int \mathbf{B} \cdot d\mathbf{s}$ the magnetic flux enclosed by a circle with the radius r, the focusing force of a solenoid is also linear with radius, if r is small enough and a Neumann's boundary condition is applied at the solenoid entrance. The focusing force will compensate the space charge defocusing one in a uniform manner across the bunch radius, preventing the beam from diluting while still keeping the charge density uniform. The solenoid compensation scheme is especially useful for round beams. This space charge compensation technique also holds for nonsymmetrical beams when appropriate focusing schemes are applied.

The above discussion assumes the polar system. If there is no rotation of particles, the conclusion is also applicable to the Cartesian system, for there exists a simple relation of rms emittance in the two systems

$$(2\epsilon_x)^2 = \epsilon_r^2 + \epsilon_\phi^2,$$

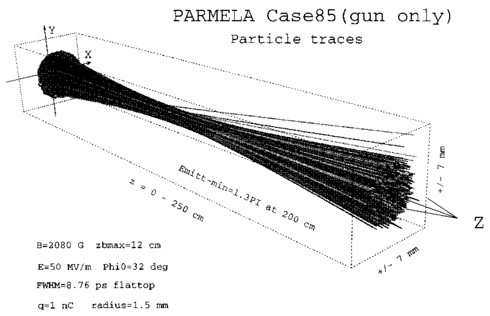

Figure 2: Particle traces in real space for the TTF FEL gun, simulated with PARMELA. Each line represents one particle. The twisting of the line bundle is caused by the solenoid field. The transition from the twisting to an inward movement can be clearly seen. The waist is at z=130 cm, while the minimum emittance occurs at z=200 cm.

where

$$\epsilon_r = \sqrt{\overline{r^2}\ \overline{p_r^2} - \overline{rp_r}^2} \quad \text{and} \quad \epsilon_\phi = \sqrt{\overline{r^2}\ \overline{p_\phi^2}}.$$

Figure 1 shows a typical setup for space charge compensation using solenoid. The Neumann's boundary condition is essential to be able to use the Busch's theorem to transfer angular momentum to a focusing force. Otherwise, the beam will become an axial-confined flow, in which each particle rotates around a "local" magnetic flux line instead of doing a global precession around the beamline axis. Minimum transverse emittance can be found where the balance between the two forces is reached. At this stagnation point, beam radius reaches its minimum and the divergence becomes zero. Figure 2 shows the particle traces in real spaince for the TTF FEL gun. The x-x' phase space development along the beamline is given in Fig. 3. Figure 4 shows the projection of the x-x' phase space development onto the x-x' plane. It is found that the actual minimal emittance occurs somewhere after the beam waist.

This is because of imperfect compensation, which is al-

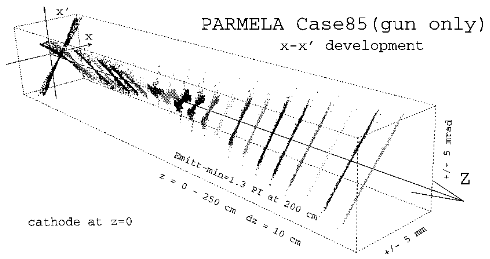

Figure 3: x-x' phase space development of the beam in Fig. 2

ways the case in practice. The above statement of space charge compensation holds true only when sources of imperfect compensation are not present. But as a qualitative approximation, this statement is still very useful in practice.

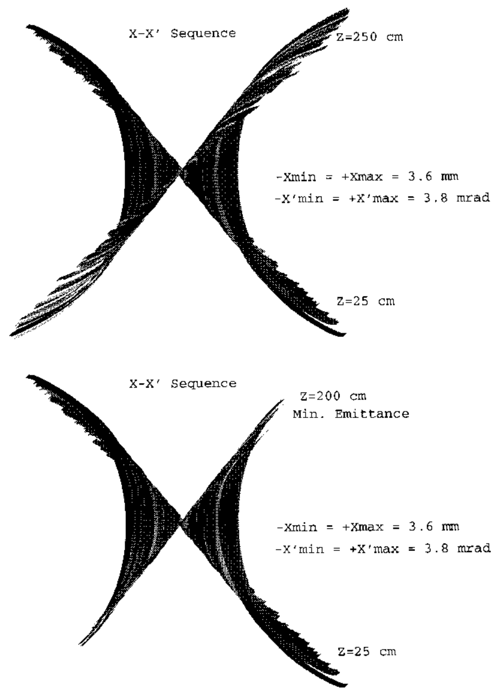

Figure 4: Projection of the x-x' phase space development. It is obtained by looking to Fig. 3 in the -z direction. The upper "X" pattern shows the full simulation range from z=25 cm down to 250 cm; while the lower one down to z=200 cm where the emittance reaches its minimum. One can see the thin area occupied by the trace front.

