LINEAR TREATMENT FOR ION TRAPPING AND ITS APPLICATION IN BEPC

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Abstract

The conditions for ion trapping are studied in the cases of uniformly distributed electron bunches and bunch trains in storage rings based on the linear theory. The formulas of the critical currents for ion trapping are derived by means of Twiss parameters in the electron-focused system. Based on the theory, the beam behavior observed in the dedicated synchrotron radiation operation in the Beijing Electron-Positron Collider (BEPC) is discussed. It is proposed to apply bunch trains in the BEPC storage ring in order to overcome the beam lifetime drop during the single electron beam operation.

1 MOTIVATION

Single electron beams are applied for the dedicated synchrotron radiation operation in BEPC with peak beam current of about 80 mA uniformly distributed in 2 bunches at 2.2 GeV. It was sometimes observed that the beam lifetime dropped from the normal 8~10 hours to less then 1 hour. Experiments indicate that the lifetime drop has following features. It did not appear at a fixed current value, but usually below 30 mA, and would not be improved along with the reduction of the beam current. No correlation between the lifetime drop and RF voltage and other machine parameters was observed, while adjusting skew quads or turning-on kickers may help sometimes. In all cases, however, the beam lifetime resumed when the ring was refilled.

Several models were proposed to explain the puzzle, such as the vacuum degrading, the influence from the synchrotron radiation beam lines, HOM effects due to change of the RF cavity temperature, ion dusts and others. All those factors may cause the lifetime problem though, the question remains how to explain the phenomena observed in BEPC. The ion dusts were found to be a reason of the lifetime drop in DCI in LURE of France and HERA in DESY of Germany [1], but this model was failed to show why the lifetime drop happened at rather low beam current in BEPC. Another model, i.e. ion trapping, is introduced in this paper to discuss the lifetime behavior of BEPC.

2 THE CONCEPT OF ION TRAPPING

Electrons moving in the vacuum chamber of a storage ring may ionize the molecules of the residual gas, and then the ions are generated. The electromagnetic effect of the electron beam to the ions can be described with an electron lens. In the general cases, the electron lens is a non-linear "thick" lens for the ions. As the bunch length of the electron beam is much shorter than the gap between the successive bunches, the thin lens is usually a good approximation. Furthermore, the linear treatment may explore the basic aspect of the problem with its simplicity. The focusing strength of the linear ion lens can be written as

$$K_{x,y} = \frac{2N_e r_p}{M\sigma_{x,y}(\sigma_x + \sigma_y)}. \quad (1)$$

Where N_e is the number of electrons in a bunch, σ_x and σ_y are horizontal and vertical beam size respectively, r_e is proton classical radius and M is the molecule number of an ion.

The motion of the ions could be stable under the potential well of the electron beams, i.e. they oscillate by the electron beam orbit. This electron beams caused ion effect is called ion trapping. In this light, the ion-trapping problem is switched to the issue of the ion stability under the focussing of the electron lenses. If the density of the trapped ions is large enough, the motion of the electron beam will be perturbed and, as an immediate effect, the lifetime of the electron beam will reduce due to the rise of the local vacuum pressure. There are few methods can be applied to avoid the ion trapping [2]: to improve vacuum, using ion-cleaning electrodes, to operate a ring with positrons and choosing a proper bunch pattern to make the ion unstable. For such an operating machine like BEPC, it is difficult to apply the first three methods. However, by taking advantage of the flexibility of the timing system in BEPC, one may find the best bunch pattern for electron operation (or worst for ion stability).

