CSR Fields from using a Direct Numerical Solution of Maxwell's Equations.

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2011 Particle Accelerator Conference
March 28 – April 1, 2011 New York, U.S.A.

NATIONAL ACCELERATOR LABORATORY
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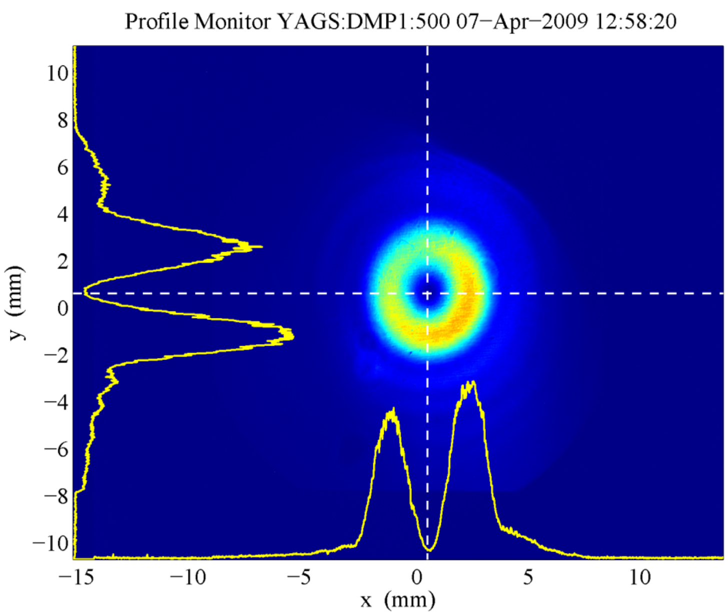
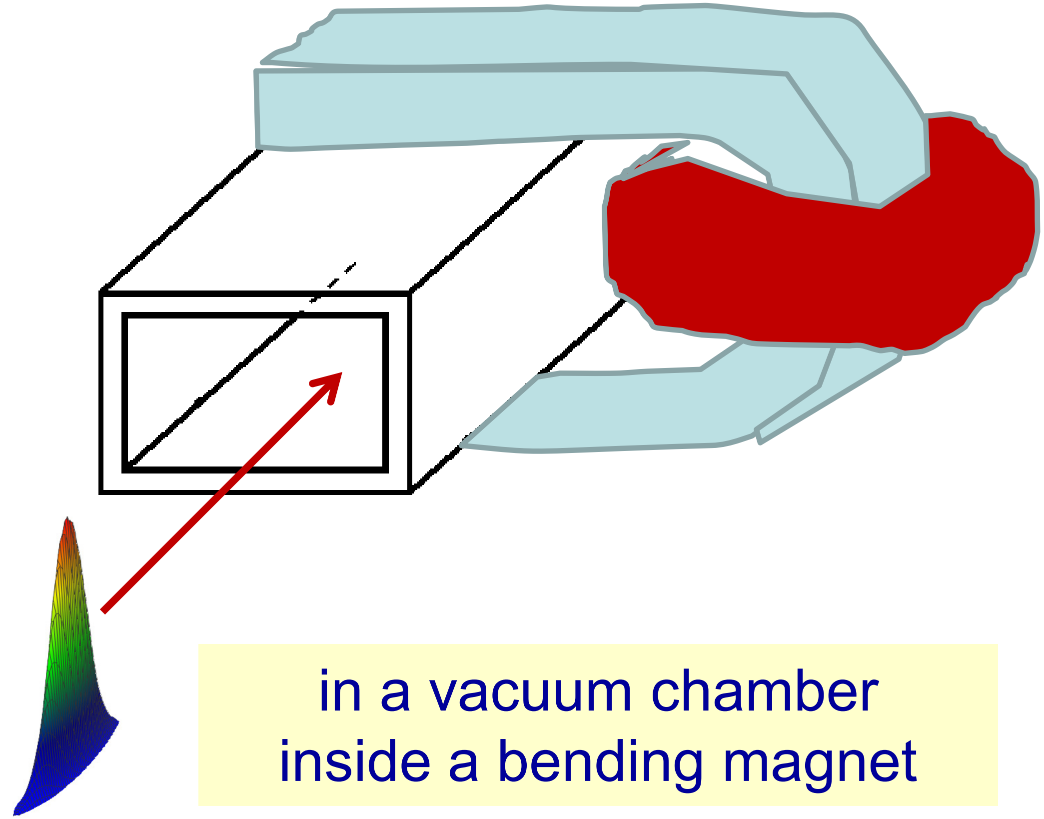
CSR – Coherent Synchrotron Radiation

electro-magnetic fields
image from LCLS



excited by
a short bunch



in a vacuum chamber
inside a bending magnet

Sasha Novokhatski March 29, 2011

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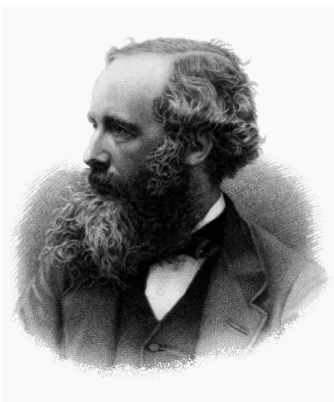
How do we analyze the problem?

Solving numerically

Maxwell's equations together with Newton’s equations



$$\nabla \times \mathbf{H} = J + \frac{\partial D}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

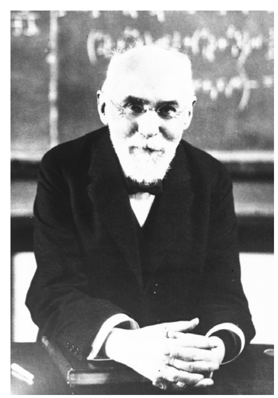
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot D = \rho .$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$$

$$\frac{d\vec{p}}{dt} = \vec{F}$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2}}$$



Lorentz's force

$$\vec{F} = e\vec{E} + \frac{e}{c}\left[\vec{v} \times \vec{H}\right]$$

How we solve Maxwell's equations for short bunches?

- **Implicit algorithm**
  - employs a more efficient use of finite element mesh techniques. This algorithm is free of frequency dispersion which means that all propagating waves have their natural phase velocity, completely independent of the simulation parameters like a mesh size or a time step. This method can produce self-consistent stable solutions for very short bunches.
- **Moving mesh**
  - To decrease the amount of needed memory. The bunch length is a micron but the distance between bends is tens of meters.
- **Fourier expansion**
  - in vertical direction
- **Parallel code***

same algorithm that we have used for wake field calculations of very short bunches

Wakefield calculations using implicit scheme, 1976.

An implicit algorithm has been used in the computer code designed in 1976 for wake field dynamics studies at the Novosibirsk Electron-Positron Linear Collider VLEPP (σ=1.8 mm).

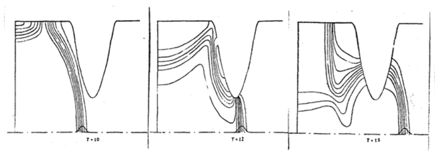
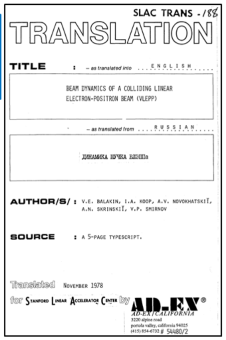
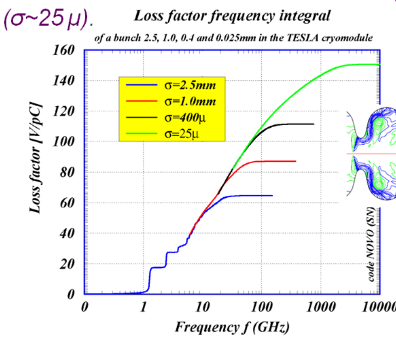


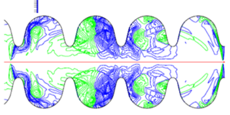
Fig. 1.



Loss integral of the TESLA cryo-module, 1996

The code was used for calculating wake fields of very short bunches at the TESLA Linear Collider (σ~0.7mm) and TTF FEL (σ~25 μ).





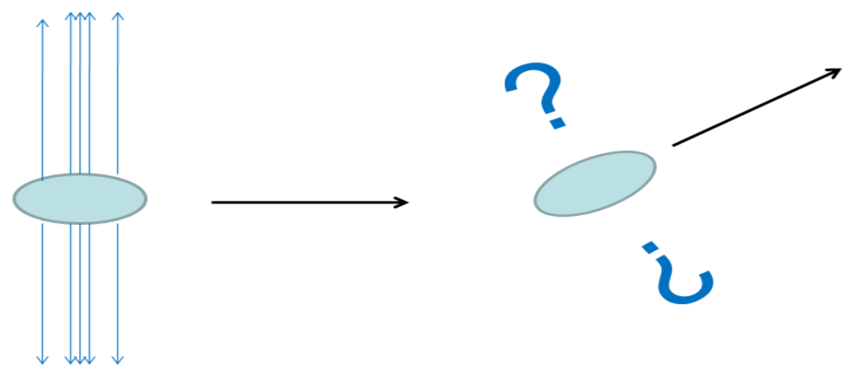
These calculations were important to prove the feasibility of the project.

How to show electromagnetic fields?

