SUPERCONDUCTING RF CAVITY MEASUREMENT FORMULAE FOR AN EXPONENTIALLY DECAYED PULSE INCIDENT POWER

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Abstract

Experimental method to evaluate the performance of a superconducting RF (SRF) cavity is through low power and high power measurements without a beam load. The most popular formulae in a pulsed mode measurement are for square-wave incident power. In practice, incident power may not be exactly squared. To understand the cavity behavior and performance more accurately, in this paper, the SRF cavity's measurement equations for an exponentially decayed and pulsed incident power are developed from a series equivalent LCR circuit. The analytical result can be directly compared with the experimental data of a SNS cavity obtained from the Cryomodule Test Facility (CMTF) at Jefferson Lab.

INTRODUCTION

RF superconductivity is an increasingly important branch of accelerator physics and technology because of its high accelerating performances and low operating cost. The low power and high power RF measurements, when after the cavities were assembled in cryomodule and before a beam is run through, are necessary steps to estimate the SRF cavity performance. At present, one-port cavity's measurement equations are the most popular used for a pulsed incident power.

The stability of the phase and amplitude during the flat top of the accelerating pulse are very important for the pulsed mode linear accelerators. To understand the cavity behavior or to control cavity's phase and amplitude more accurately during the cavity's measurements or during the beam operation, a set of two-port cavity's measurement equations has been developed by using a series equivalent LCR circuit [1].

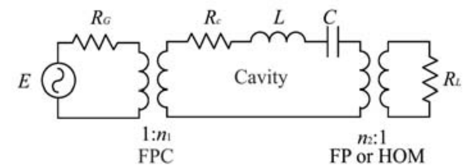
In practice, the incident power wave may not be exactly squared pulse due to the capacitor discharge in the klystron PFN, instead be an exponential decayed pulse. To understand cavity behavior and performance more accurately, in this paper, the cavity's equations for an exponentially decayed and pulsed incident power are developed further.

EQUIVALENT CIRCUIT MODEL

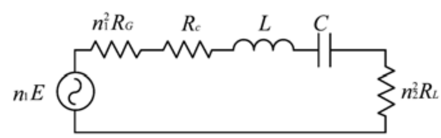
A two-port cavity with input and output couplers can be equivalent to either a parallel or a series resonance circuit. The series circuit is comparatively simpler to derive for cavity equations when no beam loading, as shown in Fig. 1. Assuming the impedance of source and load are real and given by R_G and R_L, and R_c, L, C are the resistance, inductance and capacitance of the SRF cavity. E is the equivalent generator voltage related to the incident power P_{in} with a frequency of ω. In high frequency application, the harmonic time structure is much less than the pulse modulation scale by several orders (10^{-6} for SNS case). So the cavity accelerating gradient E_{acc} or voltage V_c can be related to the circuit current $I(\omega,t)$:

$$E_{acc}(\omega,t) = \frac{V_c(\omega,t)}{d} = \frac{|I(\omega,t)|}{d\omega C} \tag{1}$$

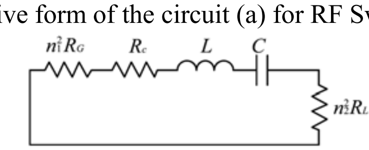
Here d is effective accelerating length.



(a) Equivalent circuit of a two-port cavity system.



(b) Alternative form of the circuit (a) for RF Switch On.



(c). Alternative form of the circuit (a) for RF Switch Off.
Figure 1: Equivalent circuits of a two-port cavity coupling system in transient state.

Normally, the cavity's parameters are defined at the cavity's resonance frequency ω_0. In following sections, the label of (t) is used to denote the physical quantity at ω=ω_0. Note that: $\omega_0^2 = 1/LC$, the cavity stored energy U is:

$$U(t) = L|I(t)|^2 = C|V_c(t)|^2 = |I(t)|^2 / \omega_0^2 C \tag{2}$$

The cavity's emitted power P_e, dissipated power P_d and transmitted power P_t are:

$$\begin{cases} P_e(t) = n_1^2 R_G |I(t)|^2 \\ P_d(t) = R_c |I(t)|^2 \\ P_t(t) = n_2^2 R_L |I(t)|^2 \end{cases} . \tag{3}$$

