STUDIES ON THE BEHAVIOR OF HIGH INTENSE BEAMS IN THE PERIODIC FIELD

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Abstract

In this paper, we study the behavior of high intense beams in the periodic field by using thin lens approximation for the space-charge forces. The main chaotic behavior and halo formation can be shown through simple calculations. Meanwhile, It is very easy for us to study the behavior of high intense beams with more complicated realistic distributions than K-V distribution by this approximation, which is rather difficult by other ways.

1 INTRODUCTION

High-intensity, high-energy proton linear accelerators are being designed for new projects around the world[1]. These projects include the accelerator transmutation of waste (ATW), accelerator-based conversion of plutonium (ABC), accelerator production of tritium (APT), and the next generation accelerator-driven spallation neutron sources. One of the important problems faced in the design of this kind of accelerators is the beam loss. Small beam loss in high energy section of the linacs can lead to radioactivation of accelerators, which can degrade accelerator components and prevent hands-on maintenance. The threat of beam loss is increased significantly by the formation of beam halo. The experiments and the numerical simulations have show that the rms mismatch of the beams is the major source of halo formation[2,3,4].

A popular model used to study the dynamics of halo formation is the particle-core model[5-9]. In this model, halo particles interacted with a beam core that is assumed to oscillate according to the rms envelope equation. The reasons of oscillation of the beam core may be an initial radial mismatch in a constant focusing channel or an initial radical matched beam in a periodical field of FODO structure. The dynamics of halo formation can be showed by this model. Moreover, the maximum amplitudes of the halo particles can be predicted quantitatively, which are basically in accord with the results drawn from PARMILA simulations[10]. Almost all the studies on this model are to write out the envelope evolution equation of beam core and the dynamic equation of the halo particles, then resolve the equation of halo particles numerically or analytically. The problem is that the dynamical equation of halo particle is very hard to get for the more complicated realistic beam distributions than K-V distribution or Gaussian distribution. In this paper we use thin lens approximation for the space-charge forces and the transfer matrix instead of dynamic equations of beam core and halo particles. The great advantage is that we can study the behavior of high intense beams with the more complicated realistic distributions. Meanwhile, It provides another way of studying the halo particles.

2 THE ALGORITHM

For simplicity, we assume that the external field and the beam are rotationally symmetrical and periodic. Take the beam line between two neighboring sites with maximum beam sizes as a period L. That is to say, if we assume that the beam radius is R then at $z=0$ or $z=L$ R $R_{\max}$ at $z=L/2$ $R=R_{\min}$. Here $R_{\max}$ and $R_{\min}$ are the maximum, minimum radius of the beam, respectively. Defining

$$\rho = R_{\max}/R_{\min}, \quad R_a = \sqrt{R_{\max}R_{\min}}, \quad \beta = R_a/\varepsilon. \quad (1)$$

Here ε is the beam emittance, ρ and β are the function of beam current I. Denote the values of ρ, β as ρ_0, β_0 when $I=0$ and assume $\rho=\rho_0$. The trajectory of the particle can be described by the following vector

$$\begin{pmatrix} r \\ r' \end{pmatrix}.$$

Here r stands for transverse displacement (x or y), $r' = dr/dz$. The action of the external field on the particle may be expressed in transfer matrix form. The transfer matrices M_{F0}, M_{L0} for the first half of the period and last half of the period are

$$M_{F0} = \begin{pmatrix} \rho^{-1}\cos(\mu_0/2) & \beta_0\sin(\mu_0/2) \\ -\beta_0^{-1}\sin(\mu_0/2) & \rho\cos(\mu_0/2) \end{pmatrix},$$

$$M_{L0} = \begin{pmatrix} \rho\cos(\mu_0/2) & \beta_0\sin(\mu_0/2) \\ -\beta_0^{-1}\sin(\mu_0/2) & \rho^{-1}\cos(\mu_0/2) \end{pmatrix}, \quad (2)$$

here μ_0 is the phase advance per period in the case of $I=0$.

