Dancing Bunches as van Kampen Modes

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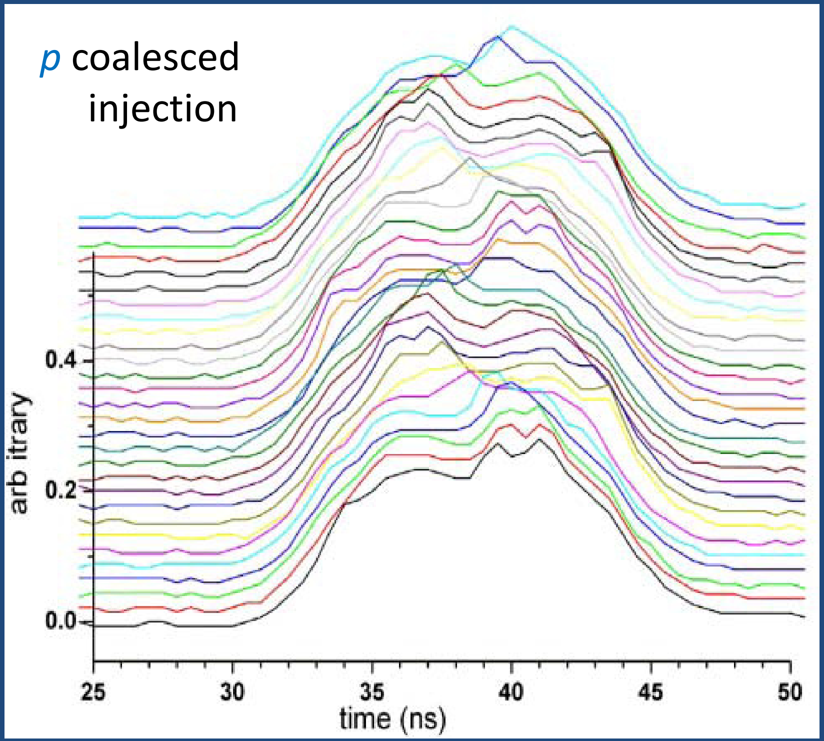
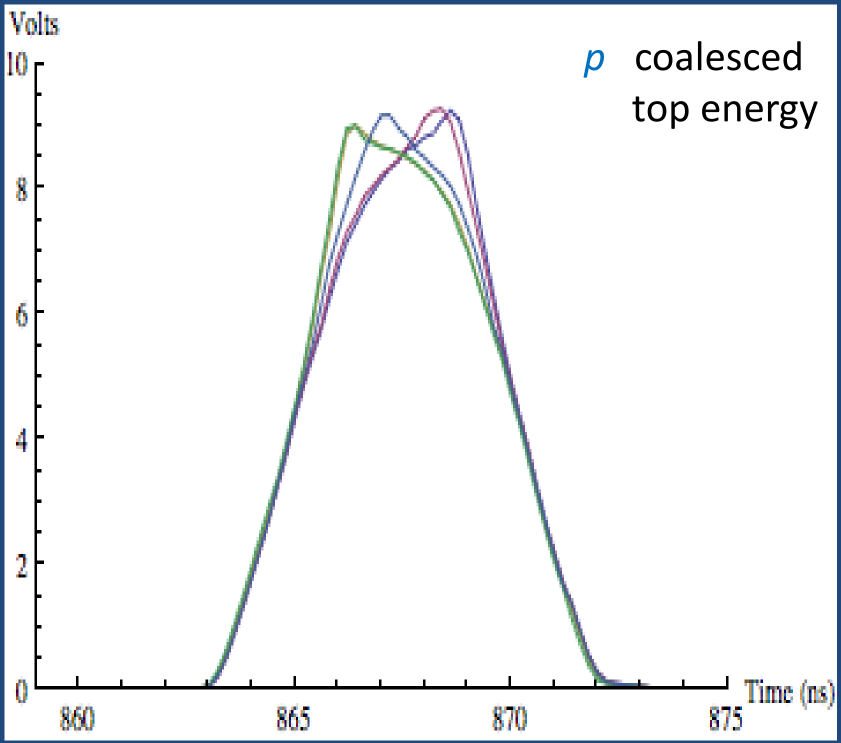
PAC'11 talk

Dancing bunches at Tevatron





R. Moore et al, PAC'03

Burov & Tan, PAC'11

At injection, the synchrotron frequency shift is estimated as low as $|\Delta\Omega_0|/\Omega_0 \approx 1\%$.

