BEAM BREAKUP INSTABILITY SUPPRESSION IN MULTI CELL SRF GUNS

V. Volkov#, BINP, Novosibirsk, Russia,

J. Knobloch, A. Matveenko, Helmholtz-Zentrum Berlin, Germany

Abstract

Beam breakup (BBU) instability analysis in superconducting multi cell cavities [1] is extended to FZD like RF guns. Also, the BBU instability was studied numerically on the basis of the dipole HOM analysis and transverse beam dynamics. An effective way to suppress the BBU instability using TE mode RF focusing is presented and proved to be technically feasible. In that way, multi cell SC cavities in powerful RF guns with stable high currents are found to be realistic.

INTRODUCTION

The BBU instability in linear accelerators and multi-cell cavities develops when the beam interacts with cavity dipole HOMs. This effect becomes particularly significant at low particle energies [1]. The existing dipole HOM coupling impedance theory [2] with a thin cavity model cannot correctly predict the BBU threshold current in multi cell cavities and only gives a rough description of this effect in ERL accelerators. In the article we try to analyse the actual beam-dipole HOM interaction in the case of a FZD like multi cell RF gun cavity.

DERIVATION OF BBU INSTABILITY FEATURES

In further computations we treat the beam as a series of point like bunches periodically appearing in the cavity. The beam current is a series of delta functions with infinite amplitudes and defined charges. Such a current is the sum of an infinite number of Fourier harmonics with equal amplitudes of twice the average current ($I_i=2I$). Since we operate with the average current, we must use the formulas for DC current and use double harmonic impedances ($R=2R_i$) to keep the voltage and power unchanged, i.e. $U_i=I_iR_i=2IR/2=IR=U$, $P_i=U_i^2/2R_i=U^2/R= =P$.

Brief Review of the Classical Theory of Coupling Impedances

The main approximation of the coupling impedance theory is visualised in Figure 1. It is for a straight line bunch trajectory having a Y offset inside the cavity (thin cavity approximation). At the cavity exit the bunch receives a kick in transversal and longitudinal directions characterized by transversal momentum (ΔPc) and longitudinal (ΔU) energy changes that depend on RF phase of the dipole HOM (φ):

$$\Delta U = U_{II} \cdot \cos(\varphi) \tag{1}$$

$$\Delta Pc = P_{\perp} c \cdot \sin(\varphi) \tag{2}$$

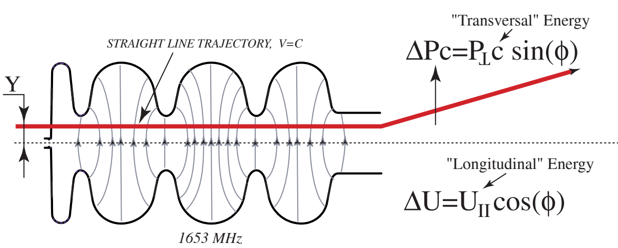

Figure 1: The demonstration of classical theory of coupling impedances on a FZD like RF gun cavity.

The stored energy of the dipole HOM RF field (J) is proportional to field amplitude squared $J\propto B^2$, and the energy change is proportional to the particle offset and the field amplitude $U_{II}\propto Y\cdot J^{1/2}$.

The excitation of the dipole HOM by the beam current (I) is characterized by longitudinal (R_{II}) and transversal ($R_{\perp}$) coupling impedances. Due to the energy conservation, the energy change of the mode is the same as energy change of the beam:

$$U_{II} = I \cdot R_{II} \tag{3}$$

$$P_{\perp} c = I \cdot R_{\perp} \tag{4}$$

These impedances are calculated numerically using the energy conservation law. The excited RF power is equal to the power dissipated in the cavity wall:

$$U_{II}^2 / R_{II} = \omega J / Q \tag{5}$$

$$(P_{\perp} c)^2 / R_{\perp} = \omega J / Q, \tag{6}$$

where Q is the quality factor of the dipole HOM, ω is its circular resonance frequency. From these equations the impedances are calculated numerically by integrating the dipole mode field along the straight line to find the energy change:

$$R_{II} / Q = U_{II}^2 / \omega J \tag{7}$$

$$R_{\perp} / Q = (P_{\perp} c)^2 / \omega J \tag{8}$$

These impedances are coupled by the Panofsky-Wenzel theorem

$$R_{\perp} = R_{II} \cdot (P_{\perp} c / U_{II})^2 = R_{II}\, c / \omega Y \tag{9}$$

It should be noted, that these impedances are calculated at constant trajectory offset and expressed in Ohms. In practice (for Fourier beam current harmonics) offset independent impedances are usually used, $R_{II}/2Y^2$ [Ohm/m^2] and $R_{\perp}/2Y$ [Ohm/m].

