# RF Phase Focusing and Asymmetric Field Shape in Standing-wave Electron Linacs \*

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#### Abstract

In order to minimize the size and weight of the low energy standing-wave linacs and to improve the beam spots as well as the transmission, the RF fields are used for radial focusing without external magnetic focusing devices. By choosing the proper phase velocity distribution and by adjusting the amplitudes of the buncher fields from cavity to cavity, the RF fields in the buncher region provide transverse focusing as well as longitudinal bunching and acceleration. The RF phase focusing is more effective by proper choice of an asymmetric cell geometry in the first cavity.

#### **1. INTRODUCTION**

Small beam spots are required at the exit of the SW linacs for both medical and industrial uses. In order to minimize the size and weight of the low energy SW linacs, the RF fields are used for radial focusing without any external magnetic focusing devices.

This paper concentrates on the effects of RF phase focusing and the influence of the first bunching cavity shape. Several SW linacs have been designed by using the RF phase focusing theory and asymmetric field shape technique in our laboratory. They have been fabricated and beam tested. One of them is expressed.

#### 2. RF PHASE FOCUSING

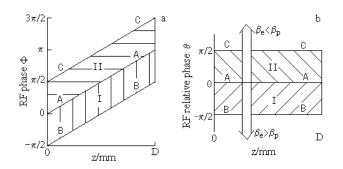


Fig. 1 Layout of the phase and relative phase in one cavity a-Layout of the phase, b-Layout of the relative phase

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The "standard particle" in a SW structure operated in the biperiod  $\pi$  mode is defined as a particle whose phase goes linearly from 0 at the entrance to  $\pi$  at the exit of one cavity along the longitudinal axis. Line A-A in Fig. 1a, b is the phase trajectory of the standard particle in one cavity. The relative phase  $\theta$  of a bunch is the difference between its RF phase and that of the "standard particle" at the same position.  $\theta = \Phi - \Phi_0$ , where  $\Phi_0 = \frac{\pi}{D} z$  is the RF phase of the standard particle;  $\Phi$  is the RF phase of the standard particle;  $\Phi$  is the RF phase of the bunch; *D* is the length of one accelerating cavity; and *z* is the longitudinal position of the bunch. Line B-B and C-C in Fig. 1a, b indicate the RF phase motion of the particles ahead of or behind the standard particle separately.

Consider an ideal circularly symmetric accelerating field in which all components are independent of the azimuthal angle. Let the space charge effects be ignored. The longitudinal electric field along the axis is of the form  $E(z,t) = E_z(r = 0, z, t)$ From divergence equation.

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} (rE_r) + \frac{\partial E_z}{\partial z} = 0$$

And from one of the curl equations,

$$(\nabla \cdot \vec{B})_z = \frac{1}{r} \frac{\partial}{\partial r} (rB_{\phi}) = \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$

the radial Lorents force of exerted on the particle to first order in r, r' and  $\phi'$  is<sup>[1]</sup>

$$F_{r} = e(E_{r} - \dot{z}B_{\phi})$$
$$= -\frac{1}{2}er\left[\frac{\partial E(z,t)}{\partial z} + \beta \frac{1}{c}\frac{\partial E(z,t)}{\partial t}\right] \qquad (1)$$

Noting that  $dE(z,t) = \frac{\partial E(z,t)}{\partial z} dz + \frac{\partial E(z,t)}{\partial t} dt$ , and using the identity  $\Phi = kct + \Phi_0$ , one may obtain

$$F_r = -\frac{1}{2} er \left[ \frac{d}{dz} - \frac{k}{\beta \gamma^2} \frac{\partial}{\partial \Phi} \right] E(z, \Phi)$$
(2)

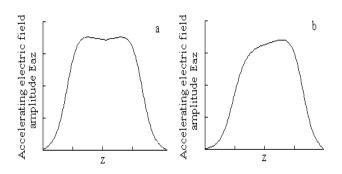


Fig. 2 Layout of the accelerating electric field amplitude distribution along the axis in one cavity a-In a symmetric cavity, b-In an asymmetric cavity

Fig. 2a shows the accelerating electric field amplitude distribution along the axis in one cavity. The first term on the right in Eq. (2) is the total derivative of the axial electric fields along the particle path. The integral effect of this term acts as an electrostatic lens<sup>[1][2]</sup>. It produces a net focusing force which is represented by

$$F_{rz} = -\frac{e^2 r}{4\beta\gamma mc^2} \left( \left\langle E^2 \right\rangle - \left\langle E \right\rangle^2 \right) \tag{3}$$

where  $\langle \rangle$  denotes an average within the structure. Noting that this term is proportional to  $(\beta\gamma)^{-1}$ .

