STRUCTURAL ANALYSIS OF SINGLE CELL SUPERCONDUCTING ELLIPTICAL CAVITY WITH STATIC LORENTZ FORCE

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Abstract

A single cell $\beta_g = 0.42$, 700 MHz superconducting elliptical cavity has been designed for high current proton accelerator [1]. The structural behavior of a single cell superconducting elliptical cavity has been studied by finite element structural analysis. The Static Lorentz Force Detuning of the cavity has been studied numerically with SUPERFISH code at an accelerating gradient of 5 MV/m and 10 MV/m. The calculated maximum Lorentz forces for 5 MV/m gradient are 302 Pa acting outward near the cell equator and 742 Pa acting inward at the cell bore. Cavity shape deformations are calculated both for copper and niobium with different material thickness. For 3 mm Copper at 5 MV/m gradient the calculated frequency shift was -243 Hz while for 5 mm copper it reduced to -86 Hz. When Niobium is concerned 3 mm thickness gives -212 Hz frequency shift and 5 mm thickness has a frequency shift of -75 Hz. For 5 mm Niobium at 10 MV/m gradient the frequency shift was -345 Hz. Three dimensional finite element models were used to determine the cavities mechanical resonant frequencies. The lowest frequency observed for 5 mm copper was 81 Hz when one end of the beam tube was held rigidly fixed in all coordinates and other beam tube free. The lowest frequency increased to 281 Hz when both end of the beam tube were kept fixed. The cavity shape was found to meet all design requirements without an annular stiffener if fabricated from 5 mm thick material operated at 5 MV/m accelerating gradient. Stiffener will be required if operated more than 5 MV/m accelerating gradient.

INTRODUCTION

A high intensity proton superconducting (SC) linac with several mA beam current and an accelerating energy of 1 GeV requires a structurally stable Niobium or Niobium coated on Copper RF accelerating structure. The frequency bandwidth of such cavities are very narrow typically a few hundreds of Hertz. Due to high frequency of these cavities, a variation of $1\ \mu m$ of cavity dimension could induce a frequency shift of several tens of Hertz [2]. SC accelerators for electron ($\beta = 1$) and heavy ions have been developed and operated successfully. But SC cavity shape of proton linac especially for $\beta < 0.5$ has some difficulties with the structural strength [3]. We have carried out these studies including consideration of static Lorentz force detuning, vacuum loading and mechanical resonant frequencies.

STRUCTURAL ANALYSIS FOR VARIOUS SHAPE PARAMETERS

The structural behavior of the single cell elliptical cavity has been studied by finite element structural analysis for various shape parameters. These analyses were done using COSMOS/M [4] with three-node shell elements. The max. Von MISES stress has been calculated under the vacuum load in the conditions of both end of the beam tube fixed or one end longitudinal direction free. The cavity material was niobium; both 3 mm and 5 mm thickness are considered. The dependence of the max. Von MISES stress on the shape parameters are listed below.

- Dependence on the equator ellipse.

  The Max. Von MISES Stress is minimum for round shape equator; it increases slightly for elliptical equator. So round shape equator is preferable.
- Dependence on wall angle.

  In the case of one end of the beam tube free the Max. Von MISES Stress is significantly decreased by increasing wall angle but in the case of both ends fixed, the Max. Von MISES Stress dose not depend so much on wall angle. So a large wall angle is preferable form structural analysis point of view. But the peak electric field also increases with wall angle [1]. So from RF point of view lower wall angle is preferable.

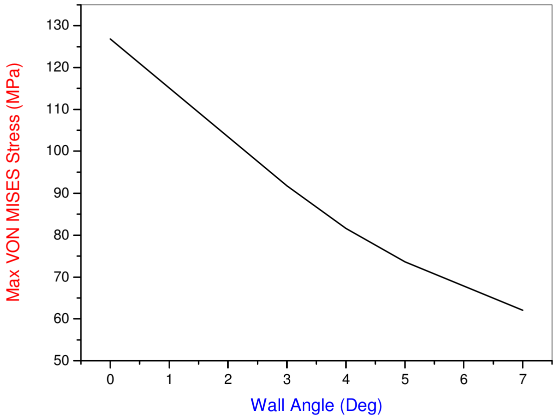

Figure 1. Max. Von MISES Stress for 3 mm Niobium with various wall angle

- Dependence on the iris ellipse.

  The Max. Von MISES Stress dose not vary so much.

- Dependence on the iris radius.
  The Max. Von MISES Stress decreases significantly with increasing iris radius when one end of the beam tube is free. In the condition of both end of the beam tube fixed the Max. Von MISES Stress decreases slightly with increasing iris radius. So a larger iris radius is preferable. This competes with the RF characteristics. With large iris radius peak electric field and power dissipation increases [1]. Again high current proton acceleration requires larger iris radius.