How may any instability happen at that low intensity (impedance)?

Boltzmann-Jeans-Vlasov (BJV) equation

- Boltzmann-Jeans-Vlasov (BJV) equation describes free oscillations of plasma or beam in given external fields, while Coulomb fields of the plasma or wake fields of the beam have to be found.

- For longitudinal motion of a bunch, BJV writes as:

$$\frac{\partial f}{\partial t} + \Omega(I)\frac{\partial f}{\partial \varphi} - \frac{\partial V}{\partial \varphi} F'(I) = 0$$

$$V(z,t) = -\int f(I',\varphi',t) W(z - z') dI' d\varphi'$$

- Here I, φ and $\Omega(I)$ are the canonical action, phase, and incoherent frequency in the distorted potential well;
- $F(I)$ and $f(I,\varphi,t)$ are the steady state and perturbation phase space density; $W(z)$ is the wake function.

- What is an eigen-system of BJV ?

van Kampen theory

- For infinite classical plasma, van Kampen found all the *eigen-modes* of BJV.

- There is always a *continuous spectrum* of them. These modes are singular, their frequencies are identical to incoherent frequencies (kv for the plasma case).

- *Landau damping* can be treated as phase mixing of the continuous van Kampen modes

- There can be *discrete modes* as well.

- $\sum_{\text{discrete}} \text{growth rate} = 0$, so some of the discrete modes do not decay.

Loss of Landau damping (LLD)

- This loss of Landau damping (LLD) has to be distinguished from an instability.

- LLD is similar to a loss of *immune system* for a living cell – although it can live in a sterile medium, even a weakest microbe would kill it.

- For LLD, a *weakest couple-bunch wake* drives an instability.

Linear focusing (parabolic RF potential)

- *Instead of BJV*, let's look at multi-particle equations:

$$\ddot{z}_i + z_i = \sum_j W'(z_i - z_j)\,; \quad i, j = 1, \ldots, N$$

- The solution can be presented as a sum of a steady-state part and a small perturbation:

$$z_i = \hat{z}_i + \tilde{z}_i$$

- Since for the steady state $\ddot{\hat{z}}_i + \hat{z}_i = \sum_j W'(\hat{z}_i - \hat{z}_j)$,

- $\tilde{z}_i = \text{const} \cdot \cos t$ is always a solution – for any wake and distribution.

(D'yachkov & Baartman, 1993)

LLD threshold = 0 for the linear focusing.

LLD scaling

For bunches, LLD was first considered by Balbekov & Ivanov (1991):

Potential well distortion incoherent tune shift scales as $\Delta\Omega \propto N \,\mathrm{Im}\, Z(l^{-1}) / l^2$

LLD happens when this shift exceeds the synchrotron tune spread $\delta\Omega \propto l^2$:

$$| \Delta\Omega | > \delta\Omega$$

From here, the threshold $N_{\mathrm{th}} \, | \,\mathrm{Im}\, Z(l^{-1}) \, | \propto l^4$

Very strong dependence on the bunch length l : for the space charge case

$$N_{\mathrm{th}} \propto l^5$$

What is the numerical factor for this scaling low?

RF ~ cos(z)

For space charge impedance

$$W(s) = -k\delta(s)\,; \quad Z(q) = -ikq$$

and Hofmann-Pedersen distribution function

$$F(I) \propto \sqrt{1 - I / I_{\text{lim}}}$$

LLD was treated analytically by Boine-Frankenheim & Shukla (2005).