The cavity's intrinsic quality factor Q_0 is:

$$Q_0 = \omega_0 U(t)/P_d(t) = \omega_0 L/R_c = 1/(\omega_0 C R_c), \tag{4}$$

The external quality factor Q_e of the cavity fundamental power coupler (FPC) is:

$$Q_e = \omega_0 U(t)/P_e(t) = \omega_0 L/(n_1^2 R_G) = 1/(\omega_0 C n_1^2 R_G), \tag{5}$$

The external quality factor Q_t of the cavity field probe (FP) or a HOM coupler is:

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$$Q_t = \omega_0 U(t)/P_t(t) = \omega_0 L/(n_2^2 R_L) = 1/(\omega_0 C n_2^2 R_L)\,. \tag{6}$$

The cavity FPC coupling coefficient β_e and the FP coupling coefficient β_t are defined as:

$$\begin{cases} \beta_e = Q_0/Q_e = P_e/P_d = n_1^2 R_G/R_c \\ \beta_t = Q_0/Q_t = P_t/P_d = n_2^2 R_L/R_c \end{cases}, \tag{7}$$

The cavity loaded quality factor Q_L is:

$$Q_L = \frac{\omega_0 L}{R_c + n_1^2 R_G + n_2^2 R_L} = \frac{Q_0}{1+\beta_e+\beta_t} \tag{8}$$

The cavity (circuit) shunt impedance R is:

$$R = \frac{|V_c|^2}{P_d} = \frac{1}{R_c \omega_0^2 C^2} \text{ and } R/Q_0 = \frac{1}{\omega_0 C} = \omega_0 L \tag{9}$$

Normally the R/Q_0 is obtained by a cavity simulation code and written as R/Q. Based on above equations, the equivalent circuit parameters can be expressed with the cavity parameters as:

$$\begin{cases} C = \dfrac{1}{(R/Q)\omega_0} \\ L = \dfrac{(R/Q)}{\omega_0} \\ R_c = \dfrac{(R/Q)}{Q_0} \end{cases} \tag{10}$$

CAVITY MEASUREMENT FORMULAE

Generally a phase lock loop (PLL) is used in the measurements, so $\omega = \omega_0$. For open-loop condition, please refer to reference [1].

A. RF Switch On

For a pulsed incident power with an exponentially decay in the form of $P_{in}(t)=P_{in}\exp(-2\alpha t)$, here α is a constant, the equivalent voltage is:

$$E(t) = E_0 \exp[(-\alpha + i\omega_0)t]\,. \tag{11}$$

The circuit current $I(t)$'s differential equation is:

$$L\frac{d^2 I(t)}{dt^2} + R_T \frac{dI(t)}{dt} + \frac{I(t)}{C} = (-\alpha + i\omega_0) n_1 E_0 \exp[(-\alpha + i\omega_0)t] \tag{12}$$

Here $R_T = n_1^2 R_G + R_1 + n_2^2 R_L$.

Using the slowly-varying amplitude approximation [2] and the RF switch-On boundary condition, the solution of the above equation is:

$$I(t) = \frac{(-\alpha + i\omega_0) n_1 E_0 \exp(i\omega_0 t)}{Q_0 R_c \left[\left(\frac{\alpha^2}{\omega_0} - \frac{\alpha}{Q_L}\right) + i\left(\frac{\omega_0}{Q_L} - 2\alpha\right)\right]} \left\{\exp(-\alpha t) - \exp\left[-\frac{\omega_0}{2Q_L}t\right]\right\} \tag{13}$$

Note that, $P_{in}(t) = |E(t)|^2/4R_G$ from reference [3], the above equation (13) becomes:

$$|I(t)|^2 = \frac{4\beta_e(\alpha^2+\omega_0^2)\left[\exp(-\alpha t) - \exp\left(-\frac{\omega_0}{2Q_L}t\right)\right]^2}{R_c(1+\beta_e+\beta_t)^2\left[\left(\frac{Q_L\alpha^2}{\omega_0} - \alpha\right)^2 + (\omega_0 - 2\alpha Q_L)^2\right]} P_{in} \tag{14}$$

Then the cavity dissipated power $P_d(t)$ is:

$$P_d(t) = \frac{4\beta_e(\alpha^2+\omega_0^2)\left[\exp(-\alpha t) - \exp\left(-\frac{\omega_0}{2Q_L}t\right)\right]^2}{(1+\beta_e+\beta_t)^2\left[\left(\frac{Q_L\alpha^2}{\omega_0} - \alpha\right)^2 + (\omega_0 - 2\alpha Q_L)^2\right]} P_{in} \tag{15}$$