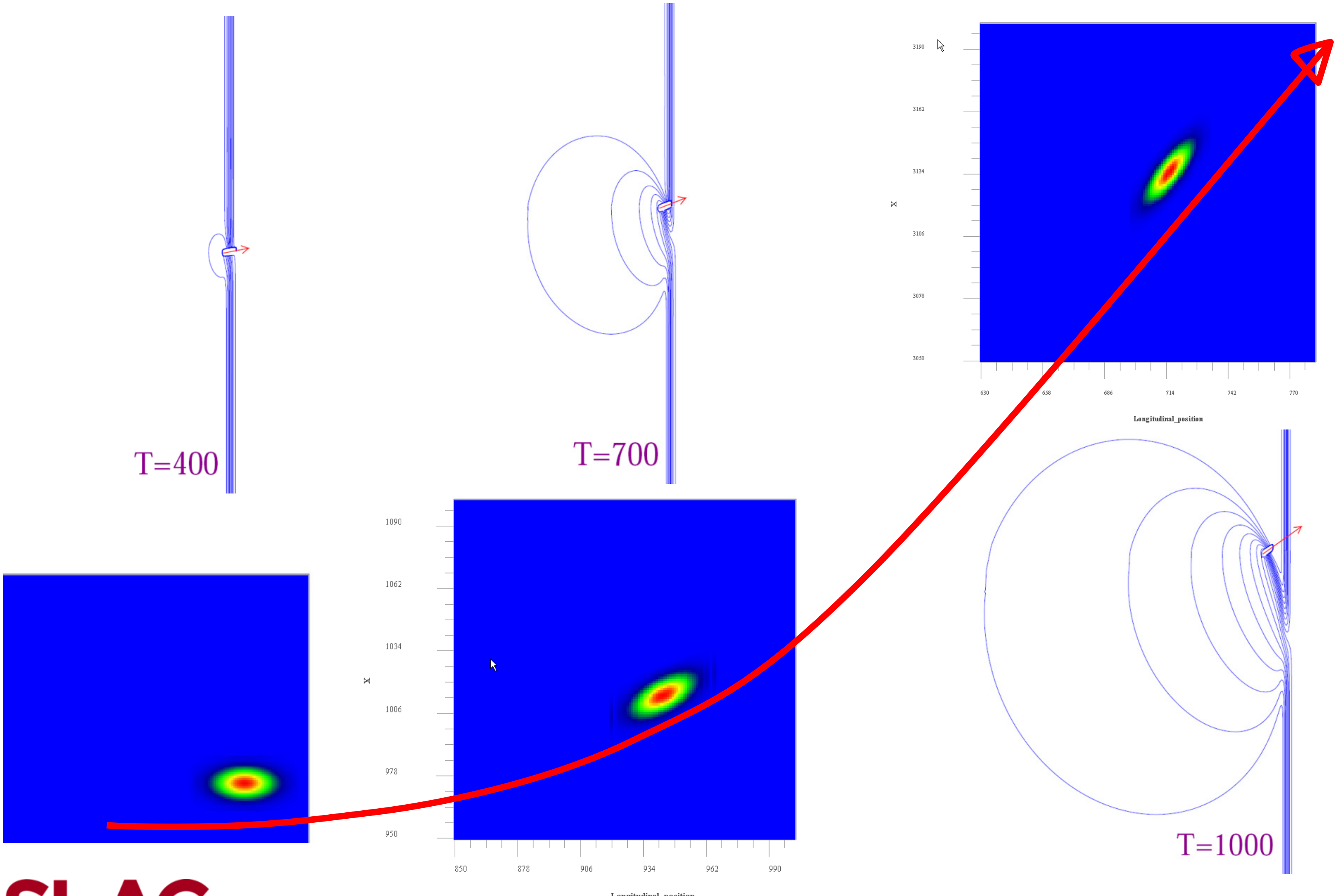
In old-fashioned classical form – plots of electrical field lines

A question:

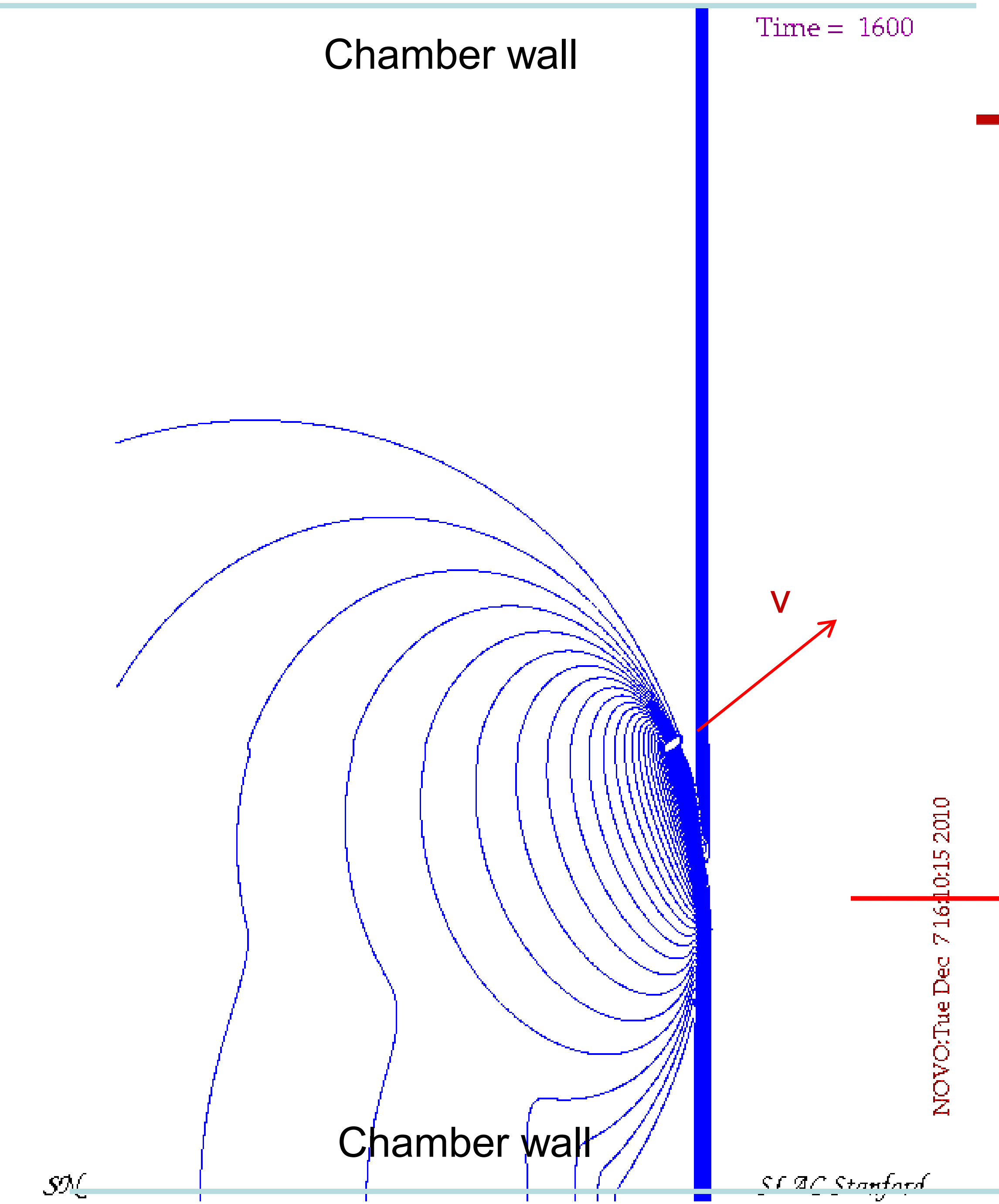
How does the bunch field change
when the bunch has been rotated in a magnetic field?

