

# STUDY OF THE SYNCHROTRON AND BETATRON OSCILLATIONS FOR THE SIAM PHOTON SOURCE

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## Abstract

The storage ring in the Siam Photon Source of National Synchrotron Research Center, THAILAND is the upgraded version of SORTEC storage ring. The magnet lattice structure is changed to obtain the low emittance and to include the long straight sections to provide space for the insertion devices. Therefore, a detailed analysis of the modified parameters of the synchrotron and betatron oscillations in longitudinal and transverse phase space, respectively have been carried out by developing the computer programs using the visual basic. The dynamics of electrons making synchrotron motion in the presence of RF accelerating field and radiation damping of synchrotron and betatron oscillations have been studied for the storage ring in the Siam Photon Source. Parameters such as the synchrotron oscillation frequency, the momentum acceptance of the RF acceleration system, the maximum phase deviation, damping times for the synchrotron and betatron oscillations have been evaluated.

## 1 INTRODUCTION

The Siam Photon Source accelerator complex consists of a 40MeV Linac, 40MeV to 1GeV Booster Synchrotron and 1GeV Storage ring. Important parameters of the Siam Photon storage ring are given in Table 1.

Table1: Main parameters of the Siam Photon Storage ring

Electron energy, $E_0$	1GeV
Circumference, $C$	81.3m
Bending radius, $\rho$	2.78m
Momentum Compaction factor, $\alpha$	0.0214
Betatron Wave numbers, $\nu_x, \nu_y$	4.758, 2.823
RF frequency, $f_{rf}$	118MHz
RF voltage, $V$	100kV
Harmonic number, $h$	32

The phase space ellipse and the separatrix obtained here are important basic data. The data obtained here will be used in calculation of practical parameters and also used in finding the appropriate RF acceleration bucket. This is necessary for the commissioning of the machine. Accurate results are obtained in calculating the different parameters as mentioned below. The dependence of the maximum momentum deviation and the maximum phase deviation on the synchronous phase angle as well as the variation of synchrotron oscillation frequency with the maximum momentum deviation is investigated. The practical calculation is made in the longitudinal and

transverse phase space by developing the Visual Basic computer programs.

## 2 DESCRIPTION

Synchronous phase  $\Phi_s$  is obtained by equating the energy gain  $eV\sin\Phi_s$  to the average energy loss per turn  $U_0$  (energy loss at  $E_0$ ).

$$U_0 = C_e E_0^4 / \rho \quad (1)$$

The obtained value of  $\Phi_s$  is  $161.437^\circ$ . At initial phase angles equal to or less than  $(\pi - \Phi_s)$  electrons make unstable trajectories. The curve that separates the stable trajectories from the unstable trajectories is called the separatrix. The maximum momentum deviation and the maximum phase deviation define the stability limits of the RF bucket [1, 2]. If an electron is injected into a storage ring with the momentum and phase deviations below the limits given by their maximum values, it circulates on a stable trajectory with in the bunch. Synchrotron oscillation (without damping) in phase space is described by the following mapping equations[3].

$$\delta_{n+1} = \delta_n + e V / \beta^2 E_0 (\sin \Phi_n - \sin \Phi_s) \quad (2)$$

$$\Phi_{n+1} = \Phi_n + 2 \pi h \alpha \delta_{n+1} \quad (3)$$

Where  $\delta_n$  is the deviation of the electron momentum from that of the synchronous electron at  $n_{th}$  revolution,  $e$  charge of electron,  $V$  peak RF voltage,  $\beta$  velocity of the electron relative to speed of light,  $E_0$  energy of synchronous electron,  $\Phi_n$  phase angle of the RF wave seen by the off-momentum electron at  $n_{th}$  revolution,  $\Phi_s$  phase angle of the RF wave seen by the synchronous electron,  $h$  harmonic number and  $\alpha$  momentum compaction factor.

The synchrotron oscillation frequency, an important characteristic parameter of the synchrotron oscillation, is determined by finding out the number of revolutions,  $n$ , taken by the electron to complete one period of synchrotron oscillation. If the time taken by the electron to complete one revolution is  $T_0$ ,  $1/(nT_0)$  gives the frequency of synchrotron oscillation.

The damping of the synchrotron oscillation is caused by the fact that the synchrotron radiation power or the rate of change of energy loss is dependent on the electron energy. The following assumption is made in the damping time calculation.

- As  $U_0$  is much smaller than the electron energy by a factor  $10^3$  or more, we consider only the effects that occur over many revolutions by neglecting the changes in electron energy during one revolution.

