THE EFFECTS OF MIRROR SURFACE ERROR ON COHERENT X-RAY PROPAGATION IN XFELO CAVITY

G.T. Park, K.-J. Kim, R.R. Lindberg, ANL, Argonne, IL, 60439, USA

Abstract

We study the propagation of coherent X-ray mode through optical cavity of X-ray FEL oscillator (XFELO) including rough grazing incidence mirror.

INTRODUCTION

Recently X-ray FELoscillator(XFELO) scheme was proposed [1] as a fully (both temporally and transversally) coherent source of X-ray with peak brightness comparable to, average brightness several orders of magnitude higher than SASE. The optical cavity in XFELO, whose schematic view is shown in Fig.1, plays an important role of propagating coherent X-ray back to undulator while giving proper focusing in the undulator for maximum gain process and loosing minimum power. More specifically, optical cavity parameters are determined so that waists and their sizes of X-ray and electron beam coincide for maximum gain. Also high reflectivity crystals are used for backscattering and grazing incidence mirrors are operated with angle of incidence θ_g less than critical angle for total reflection. The performance of optical cavity, characterized by waist fluctuation and power loss can be easily disrupted by various errors in the cavity [3]. In this paper, we focus on the the effects of mirror surface errors and simulate the propagation of X-ray beam through the cavity.

X-RAY BEAM PROPAGATION

In this simulation we ignore the effects of crystals (to be discussed elsewhere) and assume the cavity consists of the grazing incidence mirrors and vacuum only. We take initial mode to be Gaussian of wavelength $\lambda \approx 10^{-10}$(m) and

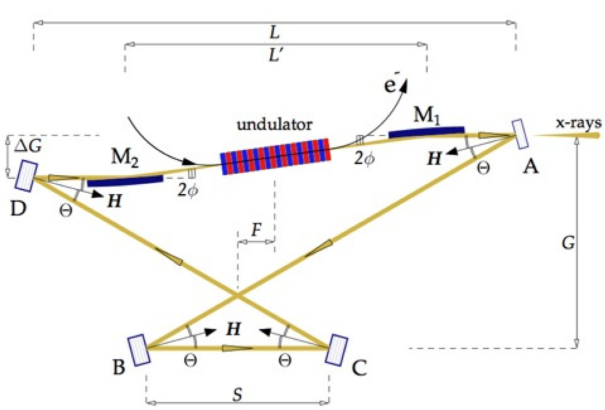
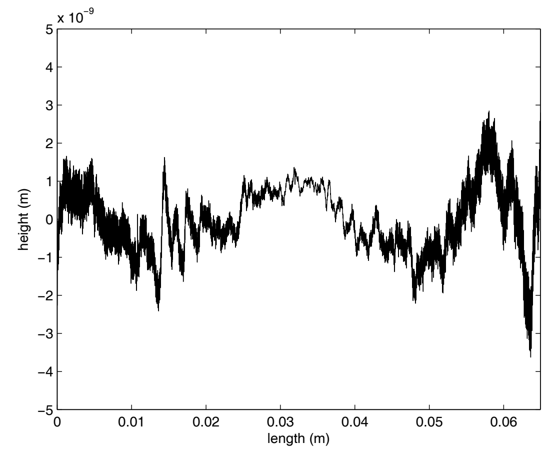
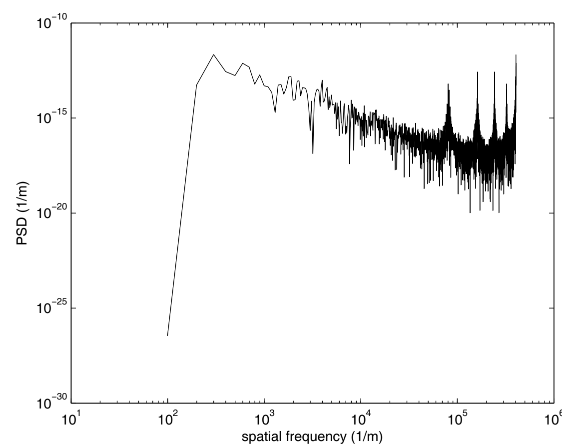


Figure 1: An X-ray FEL oscillator with a 4-crystal cavity.

waist size $w_0 \approx 1.78 \times 10^{-5}$(m). The mirror under consideration is parabolic mirror made of silica (SiO_2) whose refractive index n is $1 - 3.17 \times 10^{-6} - i(1.87 \times 10^{-8})$ at $\lambda \approx 10^{-10}$. The profile and PSD (power spectral density) of surface errors is shown in Fig. 2.



(a) Height profile



(b) Power spectral density

Figure 2: Although this is measurement from flat mirror, we assume the errors closely approximate elliptic case and treat sum of (a) and perfect elliptical surface as actual errors.