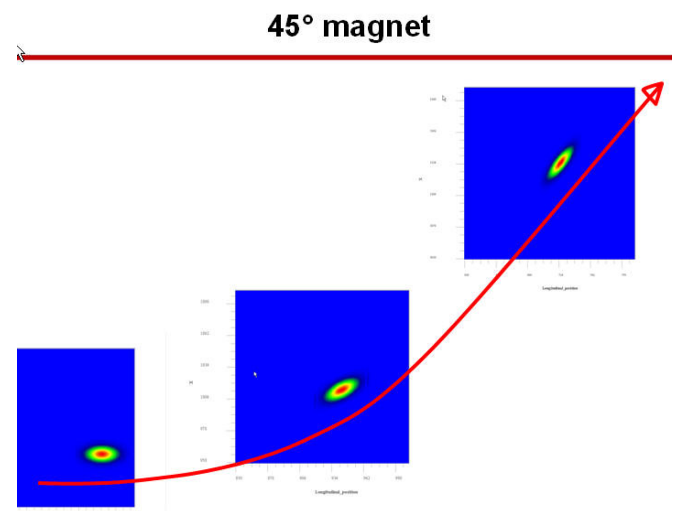


Field dynamics in a 45° magnet



Movie show

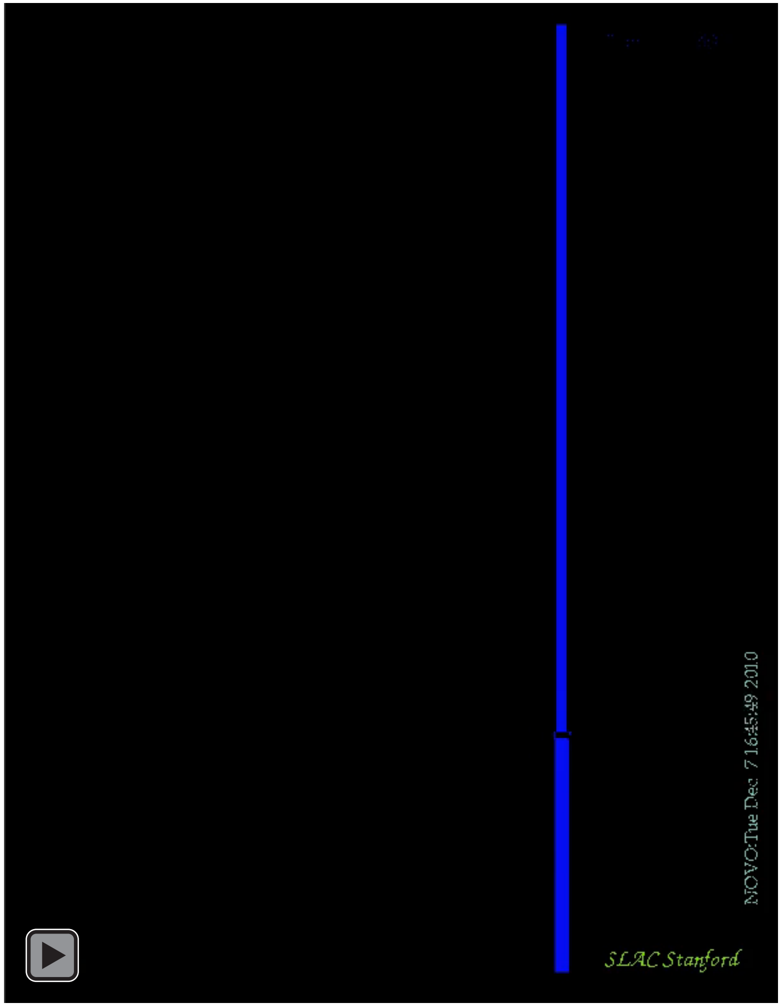


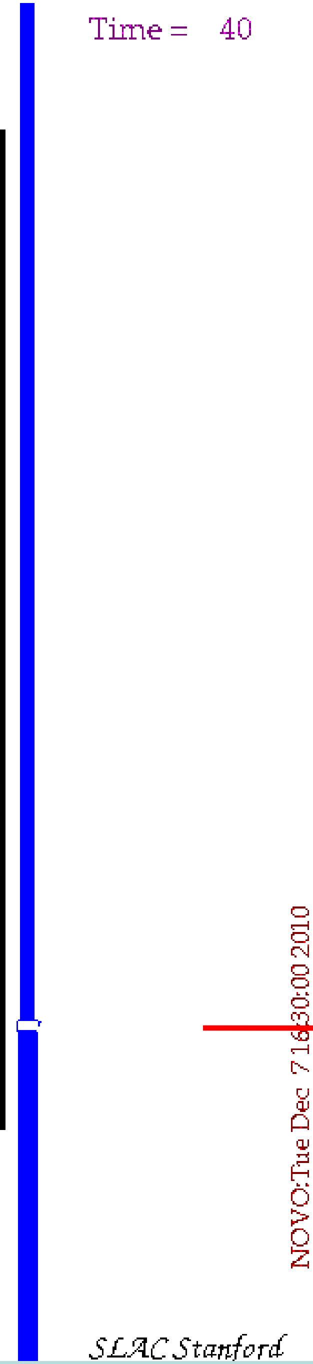


Initial direction

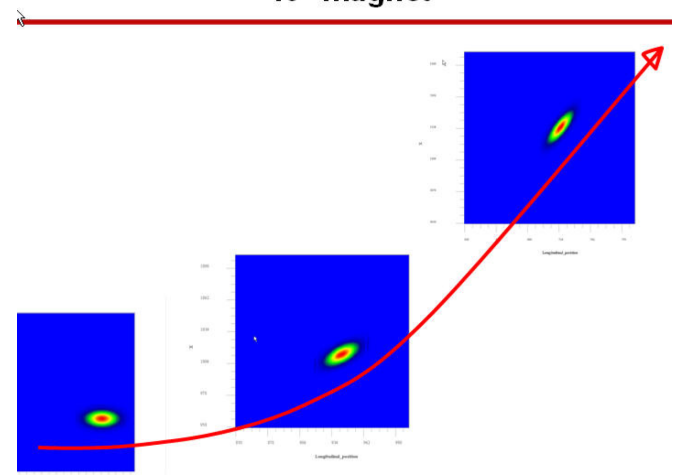
Chamber wall

Movie show





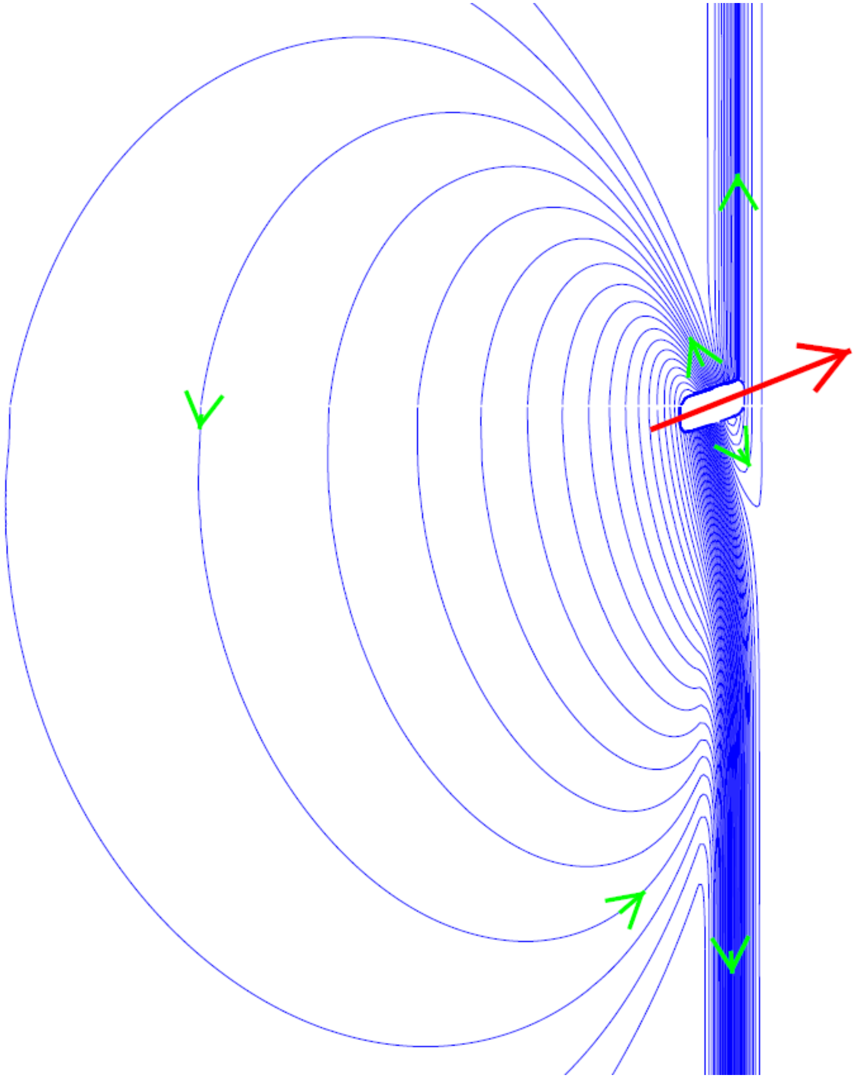
45° magnet



Initial direction

Chamber wall

Bunch self-field remakes itself



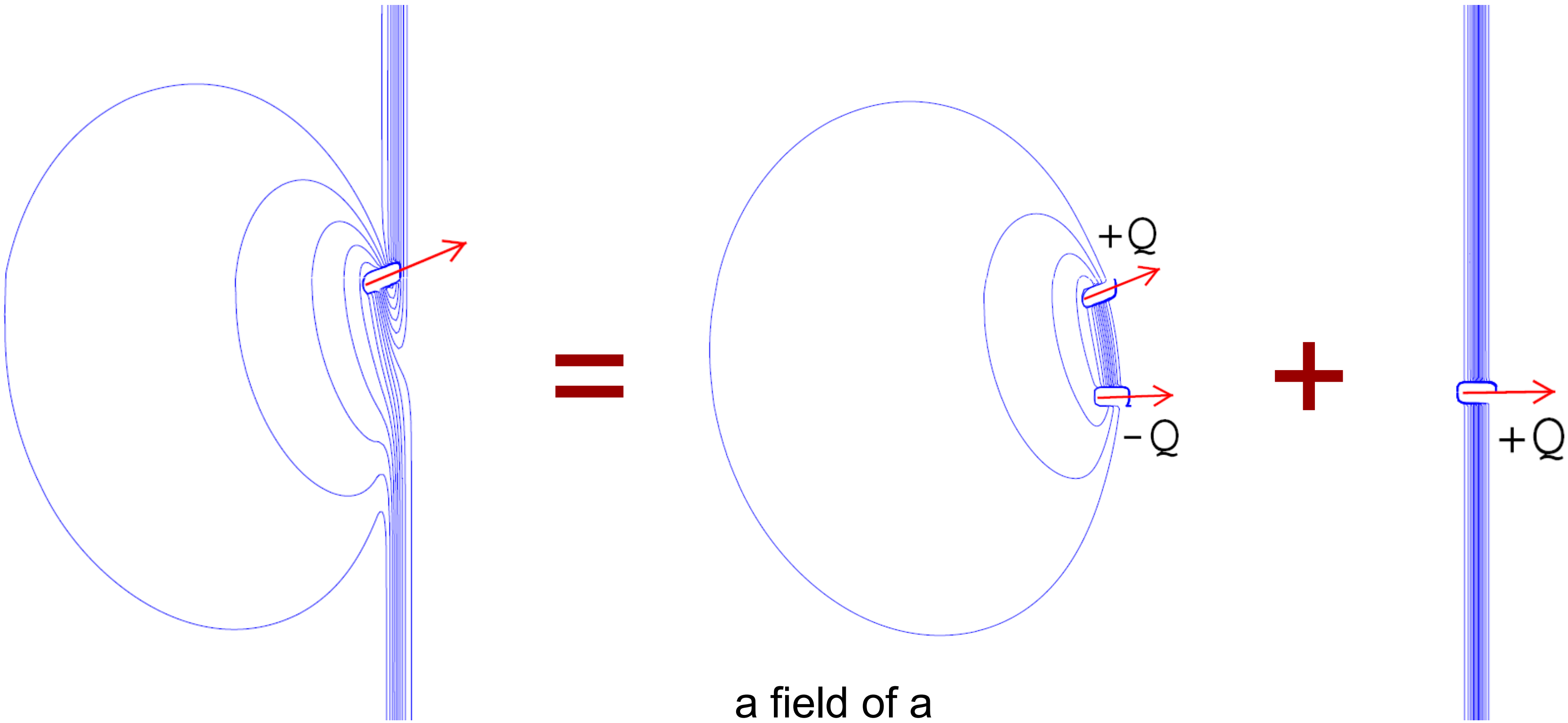
The upper field lines take the position of the lower lines