The total energy radiated in one revolution can be written as follows:

$$U_{rad} = U_0 + (2U_0/E_0) \Delta E \quad (4)$$

In case of longitudinal damping energy loss is estimated by using Eq. (4). Mapping equation for  $\Phi_{n+1}$  remain same but  $\delta_{n+1}$  is obtained as follows:

$$\delta_{n+1} = \delta_n + e V / \beta^2 E_0 (\sin \Phi_n - \sin \Phi_s(1+2\delta)) \quad (5)$$

The phase space ellipse that represents the betatron oscillation is obtained by plotting the variation of  $y$  or  $x$  (displacement) and  $y'$  or  $x'$  (slope) for  $n$  revolutions using the transfer matrix corresponding to one revolution. Damping times for the betatron oscillation [4, 5] are calculated by considering the damping of longitudinal oscillation. In case of betatron damping, the RF acceleration causes the slope  $y'$  to be decreased by the increment of the longitudinal momentum i.e.,

$$y' \rightarrow (1 - \Delta p/p_0) y' \quad (6)$$

Eq.(6) is used in the betatron damping time calculation. Computer programs developed in Visual Basic are used for the calculation of different parameters.

### 3 RESULTS

#### 3.1 Synchrotron Oscillation Frequency

Let us consider the small amplitude oscillation, in which the phase does not deviate much from  $\Phi_s$  and the fractional momentum deviation,  $\delta$  is very small. The small amplitude oscillation frequency is calculated by finding out the number of revolutions in one period of synchrotron oscillation. The value obtained by the program is 11.827kHz. In case of the large amplitude oscillation, the electron takes higher values of  $\delta$  and moves through longer path. So the number of revolutions,  $n$ , taken by the electron to complete one period of synchrotron oscillation is greater than that of the small amplitudes. This is shown in Fig.1.

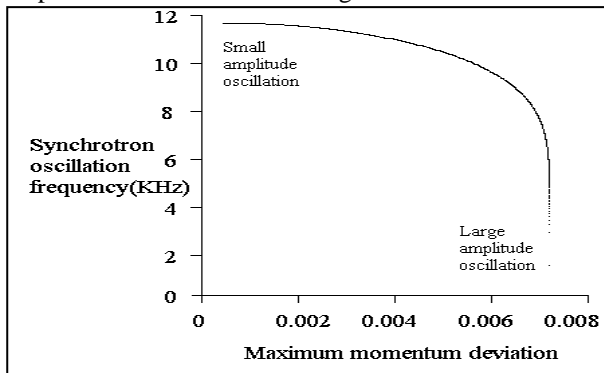


Figure 1: The variation of synchrotron oscillation frequency with the maximum momentum deviation  $\delta_{max}$ .

The data exhibits two characteristic features:

- The synchrotron oscillation frequency of the small amplitude oscillation is higher than that of the large amplitude oscillation.
- A steep edge occurs near  $\delta_{max} = 0.007157$

The region of maximum momentum deviation in which the synchrotron oscillation frequency drops rapidly with  $\delta_{max}$  is considered as the unstable oscillation region.

#### 3.2 Maximum Momentum Deviation

The maximum momentum deviation is obtained at the separatrix. The maximum momentum deviation  $\delta_{max}$  has been calculated and the obtained value is 7.157E-3 for the initial values of  $(\Phi_0, \delta_0)$  as (18.563, 0). This is shown in Fig.2. It is observed that the maximum momentum deviation or momentum acceptance is a function of synchronous phase angle and it increases with the synchronous phase angle. The relation between the maximum momentum deviation and the synchronous phase angle is shown in Fig.3.

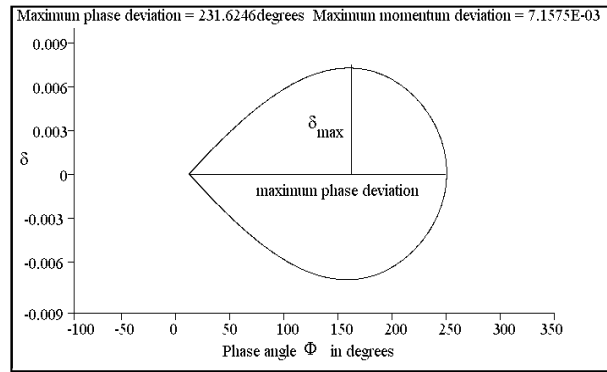


Figure 2: The phase space diagram of synchrotron oscillation indicating the maximum momentum deviation and maximum phase deviation.

#### 3.3 Maximum Phase Deviation

The maximum phase deviation is obtained at the separatrix. The maximum phase deviation has been calculated by the computer simulation and the obtained value is  $231.624^0$  or 4.0426 radians for the initial values of  $(\Phi_0, \delta_0)$  as (18.563<sup>0</sup>, 0). The result is shown in Fig.1. This parameter shows the boundary of the RF bucket in terms of  $\Phi$  i.e., the maximum phase seen by the off-momentum electron with reference to that of the longitudinal electric field. It is observed that the

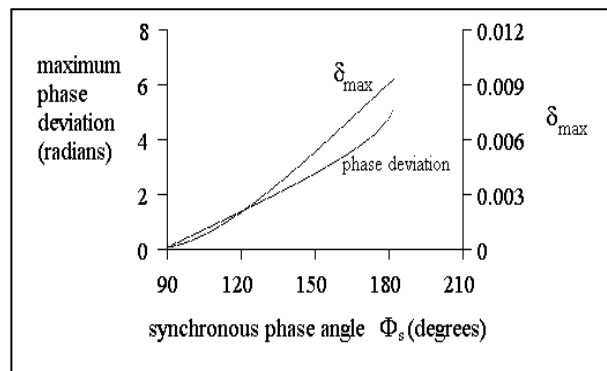


Figure 3: The variation of maximum momentum deviation and maximum phase deviation with the synchronous phase angle.

maximum phase deviation increases with the synchronous phase angle and is shown in Fig.3.