They assumed *the coherent mode as rigid-bunch*, and found for LLD threshold:

$$\frac{|\Delta\Omega|_{\text{th}}}{\Omega_0} = \begin{cases} \phi_m^2 / 20 = I_{\text{lim}} / 10, & \text{below transition} \\ \phi_m^2 / 16 = I_{\text{lim}} / 8, & \text{above transition} \end{cases}$$

$\phi_m \le \pi$ - bunch length in units of RF phase.

How big is the threshold overestimation for the rigid-bunch model?

Back to BJV: Oide-Yokoya expansion

$$\frac{\partial f}{\partial t}+\Omega(I)\frac{\partial f}{\partial \varphi}-\frac{\partial V}{\partial \varphi}F'(I)=0\,.$$

⟵ BJV equation

$$f(I,\varphi,t)=e^{-i\omega t}\sum_{m=1}^{\infty}\left[f_m(I)\cos m\varphi+g_m(I)\sin m\varphi\right]$$

⟵ Phase expansion, O&Y, 1990

$$\left[\omega^2-m^2\Omega^2(I)\right]f_m(I)=-2m^2\Omega(I)F'(I)\sum_{n=1}^{\infty}\int dI'V_{mn}(I,I')f_n(I');$$

⟵ Matrix BJV

$$V_{mn}(I,I')=-\frac{2}{\pi}\int_0^{\pi}d\varphi\int_0^{\pi}d\varphi'\cos(m\varphi)\cos(n\varphi')W(z(I,\varphi)-z(I',\varphi'))$$