3 ION TRAPPING WITH UNIFORMLY DISTRIBUTED ELECTRON BUNCHES

In the electron frame, the ions move with speed of light and undergo focusing from the electron bunches following to drift sections. In the case of the uniformly distributed electron bunches, the effect of bunches on an ion can be described with an equivalent "lattice" of N_b FO focusing cells. Here "F" denotes the electron focusing lens and "O" is a drift section, and N_b is the number of bunches in a ring. The linear transformation matrix of the cell can be written as

$$M_{FO} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} = \begin{pmatrix} 1+K \cdot l & l \\ l & 1 \end{pmatrix}, \quad (2)$$

where $l=C/N_b$ is the space between the successive bunches, C is the circumference of a ring. The trace of the matrix can easily be derived from eq. (1) and eq. (2) as

$$Tr_{x,y}(M_{FO}) = 2 + K \cdot l = 2 - \frac{2I_b r_p C^2}{Mec\sigma_{x,y}(\sigma_x + \sigma_y)} \frac{1}{N_b^2}, \quad (3)$$

where $I_b = N_b \cdot e f_0 N$ is electron beam current, e is electron charge, and f_0 is revolution frequency. The motion of ions gets unstable so that they will be swept out of the beam path when $|Tr(M)|>2$. In difference from other instabilities, the ion trapping takes place below a critical current, at least in the case of uniformly electron bunches. The critical currents of the ion trapping for the uniformly distributed N_b electron bunches can be derived from eq. (3) as

$$I_{c\ x,y} = \frac{2ecM\sigma_{x,y}(\sigma_x + \sigma_y)}{r_p C^2} N_b^2. \qquad (4)$$

It is shown in eq. (4) that the critical beam currents for ion trapping are proportional to square of the bunch number. As the beam sizes σ_x and σ_y vary along the ring circumference, the I_c is a function of the distance s. To estimate the critical currents in BEPC, an approximation of $\sigma = (\varepsilon\beta + \eta^2 \sigma_E^2)^{1/2} \approx (\varepsilon \cdot R/\nu + \eta^2 \sigma_E^2)^{1/2}$ is applied, where η being dispersion, σ_E the energy spread and $R=C/2\pi$ the ring circumference. Putting the data of BEPC into eq. (4), $\varepsilon_{x0}=7.6\times10^{-8}$ m·rad, C=240.4 m, $\nu_x=9.38$, $\nu_y=5.14$, and taking $\eta\approx0.5$ m, $\varepsilon_x/\varepsilon_y=0.1$, M=28(CO$^+$), we would expect

$$I_{c,x} = 16.5 \cdot N_b^2 ; I_{c,y} = 6.25 \cdot N_b^2 . \qquad (5)$$

Usually $\sigma_y << \sigma_x$, as shown in eq. (5), $I_{c,y}<I_{c,x}$, i.e. the vertical ion trapping is dominated. The horizontal and vertical critical currents for ion trapping are calculated for different N_b based on eq. (5), shown in Fig. 1.
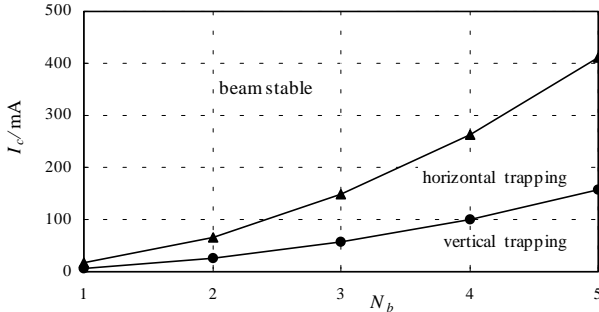

Figure 1 Critical beam currents vs. bunch number for uniformly distributed electron bunches

As it is displayed in Fig.1, the critical current I_c rises rapidly with the increase of bunch number, and $I_c \approx 25$ mA in the case of $N_b=2$, which agrees with the critical current of the beam lifetime drop below 30 mA observed in BEPC. The reason of why adjusting strength of skew quads helps beam lifetime would be explained as the change of coupling and then vertical beam size. The function of turning-on the kicker, is understood as the change of beam path.

To avoid the ion trapping, one may choose higher beam currents for operation. The beam current range of the BEPC dedicated synchrotron radiation is about 20~80 mA at this stage. In order to get higher current, one needs to use more bunches in a beam, while the critical current gets higher more rapidly. How can one get ride of ion trapping? The non-uniform distribution of the electron bunch may break the stable condition of ion motion, and therefore avoid the ion trapping.