A red arrow shows a bunch velocity vector

Green arrows show field line directions

The lower field lines take the position of the upper lines

The picture becomes clear if we decompose the field



a field of a moving dipole

Decomposition of the field of a bunch moving in a magnetic field into two fields:

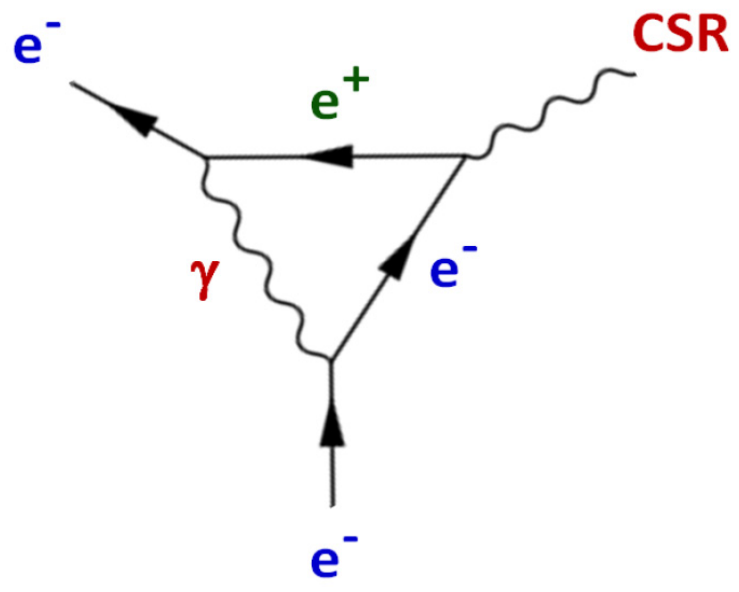
a field of a bunch moving straight in initial direction

Decomposition as an analog the Feynman diagram

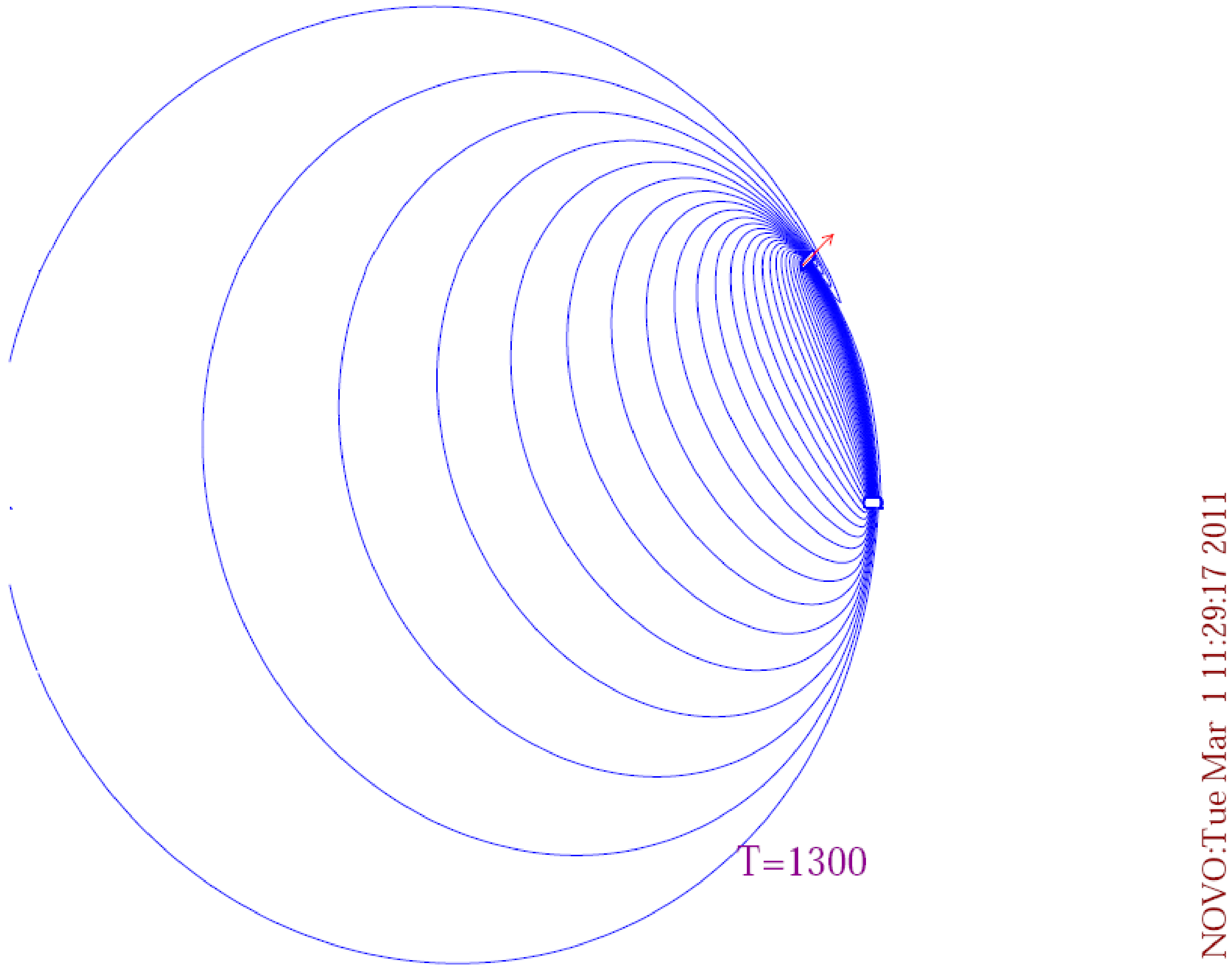
There could be a close analogy between the field decomposition and the Feynman diagram.

A real electron produces a virtual photon, which decays into electron-positron pair, corresponding to a dipole.

The positron can annihilate with the ongoing scatted electron to emit a photon. This photon corresponds to synchrotron radiation.