We use thin lens approximation for the nonlinear space-charge forces and arrange these thin lenses at the sites where the beam radius is maximum or minimum. When the particle goes through any one of these lenses, r' will increase $\Delta r'$ i.e., $r' \to r' + \Delta r'$. When the beam core is of the form of K-V distribution, at the location $z=0$ or $z=L$

$$\Delta r' = \begin{cases} \dfrac{gr}{R_a^2\rho} = \dfrac{gr}{\beta\varepsilon\rho} & if\,|r| \le R_a\sqrt{\rho}, \\ \dfrac{g}{r} & if\,|r| > R_a\sqrt{\rho}, \end{cases}$$

at $z = L/2$,

$$\Delta r' = \begin{cases} \dfrac{g\rho r}{\beta\varepsilon} & if\,|r| \le \dfrac{R_a}{\sqrt{\rho}}, \\ \dfrac{g}{r} & if\,|r| > \dfrac{R_a}{\sqrt{\rho}}, \end{cases} \qquad (3)$$

where g is a constant related to the generalized perveance. On the other hand, when we take the space-charge forces into account, the transfer matrix of the first half period may be written as

$$M_F = \begin{pmatrix} \rho^{-1}\cos(\mu/2) & \beta\sin(\mu/2) \\ -\beta^{-1}\sin(\mu/2) & \rho\cos(\mu/2) \end{pmatrix} \qquad (4)$$

where μ is the phase advance per period of the particles in the beam core. M_F should satisfy the following equation

$$M_F = \begin{pmatrix} 1 & 0 \\ \dfrac{g\rho}{2\beta\varepsilon} & 1 \end{pmatrix} M_{F0} \begin{pmatrix} 1 & 0 \\ \dfrac{g}{2\beta\varepsilon\rho} & 1 \end{pmatrix} \qquad (5)$$

From equations (4) and (5) we have

$$\frac{1}{\rho}\cos(\frac{\mu}{2}) = \frac{1}{\rho}\cos(\frac{\mu_0}{2}) + \frac{g\beta_0}{2\beta\varepsilon\rho}\sin(\frac{\mu_0}{2})$$

and

$$\beta\sin(\frac{\mu}{2}) = \beta_0\sin(\frac{\mu_0}{2}).$$

Let $G = \dfrac{g\beta_0}{\beta\varepsilon}$ we can obtain

$$G = \frac{2}{\sin(\frac{\mu_0}{2})}\left[\cos(\frac{\mu}{2}) - \cos(\frac{\mu_0}{2})\right], \qquad (6)$$

$$\beta = \beta_0 \frac{\sin(\frac{\mu_0}{2})}{\sin(\frac{\mu}{2})}. \qquad (7)$$

In a similar way, we may also get the transfer matrix of the last half period M_L.

In order to change r and r' into dimensionless parameters, we make the following transformations

$$x = \frac{r}{R_a}, \quad y = \frac{r'}{R_a}\beta_0\mu_0.$$

Then the transfer matrices M'_{F0} and M'_{L0} for x, y now become

$$M'_{F0} = \begin{pmatrix} \rho^{-1}\cos(\mu_0/2) & \mu_0^{-1}\cos(\mu_0/2) \\ -\mu_0\sin(\mu_0/2) & \rho\cos(\mu_0/2) \end{pmatrix},$$

$$M'_{L0} = \begin{pmatrix} \rho\cos(\mu_0/2) & \mu_0^{-1}\cos(\mu_0/2) \\ -\mu_0\sin(\mu_0/2) & \rho^{-1}\cos(\mu_0/2) \end{pmatrix}. \qquad (8)$$

Meanwhile, at $z=0$ or $z=L$

$$\Delta y = \begin{cases} \dfrac{G\mu_0 x}{\rho} & if\,|x| \le \sqrt{\rho}, \\ \dfrac{G\mu_0}{x} & if\,|x| > \sqrt{\rho}, \end{cases}$$

at $z = L/2$

$$\Delta y = \begin{cases} G\mu_0\rho x & if\,|x| \le \sqrt{\rho}, \\ \dfrac{G\mu_0}{x} & if\,|x| > \sqrt{\rho}, \end{cases} \qquad (9)$$

In order to introduce a more complex realistic distribution for the beam, we let

$$\Delta y = \begin{cases} \dfrac{G\mu_0 x}{\rho}(1 - h\dfrac{x^2}{\rho}) & if\,|x| \le x_S, \\ \dfrac{G\mu_0}{x} & if\,|x| > x_S, \end{cases}$$

$z=0$ or $z=L$. Where $h = 1/4$ and $x_s = \sqrt{2\rho}$ which are got from the conditions that Δy and its derivative are continuous at the boundary of $x = x_s$. So

$$\Delta y = \begin{cases} \dfrac{G\mu_0 x}{\rho}(1 - \dfrac{x^2}{4\rho}) & if\,|x| \le \sqrt{2\rho}, \\ \dfrac{G\mu_0}{x} & if\,|x| > \sqrt{2\rho}. \end{cases} \qquad (10)$$

In a similar way we may get

$$\Delta y = \begin{cases} G\mu_0 x\rho(1 - \dfrac{x^2}{4\rho}) & if\,|x| \le \sqrt{\dfrac{2}{\rho}}, \\ \dfrac{G\mu_0}{x} & if\,|x| > \sqrt{\dfrac{2}{\rho}}. \end{cases} \qquad (11)$$

at $z = L/2$.