4 ION TRAPPING WITH BUNCH TRAINS

The most common non-uniform distribution in storage ring is bunch train. For simplicity of discussion, we assume that the particle population in each bunch is the same and the trains along the entire circumference are identical. In this case, the "lattice" of a train is equivalent to N_b FO cells plus a gap between the successive trains whose length equals $(L-N_b \cdot l)$. Here, N_b is the bunch number in a train, L and l are total length of the train period and the space between the successive bunches respectively, and $L=C/N_T$, N_T being the number of trains in the ring. The transformation matrix of a cell is obtained as

$$M_b = \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix}\begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+Kl/2 & l+Kl^2/4 \\ K & 1+Kl/2 \end{pmatrix}. \quad (6)$$

Normally, the relation of $2 > Tr(M_b)=2+Kl > -2$ is held, one may introduce Twiss parameters [3] in the ion system by

$$\cos\phi = 1 + K \cdot l/2 ,$$
$$\beta = -\sin\phi/K = \frac{l \cdot \sin\phi}{2(1-\cos\phi)} = \frac{l \cdot ctg(\phi/2)}{2} . \qquad (7)$$

The matrix of the train can be written as

$$M_{N_b} = \begin{pmatrix} \cos(N_b\phi) & \beta\sin(N_b\phi) \\ -\sin(N_b\phi)/\beta & \cos(N_b\phi) \end{pmatrix}. \qquad (8)$$

The matrix of whole period be shown as

$$M = M_{l_g} \cdot M_{N_b} = \begin{pmatrix} 1 & L-N_b l \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos(N_b\phi) & \beta\sin(N_b\phi) \\ -\sin(N_b\phi)/\beta & \cos(N_b\phi) \end{pmatrix}$$
$$= \begin{pmatrix} \cos(N_b\phi)-(L-N_b l)\sin(N_b\phi)/\beta & \beta\sin(N_b\phi)+(L-N_b l)\cos(N_b\phi) \\ -\sin(N_b\phi)/\beta & \cos(N_b\phi) \end{pmatrix}. \quad (9)$$

The trace of the matrix in eq. (9) is obtained:

$$Tr(M) = 2\cos(N_b\phi)-(L-N_b l)\sin(N_b\phi)/\beta$$
$$= 2\cos(N_b\phi)-2(h/m-N_b)tg(\phi/2)\sin(N_b\phi), \qquad (10)$$

where $h=f_{rf}/(N_T/f_0)$ is harmonic number of the train, f_{rf} is RF frequency, and $m=l/\lambda_{rf}$ is the number of RF buckets per cell. The critical current for ion trapping can be derived from the equation (10) as

$$|Tr(M)|<2. \qquad (11)$$

With eq's. (11), (7) and (1), the critical current for ion trapping in the case of bunch trains can easily be obtained. As eq. (11) is an overstepping function, there are infinite solutions, or critical current values, for the ion stability. For the different beam sizes in the horizontal and vertical planes, the critical currents differ in two planes, which may help to enlarge the beam stable region, as the beam

get unstable only when the ions are trapped in both horizontal and vertical planes.