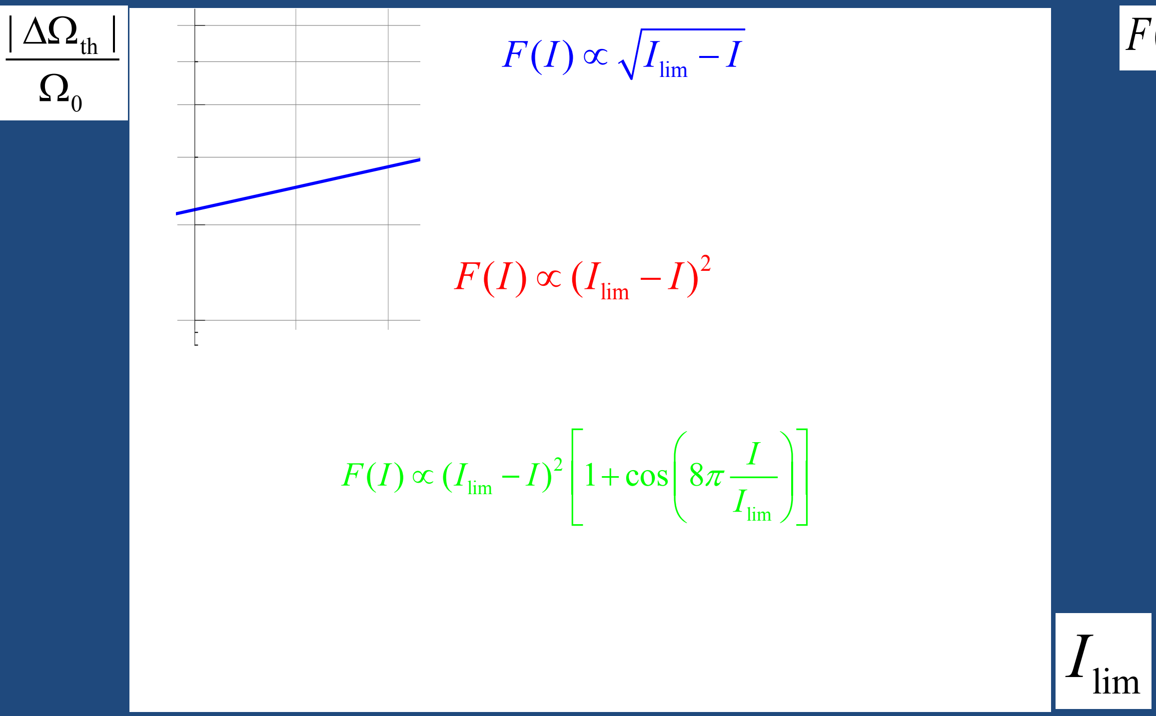
⟵ Matrix elements

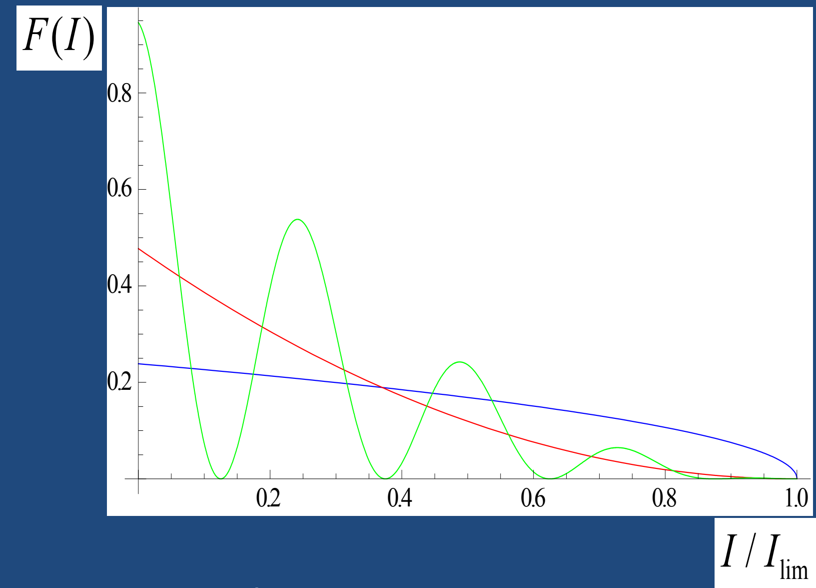
$$W(z)=-i\int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{Z(q)}{q}\exp(iqz)$$

⟵ Impedance

$$V_{mn}(I,I')=-2\,\mathrm{Im}\int_0^{\infty}dq\frac{Z(q)}{q}G_m(q,I)G_n^*(q,I');\quad G_m(q,I)\equiv\int_0^{\pi}\frac{d\varphi}{\pi}\cos(m\varphi)\exp\left[iqz(I,\varphi)\right].$$