#V.N.Volkov@inp.nsk.su

In Figure 2 the equivalent electric circuit of such a beam-dipole mode interaction is depicted.

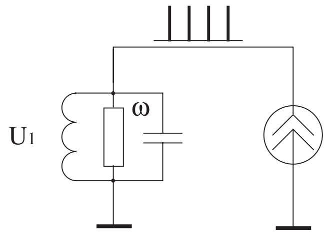

Figure 2: The equivalent electric circuit of classical beam-dipole mode interactions.

The Source of BBU Instability in Multi Cell Cavities

The actual bunch trajectory in a cavity is not a straight line as shown in Fig.3. Trajectory oscillations are the source of BBU instability in a cavity. Due to these oscillations, as it is analytically proved in [1], the actual energy dependence (1) has three components:

$$\Delta U(\varphi) = U_0 + U_1\cos(\varphi) + U_2\cos(2\varphi + \Phi). \quad (10)$$

Here, the components U_0 and U_2 are proportional to the stored energy J of the dipole HOM and are offset independent. The independent component U_1 is the same as in (1) but the integration should be made along the curve given by the focusing field trajectory as it is shown in Fig.3 (blue colour), i.e. at zero dipole HOM amplitude.

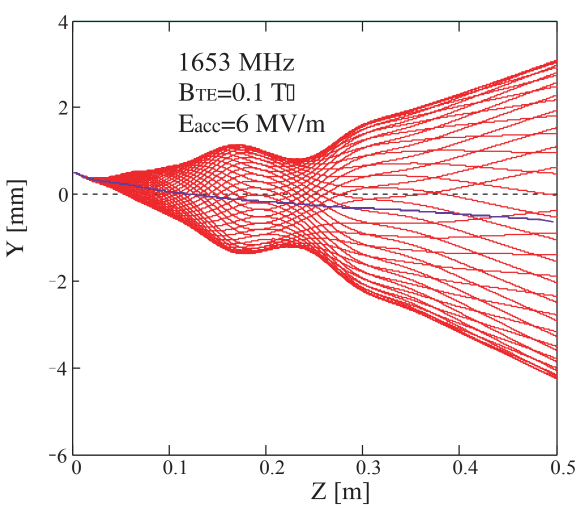

Figure 3: The actual trajectories in the RF gun cavity with 1653 MHz dipole (B_{max}=0.001T), fundamental 1300 MHz mode (E_{acc}=6 MV/m), and focusing TE 3781 MHz (B_{max}=0.1T) mode.

The energy change (see Fig.4, table 1) numerically calculated according to the algorithm described in [1] fits equation (10) with an accuracy of about 1%. All static-like focusing fields (such as the accelerating mode and focusing TE mode fields) do not change the form of equation (10) as it is analytically proven in [1].

We have to remind, Eq.10 is obtained due to linear approximated RF fields near the axis (dipole and all focusing fields are linear on Y and r).

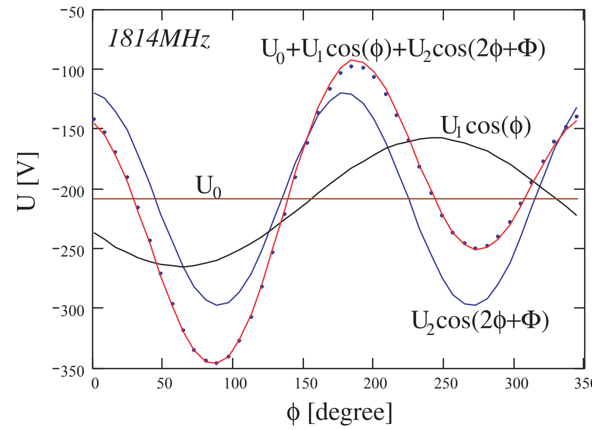

Figure 4: Numerically calculated voltage components of Eq. (10) for dipole 1814 MHz HOM

Table 1: Numerically calculated voltage components of Eq. (10) for E_{acc}=25 MV/m. B is maximal axis dipole field.

N	*F* MHz	U_0/B^2 V/ mT²	$U_1/Y/B$ V/mm /mT	U_2/B^2 V/ mT²	*Φ* degr.	Acc. %
1	1653	223	11.4	96	-183	1.4
2	1724	80	218	723	80	1.0
3	1766	-11	622	249	-17.8	0.9
4	1814	-209	108	89	126	1.2
5	1864	390	150	226	47.5	1.3
6	1873	91.4	26.3	91	-457	1.3
7	1887	-0.3	146	91	23.3	1.0
12	2699	182	58.3	112	139	1.3
20	3159	-23	263	27	-97.5	0.6
37	4466	-6	543	100	-100	0.9
89	6709	4	846	3	-87.6	0.3

The equivalent electric circuit is depicted in Fig.5. Here, the second resonance oscillating contour has double resonance frequency 2*ω*. A series-oscillating circuit has *ω* resonance frequency. The RC part of the circuit describes the interaction with the average beam current component.

