RETRIEVAL OF EFFECTIVE PARAMETERS OF METAMATERIALS FOR ACCELERATOR AND VACUUM ELECTRON DEVICE APPLICATIONS *

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Abstract

In this paper, we propose a new method to retrieve the effective material parameters, *i.e.*, the effective permittivity and permeability. We first obtain the effective permeability analytically and then retrieve the effective permittivity numerically from the structure's dispersion curve. This method is different from that one which is based on the scattering parameters for metamaterial slabs. The approach presented here offers an effective parameter retrieval for metamaterial-based accelerator and vacuum electron device applications.

INTRODUCTION

Metamaterials are artificial electromagnetic materials comprised of sub-wavelength elements. Metamaterials offer many new options for the design of components that use electromagnetic materials. A Double-Negative Metamaterial (DNM) has both negative permittivity and negative permeability and can be described by an effective medium theory [1, 2]. We investigate metamaterials at microwave, millimeter wave, and THz frequencies for application as accelerator structures, as interaction circuits of high-power microwave vacuum electron devices, and as beam diagnostics tools.

The present parameter retrieval method based on the scattering parameters (S_{11} and S_{21}) [3] is not suitable for the metamaterial-loaded waveguide case due to the waveguide dispersion and the anisotropy of the metamaterials as well as the complex mode. Up to now, the parameter retrieval method is only suitable for metamaterial slabs interacting with TEM mode in free space.

In this paper, we propose a metamaterial-loaded rectangular waveguide which is described by the effective medium theory and provide a method for the retrieval of the effective material parameters from the dispersion relation. The method presented here can offer a theoretical foundation for developing novel metamaterial-based accelerators and high-power vacuum electron devices [4]

RETRIEVAL METHOD

A square waveguide operating below the cut-off frequency which is loaded by split-ring resonators (SRRs) was first proposed by Marques et al. [5] The empty waveguide can be considered as a negative permittivity medium only when the TE mode is excited, and the SRR array forms the effective negative permeability medium. As stated in [6], when metallic rods or stripes are loaded in the transverse direction of the waveguide operating below the cut-off frequency, the TM mode can be excited. Since TM modes are of interest in accelerator and vacuum electron device applications, we propose to use a rectangular waveguide loaded by complementary split-ring resonators (CSRRs) as shown in Fig. 1 (a). As opposed to SRRs on a dielectric in a waveguide where dielectrics can easily induce breakdown or damage [5], a CSRR-loaded waveguide is also completely composed of metallic elements and thus suitable for high-power applications.

The CSRR array can be characterized by an effective negative permittivity in the y direction. In addition, when the loss of the proposed waveguide is not considered, the corresponding empty rectangular waveguide operating below the cut-off frequency can be considered as a "magnetic" plasma and its relative permeability is given by [6]

$$\text{Re}(\mu_{r,eff}) = 1 - \frac{\omega_c^2}{\omega^2},\tag{1}$$

$$\omega_c = \frac{\pi}{\sqrt{\varepsilon_0 \mu_0}}\sqrt{\frac{1}{d^2}+\frac{1}{h^2}}.\tag{2}$$

Here ω_c is the cut-off frequency for the TM$_{11}$ mode, ω is the angular frequency, and ε_0 and μ_0 are the vacuum permittivity and permeability, respectively, and d and h are the cross-section dimensions of the rectangular waveguide.

In Fig. 1 (b), two CSRR-loaded metallic plates are placed inside a rectangular waveguide and the beam is located between the two CSRR layers. In this waveguide, there is a TM-dominated hybrid mode. This mode is not pure TE or TM mode, but is a complex hybrid mode with field components both perpendicular and parallel to the electron beam direction. From the point of view of the effective medium theory, the "*macroscopic*" mode is