LLD thresholds, space charge below transition, *m=n=1*





Bucket acceptance

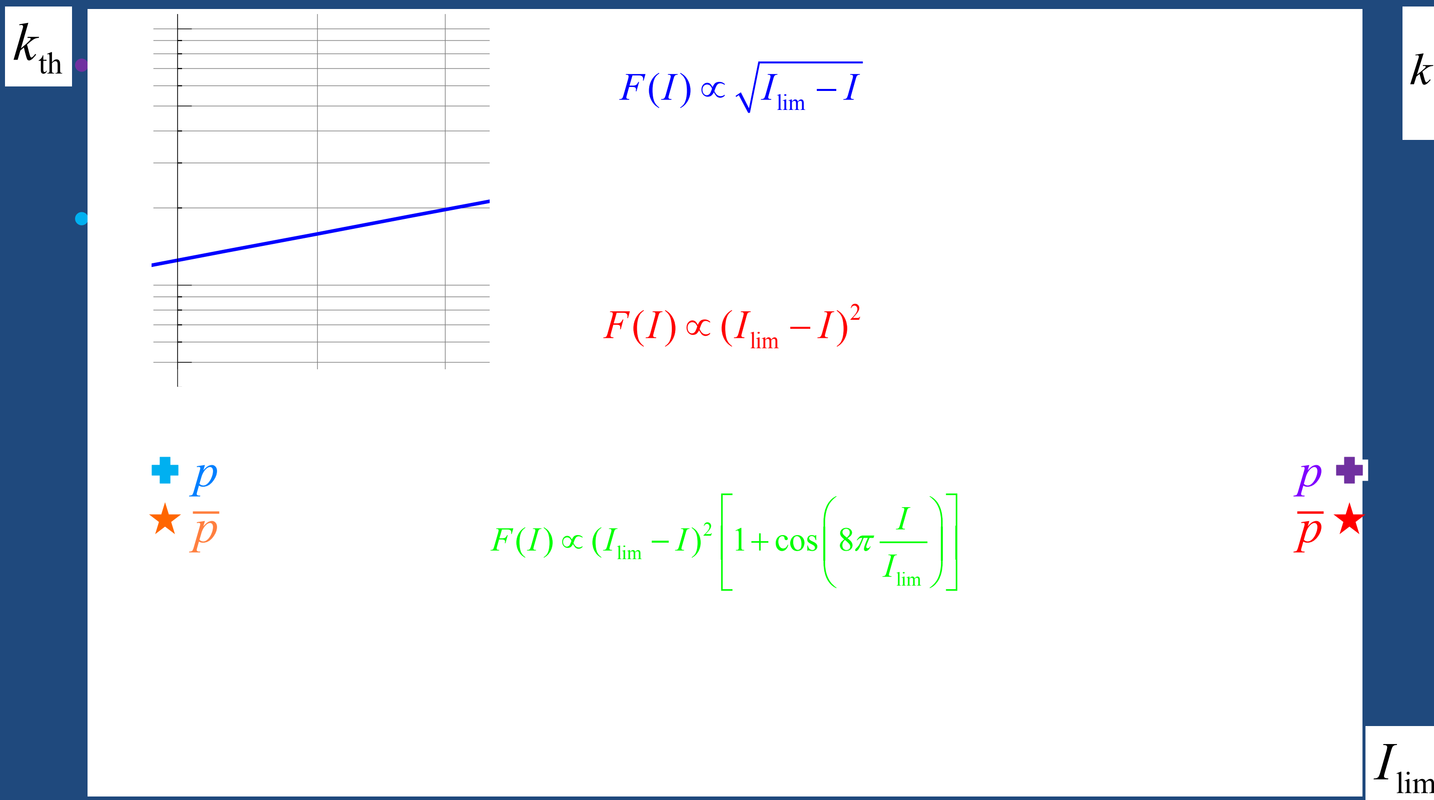
$$I_{max} = 8/\pi \approx 2.5$$

For the H-P distribution $F(I) \propto \sqrt{I_{lim} - I}$ the threshold is 3 times below rigid-bunch mode result.

For the coalesced bunch and full bucket the threshold is as low as $|\Delta\Omega_{th}|/\Omega_0 \approx 1\%$

Note how strong is dependence on the distribution function!

LLD thresholds, Tevatron



$$k = -\frac{2Nr_0\eta\omega_{\text{rf}}^3}{\gamma c \Omega_0^2} \frac{\operatorname{Im} Z_{\parallel}}{nZ_0} .$$

✚ protons, top energy

★ pbars, top energy

✚ protons, injection

★ pbars, injection

Conclusions

- The calculated LLD thresholds are in agreement with proton and antiproton dancing bunch observations and impedance model.
- Balbekov-Ivanov LLD power low is confirmed in BJV solution.
- Strong dependence on the distribution at small amplitudes suggests that some flattening would be helpful.
- This flattening can be done with resonant shaking of RF phase at zero-amplitude synchrotron frequency. Experiments confirms this idea, see

  Burov & Tan, "Bucket Shaking Stops Bunch Dancing in Tevatron",

  Wed poster session.

Acknowledgements

- Many thanks to V. Lebedev for multiple productive discussions.

Thank you all for your attention!