#### 2.2 RF Phase Focusing

The second term on the right in Eq. (2) determines the RF phase focusing. This term is 90° out of phase with  $E(z, \Phi)$  and its sign depends on the relative position of the bunch to the crest of the SW accelerating field.

In SW linac structures,  $E(z, \Phi) = E_{za}(z)\sin(\Phi)$ , the second term then is

$$F_{r\phi} = \frac{1}{2} er \frac{k}{\beta \gamma^2} E_{za}(z) \cos \Phi$$
<sup>(4)</sup>

When  $\Phi \in (-\frac{\pi}{2}, \frac{\pi}{2})$ , this force is defocusing one; when  $\Phi \in (\frac{\pi}{2}, \frac{3\pi}{2})$ , this force is focusing one. Noting that this term is proportional to  $(\beta\gamma^2)^{-1}$  and it damps out more rapidly than the first term. From Fig. 1a, the standard particle is defocused when  $\Phi \in (0, \frac{\pi}{2})$  and focused when  $\Phi \in (\frac{\pi}{2}, \pi)$ . The particles leading the standard particle, which is in the part (I) of Fig. 1a, b, are net defocused; the particles lagging the standard particle, which is in the part (II) of Fig. 1a, b, are net focused. So it is beneficial to have the bunch lagging the crest, or in the part (II) of Fig. 1a, b for transverse focusing. Careful selection of the design parameters, the phase velocity distribution, the field level arrangement within the cavities and the injection voltage, gives adequate RF focusing in SW linacs.

## 2.3 Asymmetric RF Fields<sup>[3]</sup>

In order to increase the RF focusing effect, the asymmetric RF field amplitude distribution along the axis should be designed, as shown in Fig. 2b. Then the accelerating electric field amplitude is lower when  $\Phi$  is in the defocusing domain and higher when  $\Phi$  is in the focusing domain. So that the RF phase focusing is more effective.

The asymmetric field shape can be obtained by a proper asymmetric geometry cell and should be used only in the first bunching cavity due to the factor  $(\beta \gamma^2)^{-1}$ . The design and experiment results have shown that the effects of asymmetric fields in the first bunching cavity are striking.

According to the above analysis, RF phase focusing is related with the relative position between the bunch and the standard particle. The focusing force exerted on the electron strengthens or the defocusing force weakens when it lags the standard particle. The focusing force weakens or the defocusing force strengthens when it leads the standard particle. The asymmetric fields in the first bunching cavity enhance the effects of RF phase focusing on the beam.

#### 3. DESIGN APPLICATION

Several SW guides have been designed by using the RF focusing and asymmetric shape field of the first cavity techniques in our laboratory. They have been manufactured and beam tested. The measured beam properties show that the research analysis and dynamics

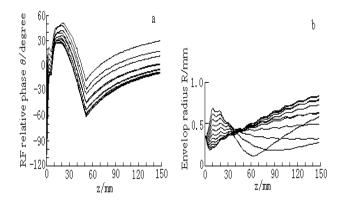


Fig. 3 Longitudinal and transverse orbits for various input phases in a 2 MeV X-band SW linac a-Longitudinal orbits, b-Transverse orbits

simulation described above are successful. The transverse focusing is very satisfactory.

For an X-band 2 MeV SW electron linac, the electron beam with an energy of 2.4 MeV and a peak current 108 mA, has been obtained. The beam spot is less than 1.4 mm (diameter) without external solenoid. Fig. 3a, b are the design results of both longitudinal and transverse dynamics. In Fig. 3a before the 40 mm, the bunch is behind the crest so that it is focused. As shown in Fig. 3b, the RF focusing effect is obvious.

### 4. CONCLUSION

The RF phase focusing, the asymmetric geometry of the first bunching cavity and the beam converging injection techniques in SW linacs are effective measures to transverse focusing the beam. Small beam spots can be obtained by these measures without external focusing devices.

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