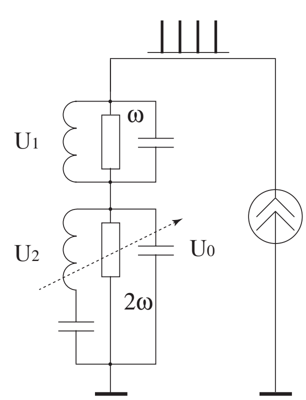

Figure 5: The equivalent electric circuit of actual beam-dipole mode interactions.

The threshold current is expressed by

$$I_{th}Q = \omega J/(-U_0 + U_2), \quad (11)$$

where $-U_0 + U_2$ is the maximum energy loss of a bunch at zero trajectory offset or U_1=0. These voltages were numerically calculated for each dipole HOMs at some stored RF energy J.

It should be noted, that equation (2) for the transversal momentum kick remains the same as it's form is a trajectory independent feature.

Approximately half of all dipole HOMs can be unstable. This fact could be explained by different phase delays in cavity cells for different HOMs designed in a simple thin cavity model [2]. Threshold currents in this model will be similar to (11) if we replace $(R/Q)_d$ in [2] with our definition $(R/Q)_d = (R_{II}/Q)/(\omega/c)^2$. The threshold current becomes:

$$I_{th}Q = \frac{-2\omega p}{qc(R_{II}/Q)m^* \sin(\omega T_r)} \quad (12)$$

Phase delays ωT_r for each HOM are unique and approximately half will have a negative sinusoidal component.

Derivation of Coupling Impedance Formulae

The coupling impedances (7) and (8) may depend on the beam current for high currents since the excited dipole HOM fields will change the trajectory.

According to (7), the coupling impedance at low beam current will have a component independent of the beam current:

$$R_1/Q = U_1^2/\omega J\ . \quad (13)$$

To estimate the coupling impedance at a higher beam current we need to formulate

$$\begin{aligned} R_{II}/Q &= \min \Delta U(\varphi)^2/\omega J \\ &\le (-U_0 + U_1 + U_2)^2/\omega J \\ &= (R_1/Q)\left(1 + \frac{-U_0 + U_2}{U_1}\right)^2 \end{aligned} \quad (14)$$

Since dissipated power must equal the excitation, we can conclude

$$\begin{aligned} \omega J/Q &= -\min \Delta U(\varphi) I \\ &\le (-U_0 + U_2 + U_1) I \\ &= (\omega J/Q)(I/I_{th})(1 + U_1/(-U_0 + U_2)) \end{aligned} \quad (15)$$

The combination of (14) and (15) gives

$$R_{II}/Q \le \frac{R_1/Q}{(1 - I/I_{th})^2} \quad (16)$$

Since the transversal momentum change in dipole mode fields is proportional to the field amplitude B or the square root of the stored RF energy $J^{1/2}$

$$P_\perp c = A(\omega J)^{1/2}, \quad (17)$$

where the constant A is calculated numerically using (17) for a given J.

Replacing $P_\perp c$ in (17) with (4) and $\omega J/Q$ with $I^2 R_{II}$ we obtain

$$R_\perp/Q = A(R_{II}/Q)^{1/2} \quad (18)$$

An estimation of the transversal coupling impedance with (16) is then

$$R_\perp/Q \le \frac{A(R_1/Q)^{1/2}}{|1 - I/I_{th}|} \quad (19)$$

SIMULATION RESULTS OF FZD LIKE RF GUN CAVITY

The geometry of FZD like RF gun cavity is shown in Fig.1. In table 2 the calculated threshold currents multiplied with the quality factors are presented for a selected of dipole HOMs. Approximately half of them can be unstable. The focusing by TE 3781 MHz mode suppresses some of them but there remain dipole modes that can not be suppressed. This is due to the absence of noticeable focusing by TE field in the last two cells of the cavity since the TE 3781 MHz mode has the field maximum in the first cell only [3].

To avoid this problem a specially shaped RF gun cavity design is applied. In this cavity each of the three full cells must have equal resonance frequencies for both the fundamental and the focusing TE modes. The example of such a cavity is depicted in Fig.6 presenting TE focusing π-mode with equal amplitudes in each cell.