We describe the propagation of optical beam in 1D by Fourier optics. In particular, we use phase difference method in the vicinity of the mirror where small interference from different mirror position can be ignored. In $\{x, z\}$ coordinate where x is coordinate in transverse direction and z is along propagation axis, E field transforms to E' after mirror as

$$E'(x, x/\theta_g) = r_0 A(x) e^{-2ik\sin\theta_g h(x/\theta_g) - \frac{ik}{2f}x^2} \times E(x, x/\theta_g) \quad (1)$$

where

$$A(x) = (1 - e^{-|x+\theta_g L/2|/\alpha})(1 - e^{-|x-\theta_g L/2|/\alpha}) \quad \alpha \approx 250\text{m} \quad (2)$$

Here r_0 is (flat) Fresnel reflectivity coefficient, $A(x)$ smooth aperture function for finite sized mirror $L \approx 6.4 \times 10^{-2}$m, k spatial wave length of initial beam, h height of mirror surface, f focal length of mirror. For the rest of the propagation we use vacuum Fresnel-Huygens integral.

$$E''(x, z) = \int_{-\infty}^{\infty} dx' E'(x', x'/\theta_g) \frac{e^{ik\rho}}{\rho} \cos\varphi \quad (3)$$

Here ρ is eikonal function and φ is oblique angle. Combining (2),(3), we have E field transform per first half-turn as,

$$\begin{aligned} E_{n+1}(x, z) &= (1+g)\int_{-\infty}^{\infty} dt' E_n(x', x'/\theta_g) \\ &\quad \times r_0 e^{-2ik\sin\theta_g h(x'/\theta_g)} A(x') \frac{e^{ik\rho}}{\rho}\cos\varphi \\ &\approx (1+g)\int_{-\infty}^{\infty} dt' E_n(x', x'/\theta_g) \\ &\quad \times r_0(1 - 2ik\sin\theta_g h(x'/\theta_g)) \\ &\quad \times A(x')\frac{e^{ik\rho}}{\rho}\cos\varphi \end{aligned} \quad (4)$$

Here g is minimum FEL gain per pass that compensates power loss and maintains stable mode. The first term on the last line represents Gaussian mode propagation and the second term its perturbation and essential in relating surface error profile h to deformation of E.

SIMULATION RESULTS AND DISCUSSION

We use MATLAB to implement Eq. 4 to our optical cavity (Fig.1) in 1D case. Primary concern is power reduction while the beam goes through the mirror. In Fig.3, we observe the incoming beam spreads out reducing the peak amplitude by $\mathcal{R}_{simulation} \approx 0.983$.

This is predicted from Eq. 4, as its Fourier transform with $q_f = q_i$ reduces to well-known result [2]

$$\mathcal{R}_{theory} = |r_0|^2 e^{-4k^2 \sin\theta_g{}^2 \sigma^2} \approx 0.982 \quad (5)$$

Here σ is rms size of the mirror roughness profile. The second factor, describing (specular) power loss by roughness is called Debye-Waller factor in analogy with solid state physics terminology.

This indicates the power reduction with the portion of the beam in "wing" region diffracting out of physical aperture. The simplest way to determine the power loss per turn is to

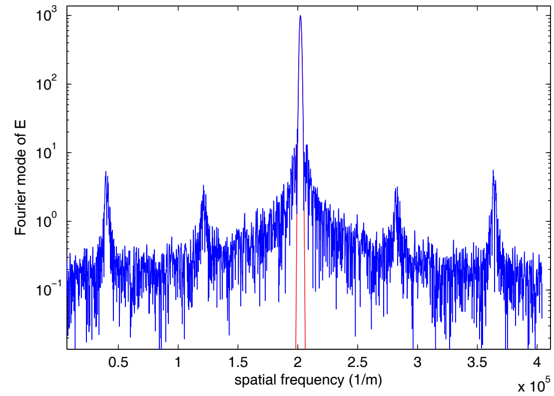


Figure 3: Fourier mode profile of $|E|$ in k space at M_1 (in log scale). Red line is incoming Gaussian beam and blue line is perturbed outgoing beam.

feed the beam with FEL gain g to compensate the loss and maintain the profile over many passes. (We found this value to be $g \approx 0.0085$) The remaining power forms equilibrium mode. However, there are in general wavefront distortions, as can be inferred from Fig. 4. The mirror could have an overall curvature error leading to an error in focal length, modifying the Rayleigh length.

Another important effect of the roughness was observed to be the small fluctuation of power profile and its growth as $\sqrt{n}$ over n passes (see Fig. 3).

This is also predicted by Eq. 4 by considering a single small Gaussian bump as representative roughness on the mirror.