The transmitted power $P_t(t)$ is written as:

$$P_t(t) = \frac{4\beta_e\beta_t(\alpha^2+\omega_0^2)\left[\exp(-\alpha t) - \exp\left(-\frac{\omega_0}{2Q_L}t\right)\right]^2}{(1+\beta_e+\beta_t)^2\left[\left(\frac{Q_L\alpha^2}{\omega_0} - \alpha\right)^2 + (\omega_0 - 2\alpha Q_L)^2\right]} P_{in} \tag{16}$$

The cavity's transmission coefficient T is:

$$T = \frac{P_t(t)}{P_{in}(t)} = \frac{4\beta_e\beta_t(\alpha^2+\omega_0^2)\left\{1 - \exp\left[\left(\alpha - \frac{\omega_0}{2Q_L}\right)t\right]\right\}^2}{(1+\beta_e+\beta_t)^2\left[\left(\frac{Q_L\alpha^2}{\omega_0} - \alpha\right)^2 + (\omega_0 - 2\alpha Q_L)^2\right]} \tag{17}$$

The cavity's emitted power $P_e(t)$ is:

$$P_e(t) = \frac{4\beta_e^2(\alpha^2+\omega_0^2)\left[\exp(-\alpha t) - \exp\left(-\frac{\omega_0}{2Q_L}t\right)\right]^2}{(1+\beta_e+\beta_t)^2\left[\left(\frac{Q_L\alpha^2}{\omega_0} - \alpha\right)^2 + (\omega_0 - 2\alpha Q_L)^2\right]} P_{in} \tag{18}$$

At this state, the emitted power here has no physical meaning. The cavity's stored energy is:

$$U(t) = L|I(t)|^2 = \frac{Q_L(1+\beta_e+\beta_t)}{\omega_0} P_d(t) \tag{19}$$

Combining equations (1), (2) and (16)-(18), the cavity accelerating gradient can be written as:

$$E_{acc}(t) = \frac{\sqrt{(R/Q)Q_0 P_d(t)}}{d} = \frac{\sqrt{(R/Q)Q_e P_e(t)}}{d} = \frac{\sqrt{(R/Q)Q_t P_t(t)}}{d} \tag{20}$$

The reflected power $P_r(t)$ is:

$$P_r(t) = P_{in}\exp(-2at) - P_d(t) - P_t(t) - \frac{dU}{dt}(t) \tag{21}$$

B. RF Switch Off

After a pulse length of τ_0, the incident power P_{in}=0. This means the $E(t)$=0 as shown in Fig. 1 (c). Then the circuit differential equation becomes:

$$L\frac{d^2 I(t)}{dt^2} + R\frac{dI(t)}{dt} + \frac{I(t)}{C} = 0\,. \tag{22}$$

Using initial and $I(t\to+\infty)$=0 boundary conditions, and the slowly-varying amplitude approximation, the circuit current $I(t)$ can be solved:

$$I(t) = I(\tau_0)\exp\left[-\frac{\omega_0}{2Q_L}(t-\tau_0) - i\omega_0(t-\tau_0)\right] \tag{23}$$

$$|I(t)|^2 = |I(\tau_0)|^2 \exp\left[-\frac{\omega_0}{Q_L}(t-\tau_0)\right] \tag{24}$$

The cavity's dissipated power $P_d(t)$, transmitted power $P_t(t)$ and emitted power $P_e(t)$ are:

$$\begin{cases} P_d(t) = P_d(\tau_0)\exp\left[-\dfrac{\omega_0}{Q_L}(t-\tau_0)\right] \\ P_t(t) = P_t(\tau_0)\exp\left[-\dfrac{\omega_0}{Q_L}(t-\tau_0)\right] = \beta_t P_d(t) \\ P_e(t) = P_e(\tau_0)\exp\left[-\dfrac{\omega_0}{Q_L}(t-\tau_0)\right] = \beta_e P_d(t) \end{cases} \tag{25}$$

The stored energy change is:

$$\frac{dU}{dt}(t) = -\frac{\omega_0 L}{Q_L}\left|I(\tau_0)\right|^2 \exp\left[-\frac{\omega_0}{Q_L}(t-\tau_0)\right] = -P_d(t) - P_e(t) - P_t(t) \tag{26}$$