3 EMITTANCE PRESERVING BEAM TRANSPORT

Figure 5 shows the layout of the TTF FEL beamline. The minimum emittance of 1 π mm-mrad from the gun is reached at z=6 m. There exists a good 5 m drift space from this point down to the entrance of the linac. In the initial

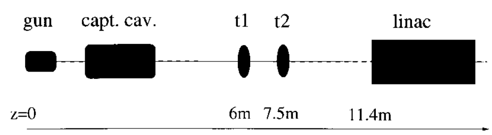

Figure 5: Schematic layout of the TTF FEL injector. t1 and t2 are triplets.

design, two triplets (t1 and t2) are used to match the beam to the linac optics. Since the focusing for triplets is spatially separated in x and y planes, they are inherently not suitable to handle the round beam which comes from the gun with respect to space charge compensation.

Different focusing schemes are studied for the TTF FEL beam transport line, including triplets, doublet, and solenoid (Table 1).

Neither triplet nor doublet scheme shows an effective

Setup	$\epsilon_x^n/\epsilon_y^n$ (π mm-mrad)
Two triplets (t1 & t2)	2.3/1.9
One doublet at t1	1.4/1.8
Solenoid Neumann BC	1.3/1.4 No fringe B_z
Solenoid Neumann 2cm hole	1.4/1.5 Small fringe B_z
Solenoid Neumann 3cm hole	1.4/1.5 Small fringe B_z

Table 1: Emittance measured at the entrance of the linac. Simulated using PARMELA with 10,000 particles. Quadruple strengths are set such that the emittance at the entrance of the linac reaches the minimum. Magnet dimensions are: 8-6-14-6-8 cm DOFOD triplets and 5-3-5 cm FOD doublet, 10 cm solenoid with the shielding plate placed 12 cm upstream.

or uniform emittance preservation in both planes. The solenoid scheme with the Neumann's boundary condition is, as expected, the best, which improves the mean emittance for the triplet case by some 35%. Since the strict Neumann's boundary condition cannot be realized in practice, we use a Neumann shielding plate with a hole of 3 cm radius at the center for the beampipe, which still shows a 30% improvement with respect to the triplet case. The particle traces and the x-x' phase space development along the beamline are shown in Figs. 6 and 7, respectively. Comparing to Figs. 2 and 3 for the gun, we find that both have the same mechanism of space charge compensation.

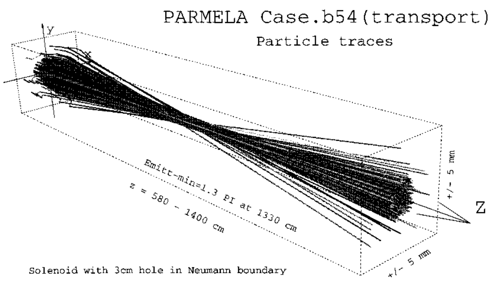

Figure 6: Particle traces in real space for the TTF FEL beamline simulated with PARMELA

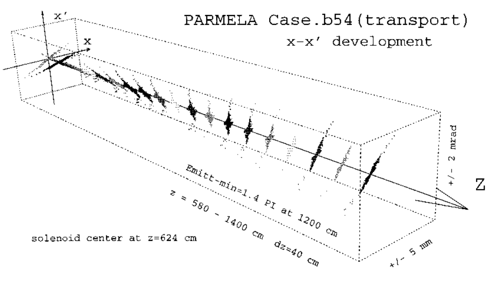

Figure 7: x-x' phase space development of the beam in Fig. 6

4 CLOSING REMARKS

Emittance compensation is obtained by the balance between two forces: the repulsion of space charges and the focusing force applied by external components. Uniform balance is necessary in order to have an optimal emittance compensation in both x and y planes. Minimum emittance occurs at the point where the two forces reach a balance. A subsequent instant acceleration of the beam to infinity will result in a zero emittance beam. This is the intuitive description of the lower bound limit of the emittance compensation technique.

The intuition of the applicability of solenoid focusing leads to some 30% reduction of emittance growth in the TTF FEL beam transport line.

5 REFERENCES

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