$$h(x) = be^{-\frac{(t/\theta_g - x_0)^2}{2a^2}} \quad (6)$$

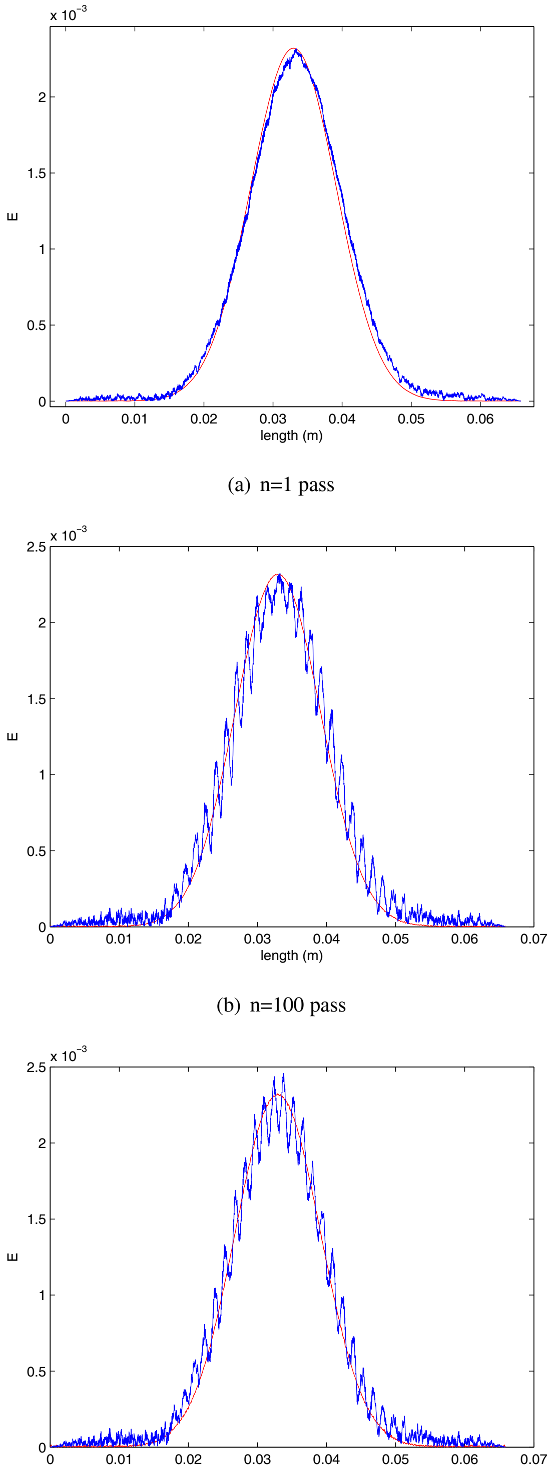
Then Eq. 4 gives an approximate power formula as

$$|E|^2 \approx \mathcal{E}_0^2 (1 + \frac{4k^2\theta_g^2 a^2 b}{L} \cos\{\frac{k}{2L}(t - x_0\theta_g)^2\}) \quad (7)$$

Its fluctuation amplitude term $4\mathcal{E}_0 k^2\theta_g^2 a^2 b/L$ can be interpreted as follows: Incoming field upon bump has power $a\theta_g\mathcal{E}_0^2$ and reflected field $4bk\theta_g a\mathcal{E}_0^2$, which diffracts out with angle $\varphi \approx \lambda/(a\theta_g)$. So at distance L down the optical path the power is spread over the range $a' \approx L\varphi \approx L\lambda/(a\theta_g)$. Therefore conserved power is given as $\mathcal{P} \approx 4\mathcal{E}_0^2 a^2 k^2 \theta_g^2 b/L$. If we now assume random distribution of N bumps, the power after n passes would roughly be given as

$$\mathcal{P}_n \approx \sqrt{n}\frac{4a^2k^2\theta_g^2 b}{L\sqrt{N}} \quad (8)$$

In principle, from the power profile, one could conversely get some information about "effective" bump size a and number N on the mirror. ($a^2/\sqrt{N}$ is determined from Eq. 8).



(a) n=1 pass

(b) n=100 pass

(c) n=400 pass

Figure 4: $|E|$ field at M_2 after n passes. We observe the beam reaches stable mode about after n=100 (fluctuation of the mode would keep magnifying without aperture).

CONCLUSION

In conclusion, we find low compensation factor $g \sim 0.01$ shows power loss due to rough mirror is small enough and easily can be overcome by FEL gain which ranges over $0.3 \sim 0.4$. In terms of controlling the fluctuations, large aperture size reduces power loss but allows more fluctuation to propagate. Compromise has to be made for optimal performance.

Further factors that affect the X-ray propagation are orientation misalignments of mirrors and crystals, focal length errors and bunch timing errors. These are discussed in detail elsewhere by matrix method in phase space and supermode theory [3], but its main results can be summarized as follows: the orientation errors deviate optical axis of propagation and the stability condition should be treated as an eigenvalue problem. Quantitative computation shows the angle error must not exceed 2×10^{-8}. Computing tune shift due to focal length error shows that the error should be less than 5%. Beyond stability issue, these errors and bunch timing errors disturb overlap of the radiation with electron beam undermining gain process. In case of bunch timing error, solution to supermode theory shows timing should be kept within ~ 50 fs ($\sim 12\mu$m).

REFERENCES

[1] K-J Kim, Y. Shvyd'ko and S. Reiche, Phys. Rev. Lett 100, 244802 (2008).

[2] S.K Sinha, E.B. Sirota, S. Garoff, Phys. Rev. B 38, 2297 (1988).

[3] G.T Park, K-J. Kim, R.R Lindberg, in preparation.