The reflected power now becomes emitted power which can be measured and has the physical meaning:

$$P_r(t) = -P_d(t) - P_t(t) - \frac{dU}{dt}(t) = P_e(t) = \beta_e P_d(t) \tag{27}$$

According above results, the E_{acc} is:

$$E_{acc}(t) = E_{acc}(\tau_0)\exp\left[-\frac{\omega_0}{2Q_L}(t-\tau_0)\right] \tag{28}$$

From equation (25) and (27), we found

$$\log P_e(t) = \log P_t(t) + \log(\beta_e/\beta_t) = -k(t-\tau_0) + b \tag{29}$$

Here $k=\log(e)\cdot(\omega_0/Q_L)=0.4343\omega_0/Q_L$, $b=\log(Q_tP_t(\tau_0)/Q_e)$ are constants. This equation can be used to exactly measure the loaded Q_L at pulse mode by fitting the slop of equation (29). We can also measure the ratio of emitted power P_e and transmitted power P_t after the RF switch off:

$$\frac{P_e(t)}{P_t(t)} = \frac{\beta_e}{\beta_t} = \frac{Q_t}{Q_e} = K^2 \tag{29}$$

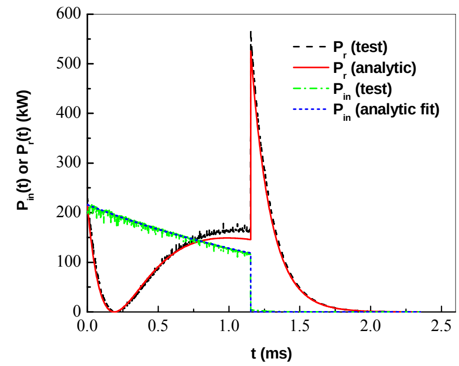
If the measurement circuit is carefully calibrated, the K constant can be used to approximate the FP coupling:

$$Q_t = Q_L K^2 \tag{30}$$

The incident power used to measure the SNS cavities at the CMTF in JLab was a decayed pulse. After fitting it into the equation (11), we fund the decay rate is $\alpha=1/3.84$ $(\mathrm{ms})^{-1}$. The measurement data used in this analysis were taken from the medium beta No. 8 cryomodule, cavity #2 measured at 9:36AM on April 8, 2004. Substituting this α and P_{in}=216kW into equations (16), (20), (21), (25), (27), and (28), we find that the analytic results of the reflected, transmitted power can fit the test dada very well as shown in Fig.2. The discrepancy in the reflected power near the end of the RF pulse could be a clue of the Lorentz force detuning which this analysis doesn't include. The major discrepancy is on the E_{acc}, the analytical data is about 3.7% lower than the measured data at the top. The test data was calculated online in the data acquisition system using the Labview, in which the one-port measurement equations were implemented [4].

CONCLUSION

The two-port RF cavity's equations, developed by a series LCR equivalent circuit, can accurately describe the cavity electric behaviors under pulse incident power mode. These equations can be used to measure some SRF cavity's parameters, except cavity instinct quality factor Q_0 when the FPC is heavily over-coupled. The two-port equations can be simplified into one-port cavity equations. The cavity stored energy change dU/dt is the cause of the reflected power, emitted power and transmitted power transients. A general case of measurement formulae with exponential decayed incident power has been developed and could be used in the CMTF type of experiments. A special case is interesting. When $\alpha=\omega_0/(2Q_L)$, the incident power decay is just fast enough that the cavity will behave like detuned, no RF power can be input into the cavity. Although a faster decay than this rate can make a difference, the power decay in these rates could not happen in practice.



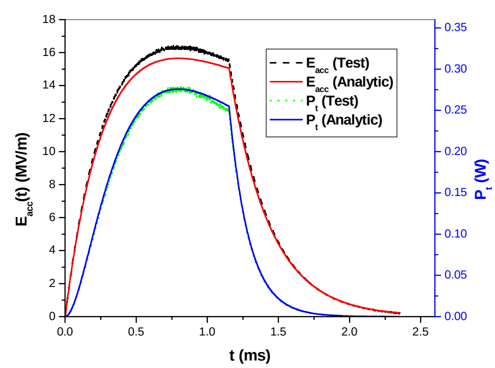


Figure 2: SNS medium beta cavity M082 test data in JLab CMTF and comparison with the analytic fitted data. The E_{acc}(Test) was calculated by one-port measurement equations.

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