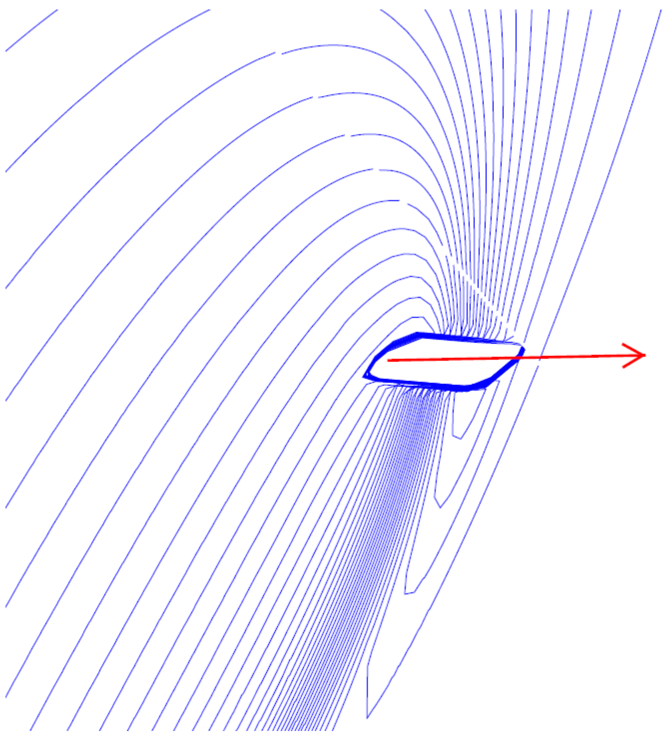


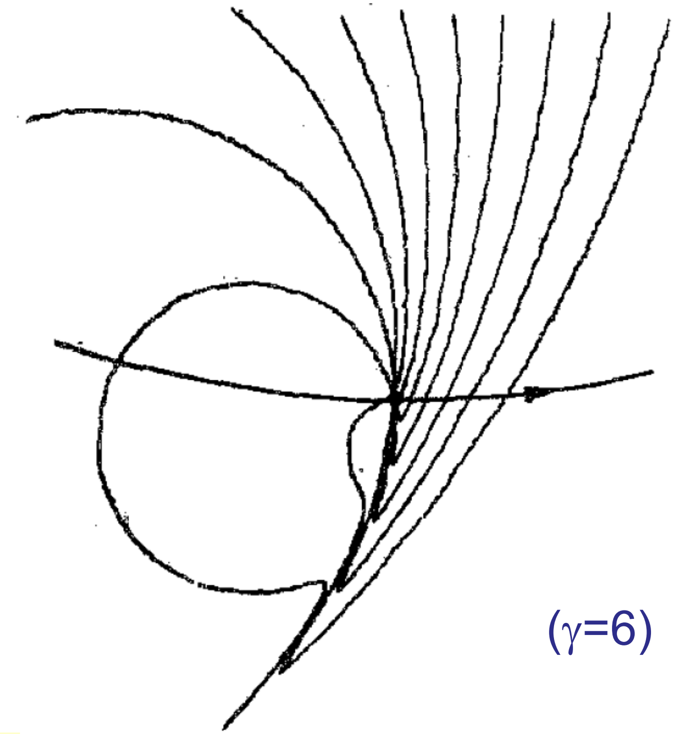
Detailed plot of a dipole field



Comparison with a classical synchrotron radiation

γ - region in front of a particle





equivalent bunch length

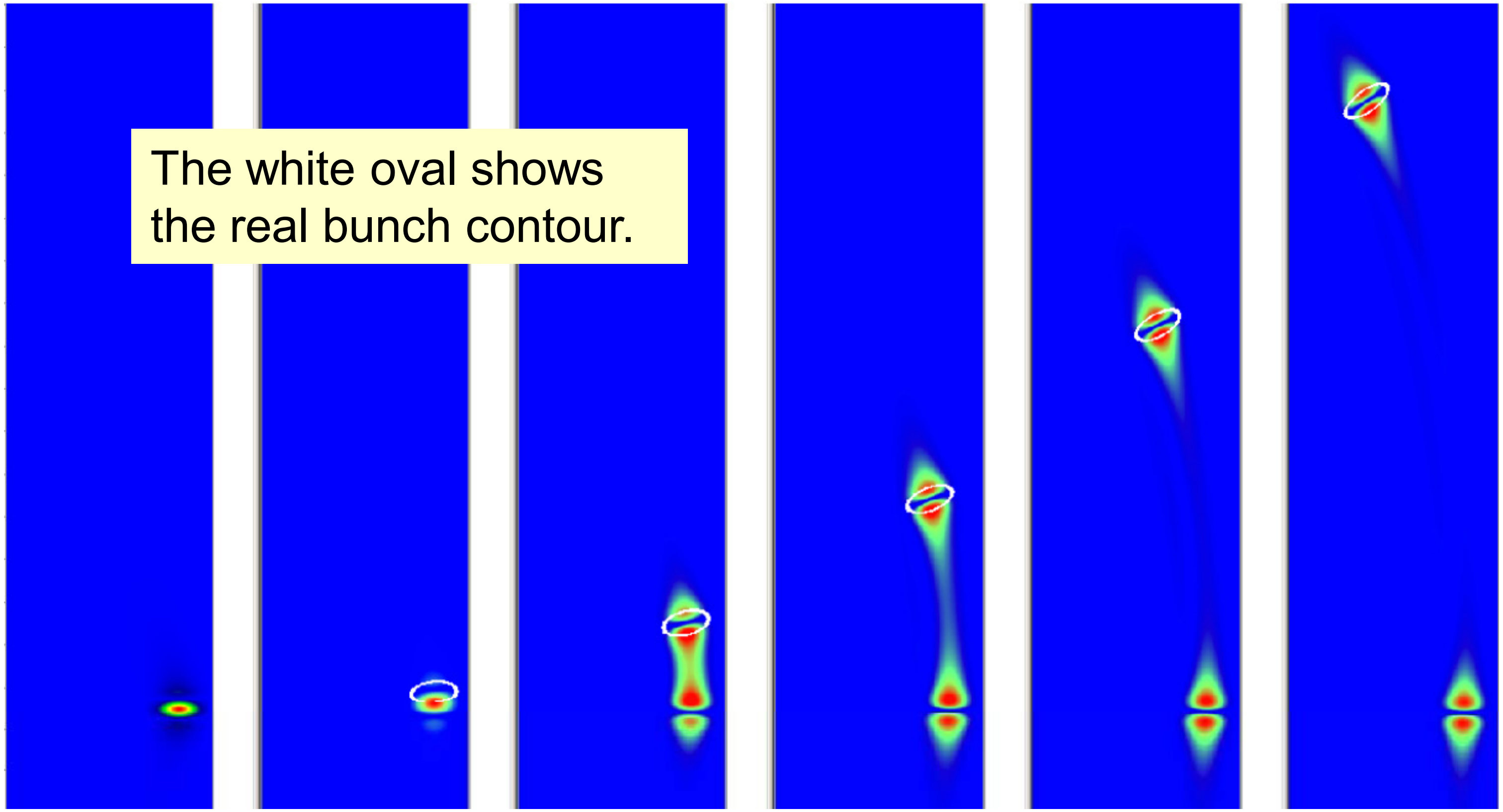
for a bending radius ρ

$$\sigma \approx \frac{2}{3}\frac{\rho}{\gamma^3}$$

FORCE LINES OF ELECTRIC AND MAGNETIC FIELDS
OF AN ARBITRARILY MOVING CHARGE

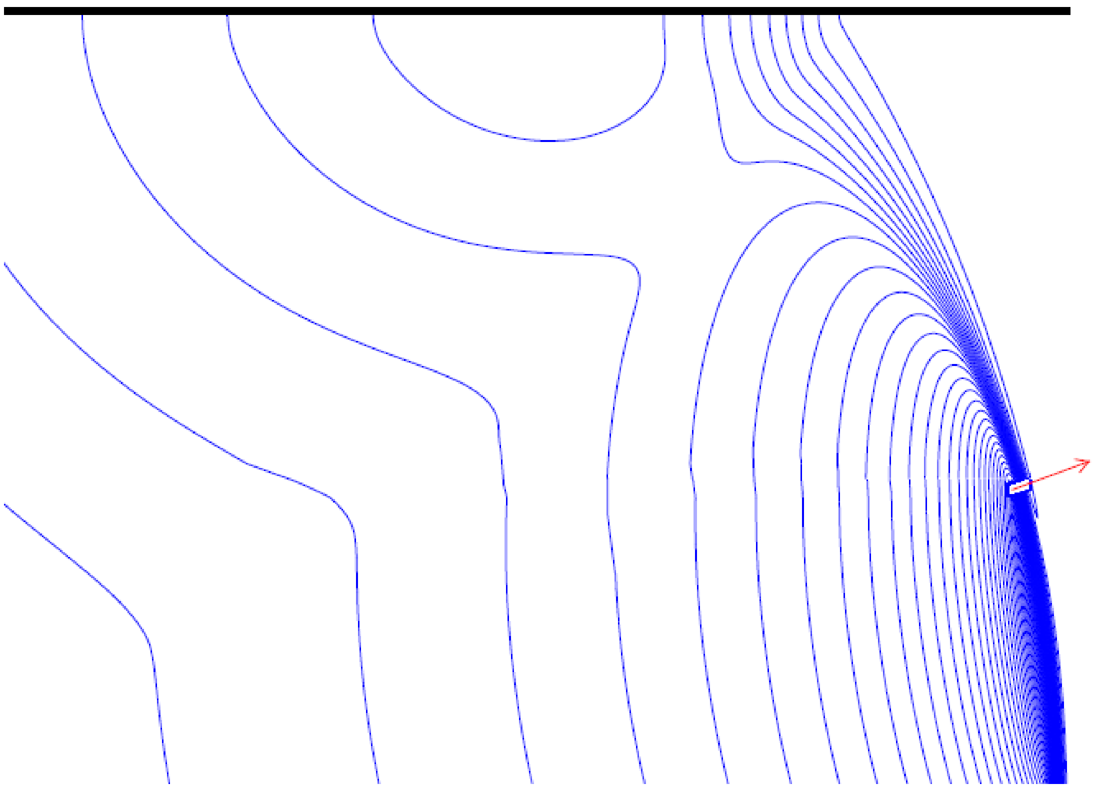
S. G. Arutyunyan

An absolute dipole electric field in time



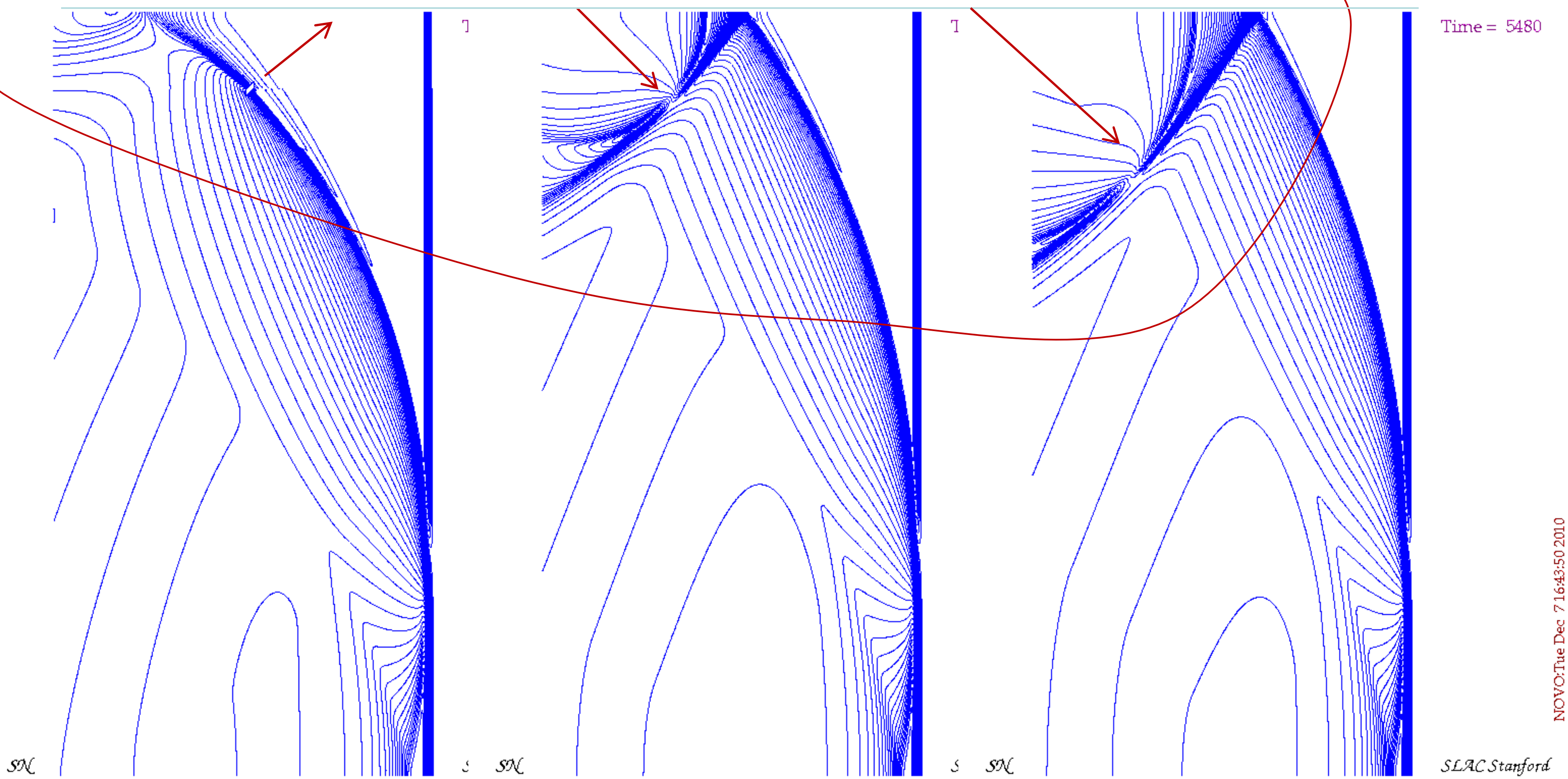
When a dipole is created an electric field appears between a real bunch and a virtual bunch. This field increases in value and reaches a maximum value when the bunches are completely separated and then it goes down as the bunches more apart leaving fields only around the bunches.