### 3.4 Radiation Damping of the Synchrotron Oscillation

The radiation damping time is calculated and the obtained value is 8.19ms. With damping, the size of the ellipse decreases and phase space trajectory is an inward spiral shown in Fig.4. The centre of the spiral motion represents the synchronous electron or the center of the bucket. Radiation excitation is not considered in the calculation. Therefore synchrotron oscillation damps continuously.

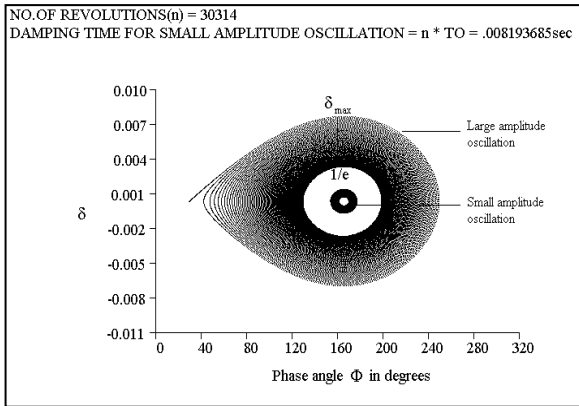


Figure 4: Radiation damping of the synchrotron oscillation for the small amplitude and large amplitude.

### 3.5 Damping of Vertical Betatron Oscillation

As explained above, a program has been developed to study the radiation damping of the vertical betatron oscillation in the phase space. The calculated damping time is 16.96ms. The damped betatron oscillation in the phase space makes the inward spiral as shown in Fig.5. In the present case, damping time calculation is done for the ideal situation by neglecting the magnet imperfections. The damping time is taken as the time at which the amplitude of the oscillation is reduced to 1/e from its maximum value.

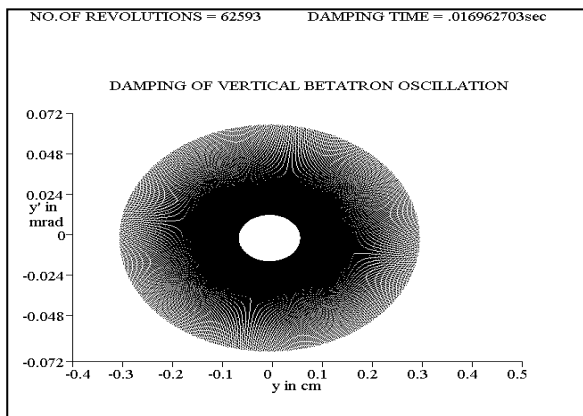


Figure 5: Damping of vertical betatron oscillation.

### 3.6 Damping of Horizontal Betatron Oscillation

The damping time for the horizontal betatron motion has been calculated in the same way as explained in the case of vertical betatron oscillation. The amplitude damping of horizontal betatron oscillation in phase space is shown in Fig.6. The damping time is found as 17.038msec(maximum). It has been observed that variation of damping time is very much dependent on the initial values of the particle position i.e.,  $x_0$  and  $x_0'$ . Dispersion effect is not included in the calculation.

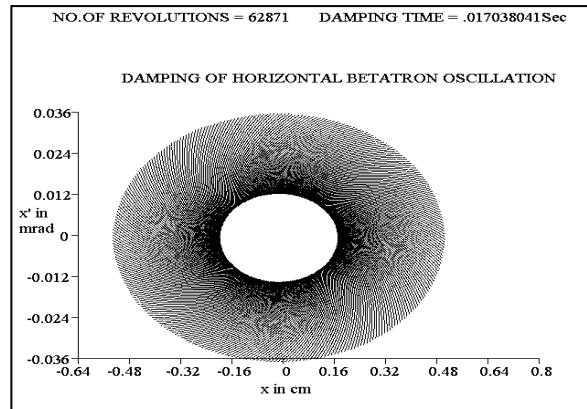


Figure 6: Damping of horizontal betatron oscillation

## 4 DISCUSSION

The results are in agreement with the values calculated analytically. It has been observed that the synchrotron oscillation frequency varies inversely with the  $\delta_{max}$  and the maximum phase deviation and maximum momentum deviation increases linearly with the synchronous phase angle. These are in agreement with the theory. The damping time for the betatron oscillation is calculated with the assumption that energy loss during one revolution is constant. In case of the horizontal betatron oscillation the effect of dispersion is not taken into account. These are the factors that could have caused error in the calculation of the damping time.

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