The critical currents for ion trapping are calculated for different bunch number N_b and various bunch space ($m=1,2,3,4$) in BEPC based on the same parameters applied in the last section and there is one train in the ring. The results are plotted in Fig. 2. The triangles in the figures represent the horizontal critical currents, while the circles are for the vertical. On the other hand, the solid points stand for the down-critical current values, below which the ions get trapped, while the hollow points for the up-critical values.
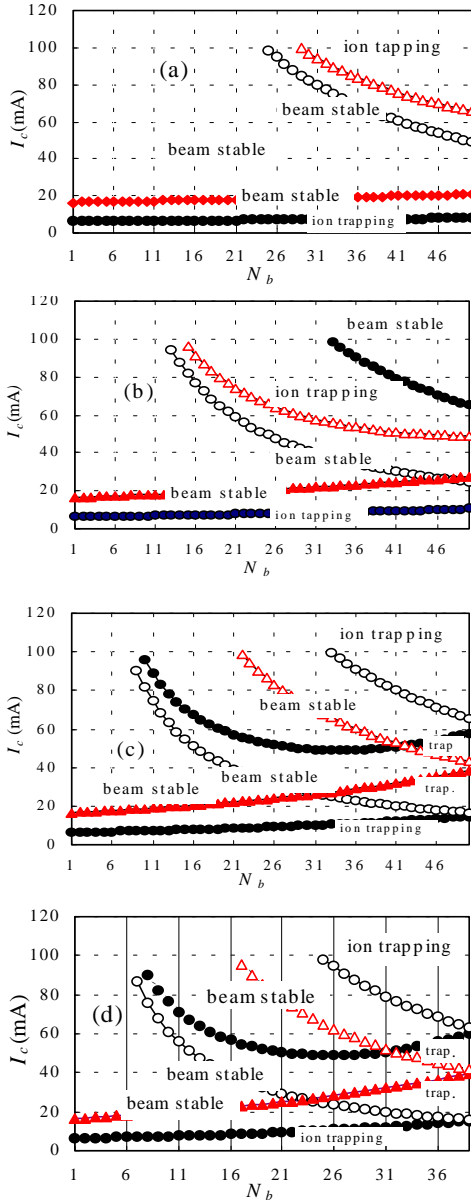

Figure 2 Critical beam currents vs. N_b for bunch train
(a) $m=1$ (b) $m=2$ (c) $m=3$ (d) $m=4$

It can be seen from the figures that the up and down critical current curves meet each other when the bunch distribution gets uniform ($m=3$, $N_b\sim53$ in Fig. 2c and $m=4$, $N_b=40$ in Fig.2d). It is clear that the non-uniformly distributed bunches may help to break the condition of ion stability so that the beam stable area becomes larger for the non-uniformly distributed bunch system than it is in the uniform case.

Figure 3 displays the critical currents as functions of bunch space for a bunch train containing two bunches, which is the case of the present synchrotron radiation operation of BEPC.
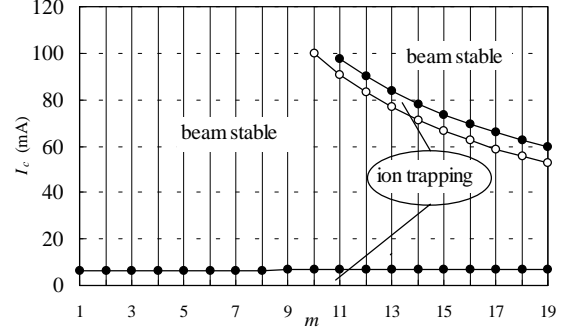

Figure 3 Critical currents vs. bunch space for N_b=2

It can be found from the figure that the ion trapping takes place below ~ 7 mA and it almost does dot change with the bunch space characterized by m; the second and third critical currents appear when $m \geq 10$, and ions are trapped at higher beam currents. This situation allows us to choose a suitable bunch space in order to avoid coupled bunch instability and get ride of ion trapping.

5 DISCUSSION

A train of two bunches is applied in BEPC synchrotron radiation operation with the bunch space of 2, 4, 6, 8 buckets (m=2, 4, 6, 8). The experiments have indicated that the frequency of beam lifetime drop has significantly reduced. Occasionally, beam lifetime dropped from 8~10 hours to about 2 hours, which could be resumed itself or by trimming skew quads or other machine parameters. This could be explained as the local ion trapping, where the beam size is large and vacuum is relatively worse. On the other hand, the linear approximation may also be a reason of the partial agreement between the theory and experiments. In our further study, the non-linear effects will be taken into consideration.

6 ACKNOWLEDGMENTS

7 REFERENCES

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