A new bunch field

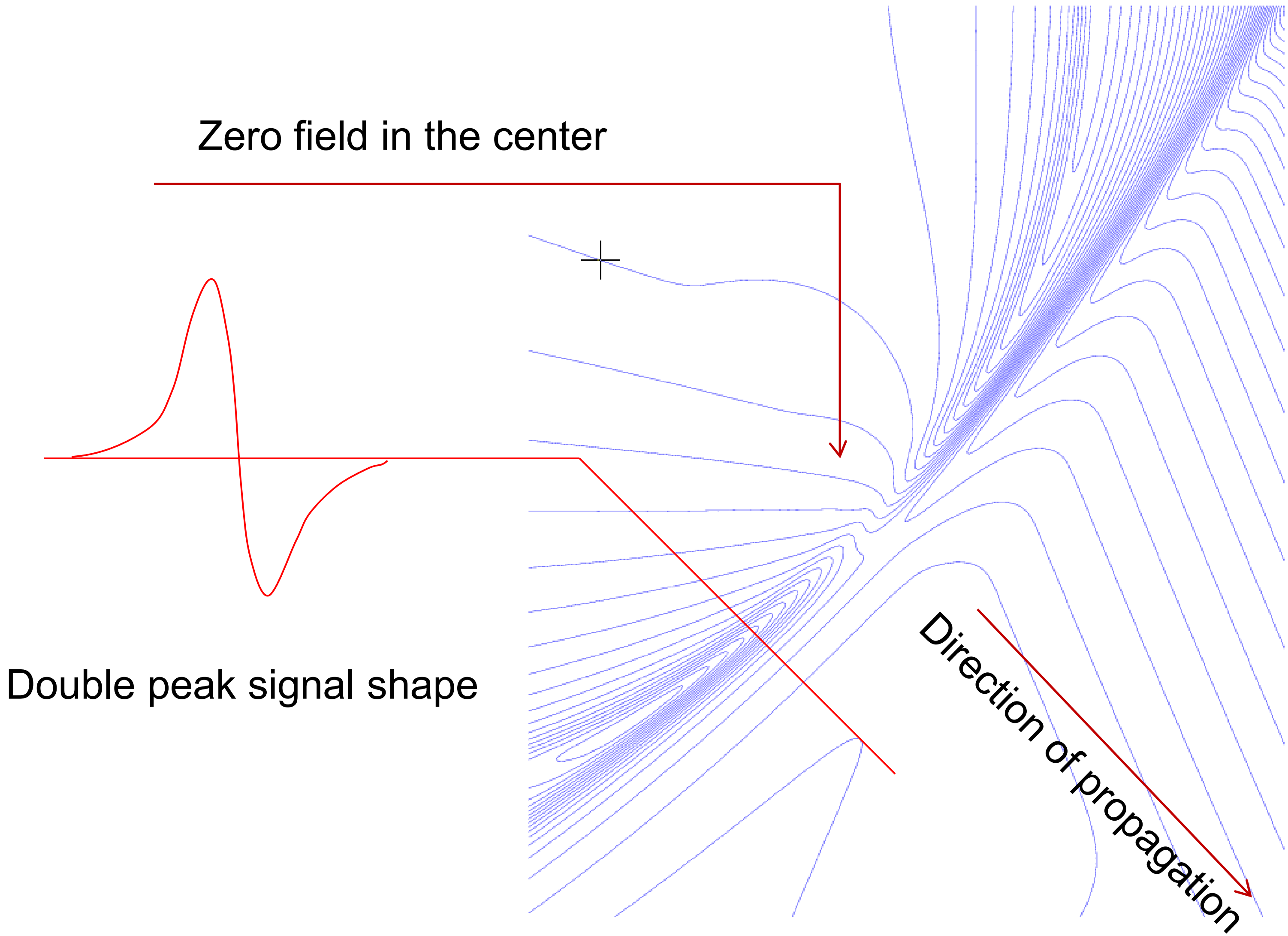


A long way to a steady-state regime

Transition radiation



Transition radiation. Details.