Since the yield stress of Niobium after the heat treatment at about 700 deg C is considered to be 70~100 MPa [2], additional stiffening structure or a thickness of 5 mm is required.

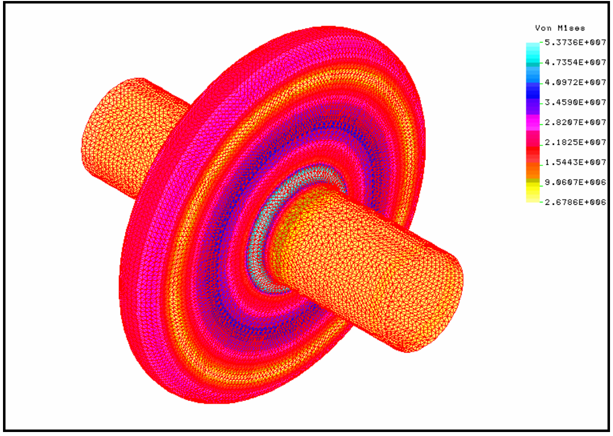

Figure 2. Stress distribution in a 3 mm Niobium cavity without stiffener.

We have also studied the effect of conical stiffener at various radiuses for 3 mm material thickness. It has been observed that the Max. Von MISES Stress and the resultant displacements of the cavity reduced considerably at 11.8 cm stiffener radius.

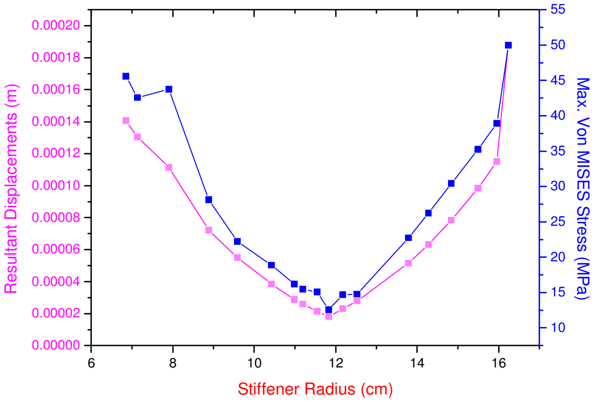

Figure 3. Stiffener radius optimization for 3 mm Niobium Cavity.

The resonant frequency shift due to vacuum load has been calculated from the frequency sensitivity data of SUPERFISH code. For 3 mm Niobium cavity the calculated frequency shift was 306 Hz/millibar and for 5 mm Niobium it is 74 Hz/millibar.

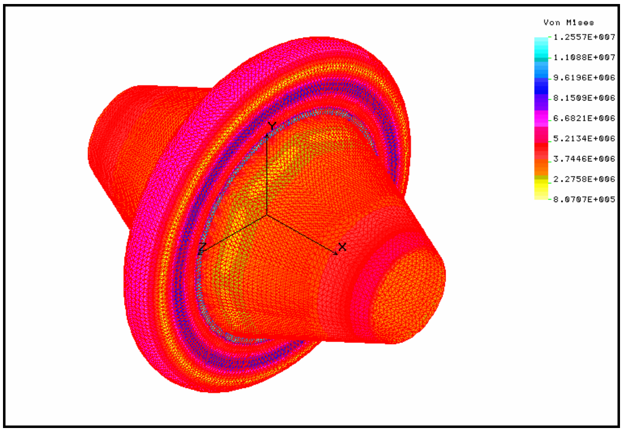

Figure 4. Stress distribution in a 3 mm Niobium cavity with conical stiffener.

STATIC LORENTZ FORCE DETUNING ANALYSIS

RF power in the cavity produces radiation pressure that act on the cavity wall. The pressures are a function of the surface electric and magnetic fields as given by the equation [6]

$$P = \frac{1}{4}(\mu_o H^2 - \varepsilon_o E^2)$$

These pressures deform the cavity wall, tending to act outward near the equator and inward near the iris. The cavity deformations produces a frequency shift depends quadratically on accelerating field E_{acc}. By assuming linearity between the deformation and detuning we can right.

$$\Delta f = K_L E_{acc}{}^2$$

Where K_L is the Lorentz Force detuning coefficient.

For a CW machine, the cavities can simply be detuned to compensate the static frequency shift.