Table 2: Threshold currents multiplied with quality factors at different focusing strengths B` for the focusing 3781 MHz mode. Eacc=25 MV/m

N	F MHz	$IQ_{B`=0.0}$ A	$IQ_{B`=0.1}$ A	$IQ_{B`=0.2}$ A	$IQ_{B`=0.3}$ A
1	1653				
2	1724				
3	1766	1.7e+6			
4	1813	3.2e+4	3.5e+4	4.4e+4	6.7e+4
5	1864				
6	1873				
7	1887	1.2e+7	1.2e+6	3.4e+5	1.9e+5
12	2699				
20	3159	5.2e+5	3.5e+5	1.9e+5	1.3e+5
37	4466	2.5e+6			
89	6709				3.3e+6

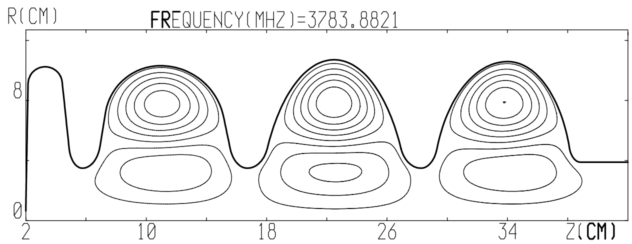

Figure 6: TE focusing π-mode in the specially shaped cavity. TE_{021} mode magnetic force lines are depicted.

Table 3 demonstrates the effect of HOM loading with the insert described in [4]. There the threshold currents become large for untrapped modes. All higher quality trapped modes turn out to be stable with the exception of 4466 MHz that only becomes stable due to TE focusing (see table 2).

In the table 4 the calculated transversal impedances for Fourier components of a low beam current ($R_1/Q/2Y$) at different focusing TE field strengths are presented. CLANS calculated impedances (at B`=0) according to classical theory are also shown for comparison.

Table 3: Threshold Currents at E_{acc}=25 MV/m with and without the Insert

N	f	Without Insert		With insert	
		I_{th}	Q_{ext}	I_{th}	Q_{ext}
1	1653		6.20e+7		1.95e+5
2	1723		1.77e+7		1.32e+5
3	1765	215	7.71e+6	20746	7.97e+4
4	1814	33.4	9.72e+5	2238	1.45e+4
5	1864		4.65e+8		1.39e+7
6	1873		2.06e+8		2.00e+8
7	1887	417	8.06e+7	18147	1.85e+6
12	2698		6.73e+8		1.49e+9
20	3158	663	7.72e+5	138	3.70e+6
37	4466	0.46	4.61e+9	7.3	2.94e+8
89	6707		2.83e+4		2.43e+4

Table 4: Transversal Impedances for E_{acc}=25 MV/m

N	**f**	**$R_1/Q/2Y$ [Ω/cm]**				
		CLANS	**B` [T]**			
		B`=0	**0**	**0.1**	**0.2**	**0.3**
1	1653	0.84	0.98	3.03	2.91	2.73
2	1723	2.41	7.22	6.12	9.36	9.52
3	1765	19.4	6.24	6.84	6.75	5.80
4	1814	3.71	4.22	7.37	12.3	7.68
5	1864	2.89	2.58	5.17	4.47	9.34
6	1873	1.00	1.00	0.88	1.87	3.61
7	1887	0.57	0.88	0.82	1.68	1.04
12	2698	2.08	1.56	2.44	4.65	4.4
20	3158	0.51	1.08	1.66	2.29	1.84
37	4466	0.12	0.04	0.21	0.65	0.81
89	6707	0.75	0.03	0.03	0.16	0.17

CONCLUSION

The analysis of BBU instability in SRF guns reveals the unstable dipole HOMs and its associated threshold current. The formulae for coupling impedances of dipole HOMs depending on the beam current are derived. The effect of the threshold current increase for untrapped dipole HOMs through coupling with the external load insert is shown. Also, the suppressing of BBU instability by TE focusing is demonstrated. The possibility of TE focusing in all cavity cells is predicted.

ACKNOWLEDGMENTS

The authors are thankful to T. Atkinson and other colleagues from Helmholtz-Zentrum for interest in this work, for his important comments and a lot of number corrections in this work.

REFERENCES

[1] V. Volkov, Phys. Rev. ST Accel. Beams **12**, 011301 (2009).

[2] E. Pozdeev et al., Nuclear Instruments and Methods in Physics Research, A 557 (2006), 176-188.

[3] V. Volkov, K. Floettmann, D. Janssen. Superconducting RF gun cavities for large bunch charges. PAC07, 2007.

[4] “Dipole high order mode analysis in FZD like SRF guns”, in these proceedings.