Electrical forces inside a bunch

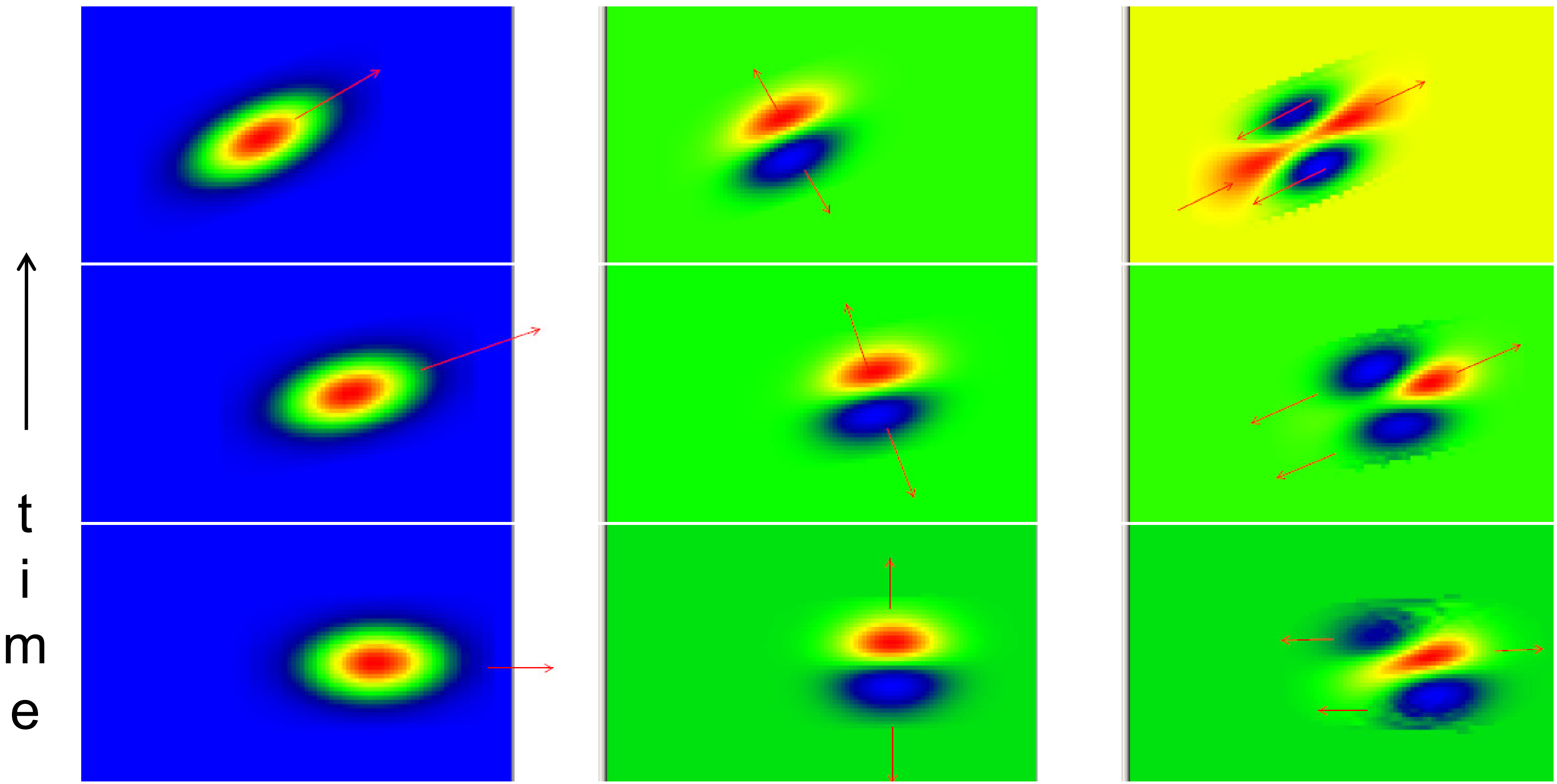
Bunch shape

transverse force

$$F_{\perp}^{e} = \left[\vec{J} \times \vec{E}\right]$$

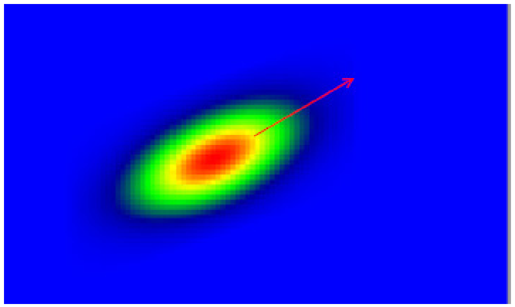
collinear force

$$F_{\parallel}^{e} = \left(\vec{J} \cdot \vec{E}\right)$$



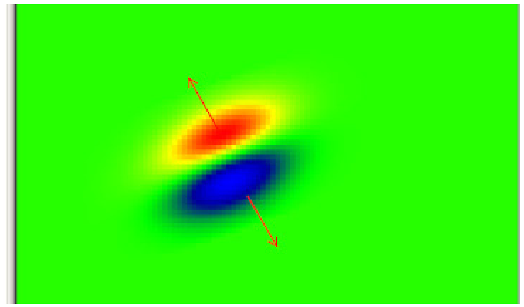
Electrical forces inside a bunch

Bunch shape



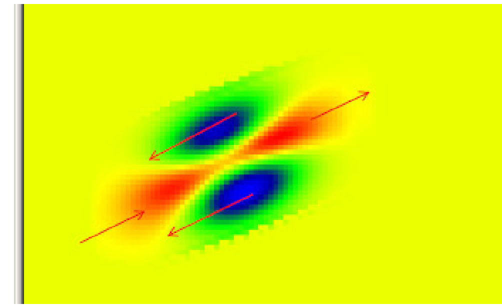
transverse force

$$F_{\perp}^{e} = \left[\vec{J} \times \vec{E}\right]$$



collinear force

$$F_{\parallel}^{e} = \left(\vec{J} \cdot \vec{E}\right)$$



The transverse force is the well known space-charge force, which probably is compensated by a magnetic force in the ultra-relativistic case.

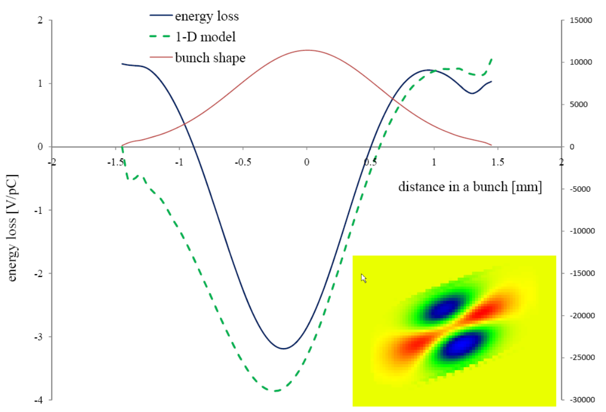
The collinear force is responsible for an energy gain or an energy loss. The particles, which are in the center, in front and at the end of the bunch are accelerating, whereas the particles at the boundaries are decelerating.

The total effect is deceleration and the bunch loses energy, however the bunch gets an additional energy spread in the transverse direction.

Intermediate summary

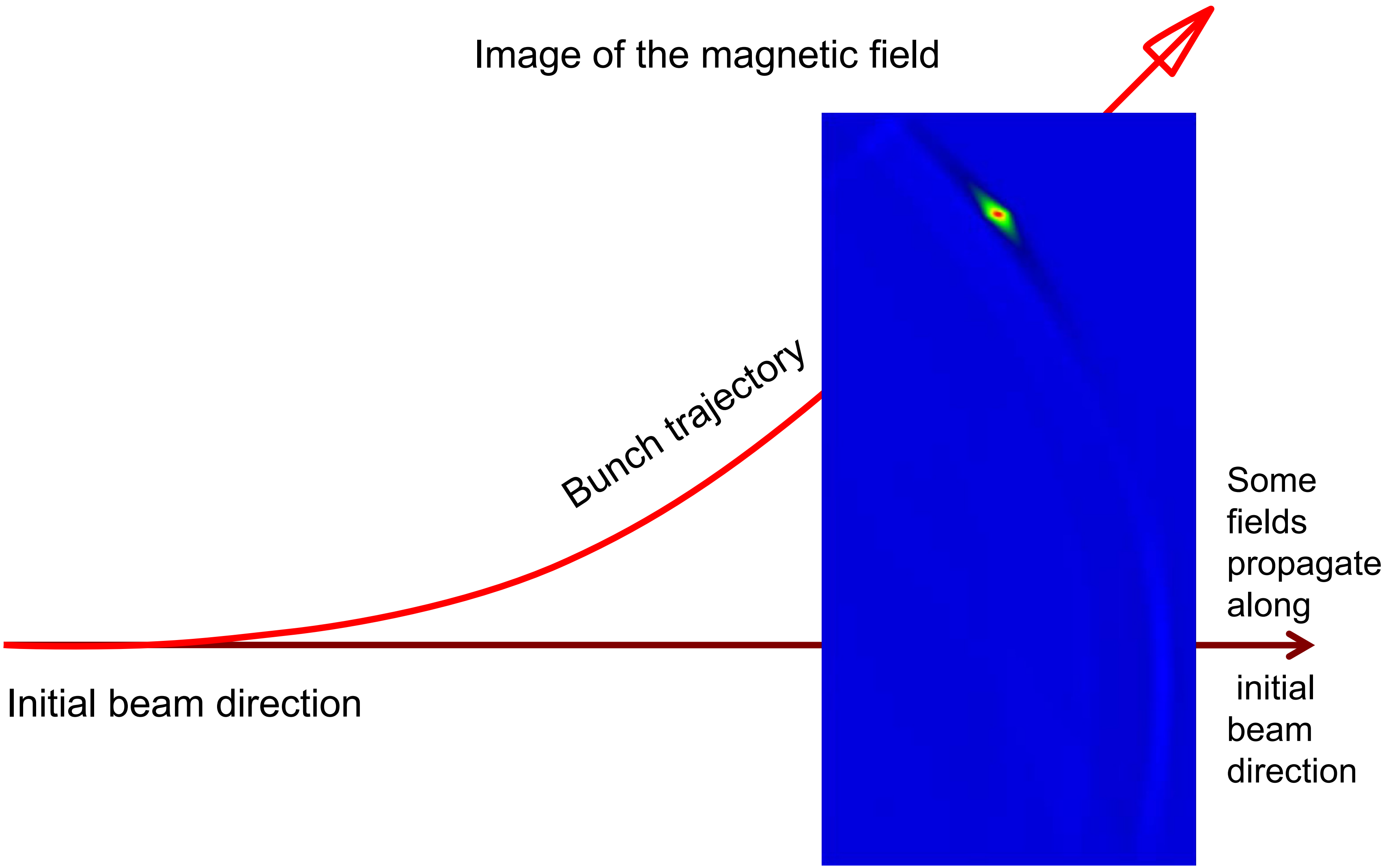
- This complicated structure of the collinear field is very important.
- A bunch will get an additional transverse energy spread, which may not be compensated.
- This energy spread in the magnetic field immediately generates emittance growth.

Integrated in X

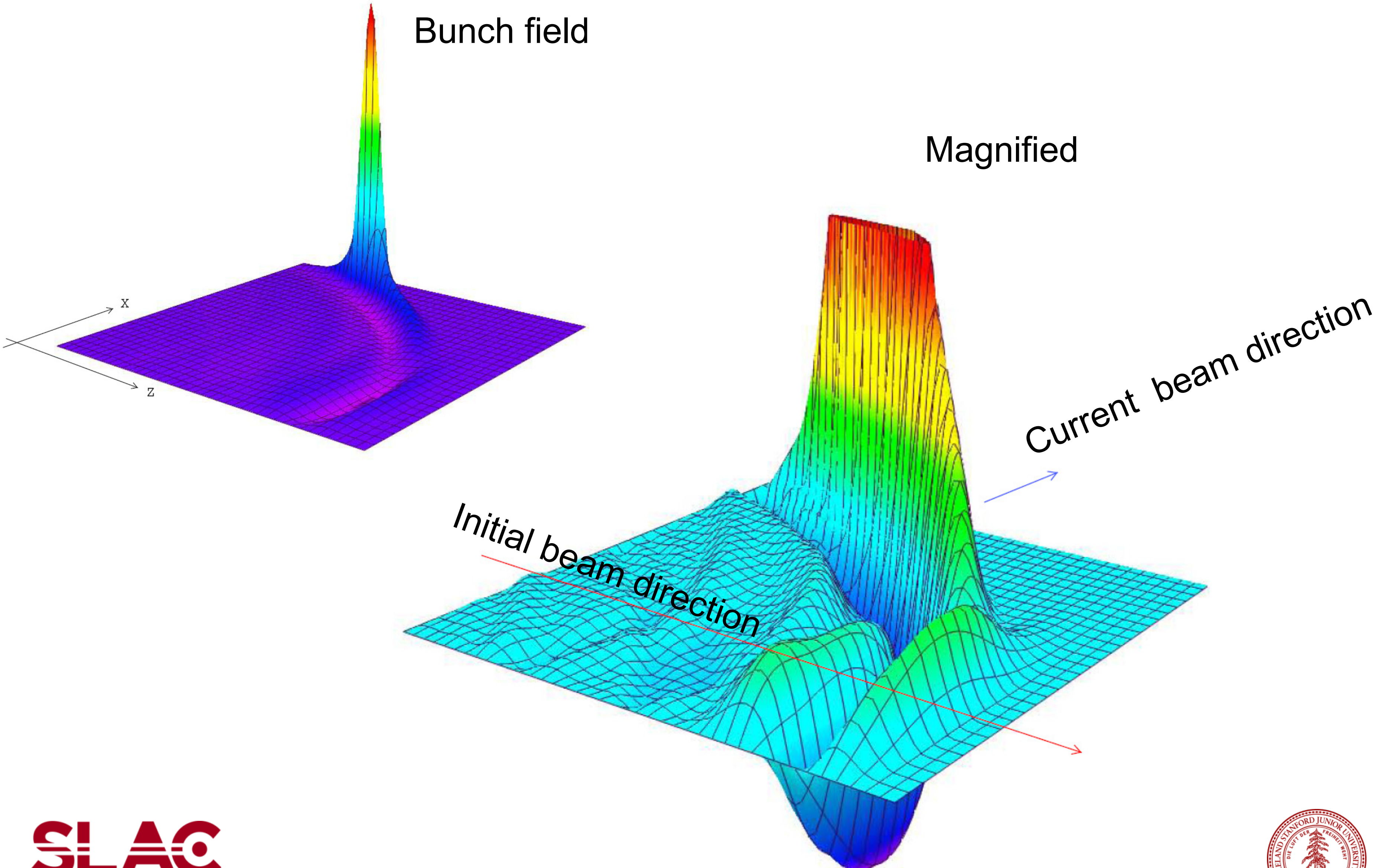


- Transverse energy spread may increase decoherence .
- This effect can limit the efficiency of the magnetic bunch compressors and as a result the efficiency of FELs.