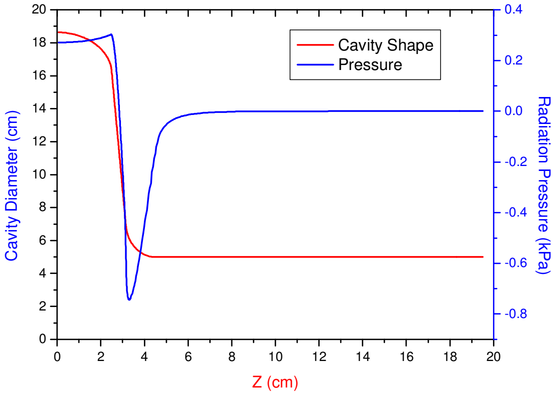

Figure.5. Radiation pressure computed from SUPERFISH code at 5 MV/m accelerating gradient.

SUPERFISH is used to compute the radiation pressure for each mesh element. The finite element code COSMOS/M computes the displacements for each mesh elements. Two-dimensional axi-symmetric elements [5] were used to compute the displacements due to Lorentz force at 5

MV/m and 10 MV/m accelerating gradient. The frequency shifts were determined from the output of SUPERFISH code. Deformation and frequency shifts were calculated both for Copper and Niobium as cavity material.

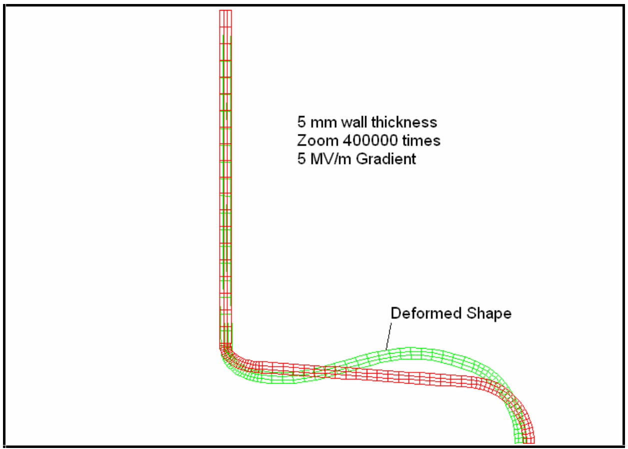

Figure 6. Lorentz Pressure Deformation for 5 mm wall thickness.

The calculated frequency shifts due to static Lorentz force are listed in the table below.

Table 1: Static Lorentz Force Detuning.

Material	**Wall Thick**	Δf **(Hz) At 5 MV/m**	Δf **(Hz) At 10 MV/m**	K_L **Hz/ MV/m**
Copper	3 mm	-243	-1159	-10.7
Niobium	3 mm	-212	-1007	-9.3
Copper	4 mm	-135	-625	-5.8
Niobium	4 mm	-118	-545	-5.1
Copper	5 mm	-86	-396	-3.7
Niobium	5 mm	-75	-345	-3.2

DYNAMIC ANALYSIS OF SINGLE CELL CAVITY

Cavities mechanical resonant frequencies were calculated for two different boundary conditions. In one condition both end of the beam tubes were held rigidly fixed in all coordinates. In another case one end of the beam tube was kept fixed while other end free in all coordinates. Four-node shell elements were used to determine the mechanical resonant frequencies. Results of these analyses are listed in table 2. Addition of an annular conical stiffener increases the lowest mechanical resonant frequency. These analyses were done for simple cavity without power couplers, HOM couplers and stiffness of the cavity support structures. Inclusion of any or all of these items will reduce the mechanical resonant frequencies [5].

Table 2: Mechanical Resonant frequencies of the single cell cavity for various wall thicknesses.

Material	**Wall Thick**	**Lowest Frequency**		
		Both end Fixed Hz	**One end Free Hz**	**One end Free with Conical stiffener Hz**
Copper	3 mm	212	61	130
	4 mm	245	71	153
	5 mm	281	81	172
Niobium	3 mm	231	66	141
	4 mm	267	77	166
	5 mm	305	88	187

CONCLUSIONS

Structural analysis have been done for a single cell $\beta_g = 0.42$, 700 MHz superconducting elliptical cavity with attach beam tube. Structural properties of Niobium is slightly better than Copper. Frequency change due to static Lorentz force is quite low for 4 mm and 5 mm wall thickness. For 3 mm wall thickness stiffener is essential. The cavity shape was found to meet all design requirements without an annular stiffener if fabricated from 5 mm thick material operated at 5 MV/m accelerating gradient. Stiffener will be required if operated more than 5 MV/m accelerating gradient.

ACKNOWLEDGEMENT

Authors would like to thank Dr. R. C. Sethi, Head APPD and Dr. A. K. Ray, Director, Beam Technology Development Group, BARC for their keen interest in this programme.

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