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approximately a "TEM" mode because the integrals of the longitudinal electric- and magnetic- fields in a period approximately vanish, respectively. Thus, from the simple physical model, once the dispersion is obtained, the phase velocity is determined. Therefore, for the approximate "macroscopic TEM" mode, the phase velocity can be easily written as

$$v_p = \frac{c_0}{\sqrt{\mathrm{Re}(\varepsilon_{r,eff})\,\mathrm{Re}(\mu_{r,eff})}}, \qquad (3)$$

where c_0 is the light velocity in vacuum. Thus we can write the real part of the negative relative permeability as:

$$\mathrm{Re}(\varepsilon_{r,eff}) = \frac{c_0^2}{v_p^2\,\mathrm{Re}(\mu_{r,eff})}. \qquad (4)$$
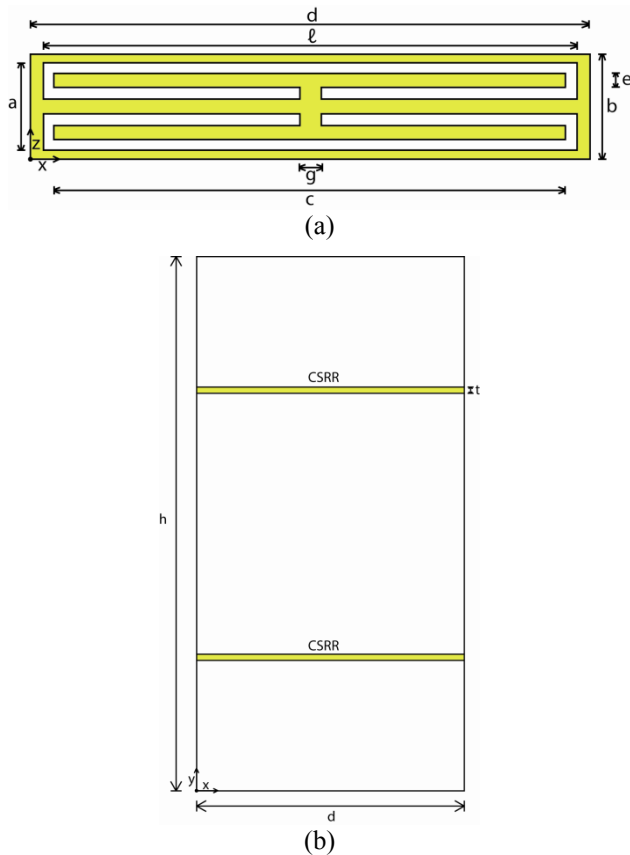

(a)

(b)

Figure 1: (a) a CSRR unit-cell, a=7.25 mm, b=8 mm, c=39.18 mm, d=43.18 mm, e=0.75 mm, g=2.5 mm, l=41.38 mm; (b) the cross section of the proposed CSRRs-loaded rectangular waveguide, h=86.36 mm, t=1 mm.

Therefore, knowing the dispersion of the proposed waveguide and using the permeability given in Equation 1 we can completely determine the effective permittivity and permeability for the CSRR-loaded rectangular waveguide.

SIMULATIONS AND RESULTS

The two CSRR layers are symmetrically located in the waveguide along the z direction and the distance between the two CSRR layers is set to be 42 mm. The CSRRs are periodically placed along the axis of the structure with period 8 mm. Using the Eigen-mode solver of HFSS [7], we can get the dispersion curve for this structure which is shown in Fig. 2. Note that the material loss is not considered in the HFSS simulation. The cut-off frequency is ~3.89 GHz for the TM$_{11}$mode in the empty rectangular waveguide, as shown in Fig. 3. From the dispersion curve the phase velocity can easily be obtained by $v_p = \mathrm{Re}(\omega)/\beta$. For example, we can see that the double-negative frequency band is only from ~2.3 to 2.84 GHz for the l=41.18 mm case, where the directions of $\bar{v}_p$ and $\bar{v}_g = d\omega/d\beta$ (group velocity) are opposite. Here β is the phase constant.