3 THE RESULTS OF CALCULATION

Before calculation, let us first analyze the motion of the beam particles qualitatively. When the beam core is K-V distribution, for the particles with $r \le R$, the motion of the particles can be described by the transfer matrices M_F and M_L, μ is the phase advance per period of the particles with $r \le R$ For the particles with $r >> R$ since the phase-charge forces act on the particles can be omitted, the motion of the particles can be described by matrices M_{F0} and M_{L0}, μ_0 is the phase advance per period of the particles with $r >> R$. The phase advance per period increases from μ to μ_0 when r changes from $r \le R$ to $r >> R$. If $\mu_0 > 180^\circ$ $\mu < 180^\circ$ half-integer resonance will occur, which must be voided in the design of the accelerator. If $\mu_0 > 90^\circ$ $\mu < 90^\circ$ there will give rise to the fourth order integer resonance and may lead to other larger order integer resonance, which also make the beam unstable as will be shown in the calculations. When the beam core is the other kind of distribution introduced in this paper, since the beam density decreases gradually as r increases and it is zero at $r = \sqrt{2}R$, the beam is comparatively more stable than that with K-V distribution in the same case of μ μ_0 which is in accordance with the result that the K-V distribution beam is somewhat more susceptible to instability than a more realistic distribution beam[11].

In the case of $\mu_0 = 60^\circ$ and $\mu = 40^\circ$ the plot of the Poincare surface of section[12] (one point is plotted in the (x, y) phase space at every site where the beam has the maximum radius) for the K-V distribution shows that islands around the resonance $1/8$ ($\mu = 45^\circ$) is very obvious and comparatively large. Islands of other orders resonance can also be told apart. For the other distribution introduced here, we can just see the islands of $1/7$ ($\mu = 51.4^\circ$) resonance. The islands of $1/8$ resonance should be more inside, but they are not found, perhaps because these islands are too small. Stochastic trajectory is not found in both figures. But we can see that K-V distribution is more regular. Both figures are drawn with $\rho = 1.44$. The figures with other values of ρ have the same topology as above two figures. So we keep $\rho = 1.44$ in all calculations afterwards.

For $\mu_0 = 100^\circ$ and $\mu = 70^\circ$, according to the above analysis, there should exist $1/4$ ($\mu = 90^\circ$) and $1/5$ ($\mu = 72^\circ$) resonance. From the Poincare surface of section for K-V distribution, islands of $1/4$ resonance can be clearly seen, but the islands of $1/5$ resonance are not found, which are overlapped with the islands of $1/4$ resonance. In the vicinity of the islands of $1/4$ resonance there are a large of area of chaos, via this area the inner particles can go out to become halo particles, *vice versa*, the outside halo particles can also go through this chaotic area to become inner beam core particles. Contrary to K-V distribution, for the more realistic distribution introduced here, though the islands of $1/4$ resonance also appear, one can not find any chaotic area from the Poincare surface of section for this distribution. So, for K-V distribution, in order to get stable intense beams, the phase advance μ_0 per period must be less than 90°. However, for a realistic beam distribution this limit may not be so strict.

For the same μ_0 when μ decreases further, the situation is basically not changed. From the Poinare surface of section at $\mu_0 = 100^\circ$ $\mu = 50^\circ$ for K-V distribution, one can see not only the islands of $1/4$ resonance but also the islands of $1/6$ resonance. A large chaotic area still exists between this two kinds of islands and outside the islands of $1/4$ resonance. The beam halo can also be formed in this case. For the other distribution introduced in this paper, islands of $1/4$ resonance can be seen, the chaotic area is still not seen and the beam halo can not be formed.

4 CONCLUSION

The behavior of intense beams in the periodic field is studied by using thin lens approximation for the space-charge forces for two kinds of beam distribution. Through calculation the chaotic behavior and the kinetic of beam halo formation can be shown. Moreover, the property that K-V distribution is more susceptible to instability is clearly shown. Therefore, the algorithm introduced in this paper provides another good way to study the dynamics of beam halo formation.

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