Coherent edge radiation?

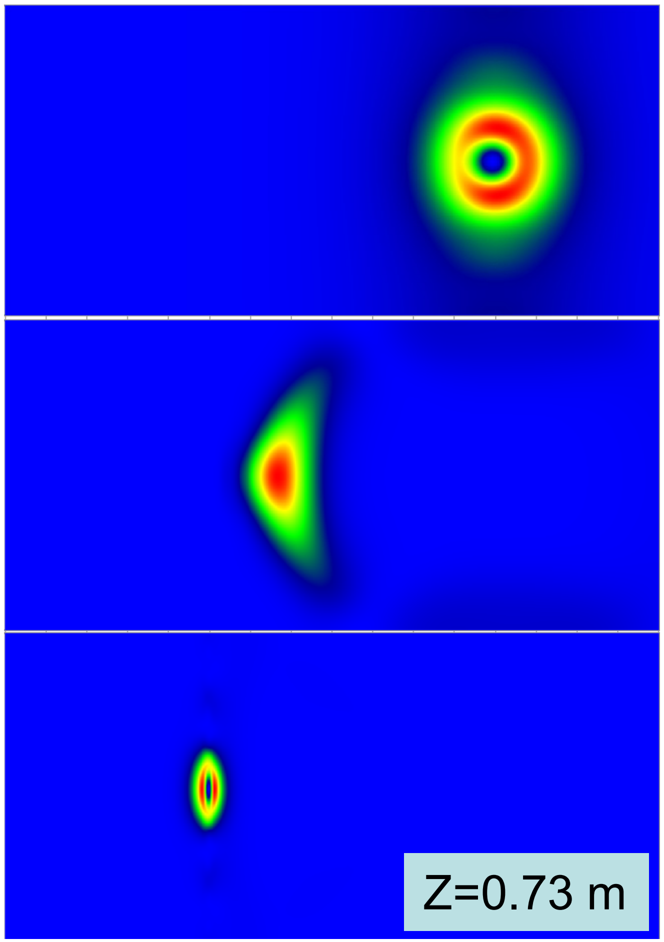


Magnetic field plots



Images of radiation (transverse magnetic field)

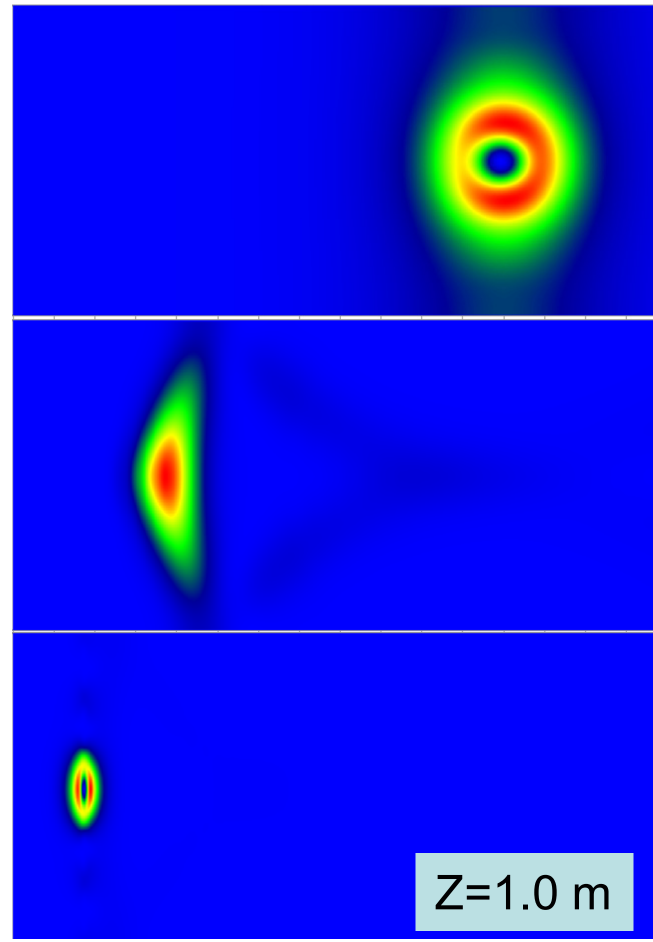
very similar to the images, which we have seen on the YAG screen after the dump magnets, which bend down the beam at LCLS.



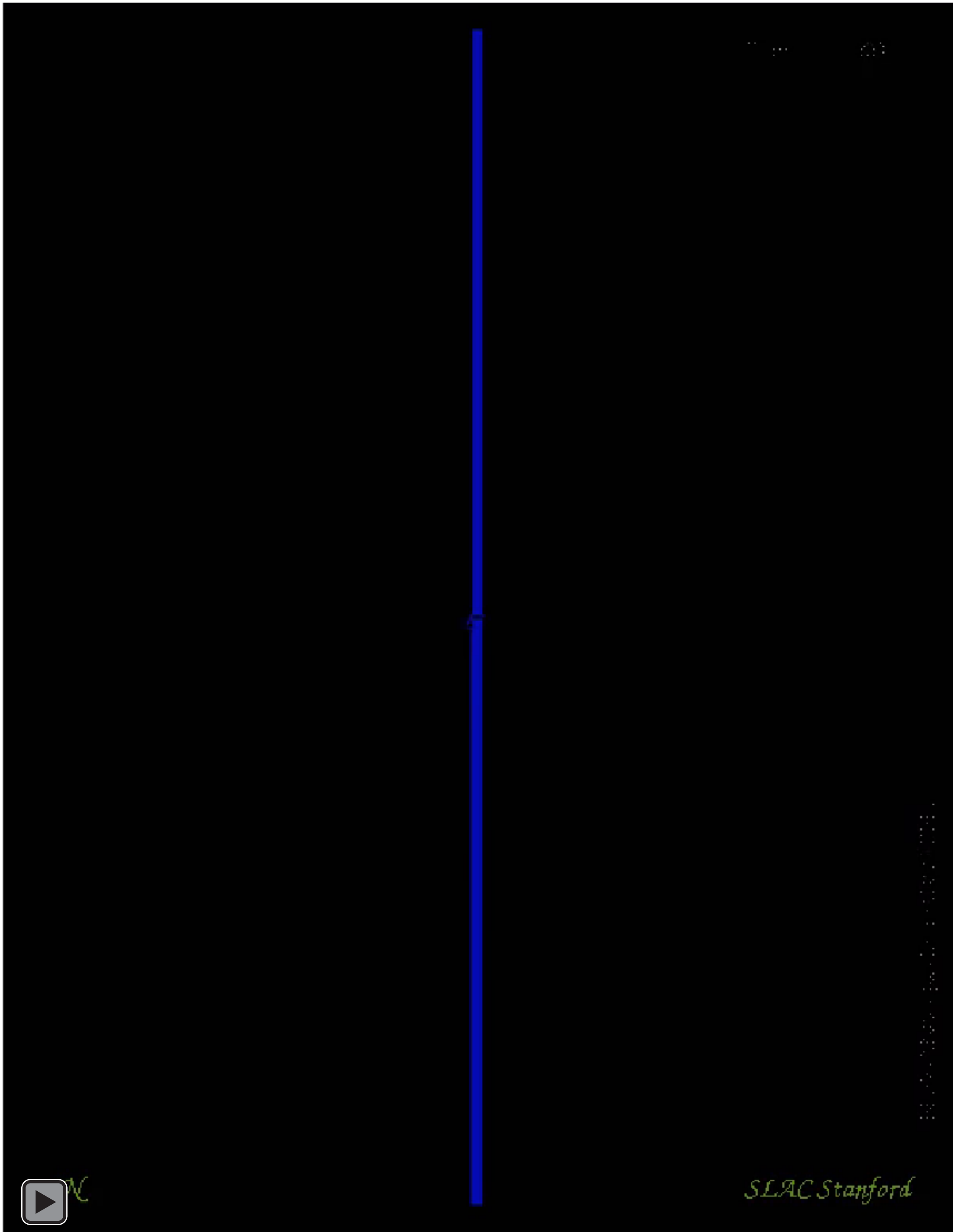
Edge radiation

Synchrotron radiation

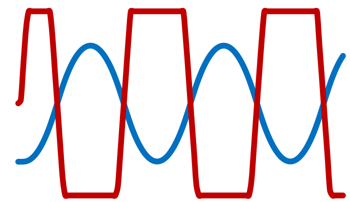
Bunch field



Movie show



Undulator



Acknowledgments

The author would like to thank

Mike Sullivan and R. Clive Field

for help and valuable comments;

Franz-Josef Decker, Paul J. Emma and Yunhai Cai

for support and interest in this work;

Physicists of the SLAC Beam Physics Department

for useful discussions.