Applying the parameter retrieval method presented here, the effective permeability can be found according to Equations (1) and (2), and it is shown in Fig. 3. Also, using Equations (3) and (4) and simulation results of the dispersion (from Fig. 2), we can retrieve the effective permittivity in the double-negative frequency band which is shown in Fig. 4. In the simulations d is kept as a constant and $c = l - 2$ is varied for three different values. Decreasing the resonator length l increases the resonant frequency of the CSRRs and thus the dispersion curve moves to the high frequencies (Fig. 2). The retrieved effective permittivity is dramatically changed (Fig. 4).
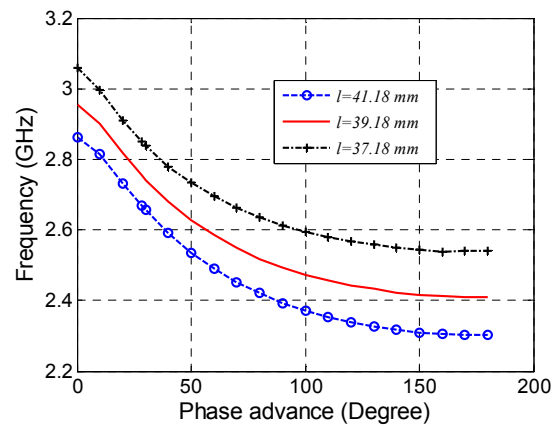

Figure 2: The dispersion curve for the proposed CSRR-loaded rectangular waveguide.

Here we are not interested in the frequencies beyond the double-negative frequency band, since this method is based on the effective medium concept and the dispersion characteristics. Note that this method is not rigorous, but it is approximately correct for the effective material parameters. Currently, we are developing another method based on the field analysis to retrieve the effective permittivity for the CSRRs which are filled in the waveguide.
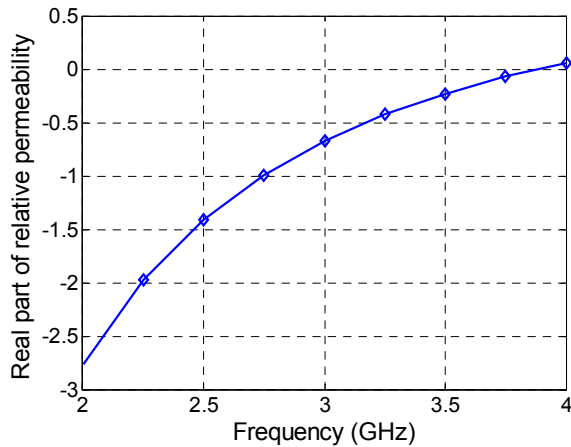
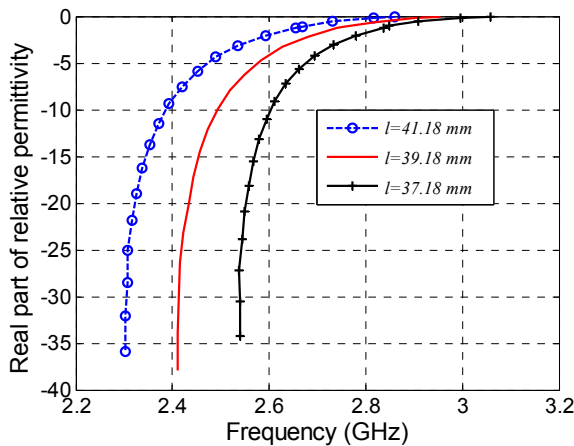Figure 3: The real part of relative permeability vs. frequency.

Figure 4: The real part of relative permittivity vs. frequency.

CONCLUSIONS

In this paper, we have proposed a CSRR-loaded waveguide. Using the effective medium theory and setting up a simple physical model, we have developed a method to retrieve the effective parameters for the CSRR-loaded waveguide. This method is based on the "magnetic plasma" concept and the structure's dispersion characteristics. It is different from the widely used scattering parameter method which is suitable for metamaterial slabs excited by TEM mode, but is not suitable for the metamaterial-loaded waveguide, where the waveguide affects the electromagnetic behaviour of the metamaterial.

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