

# NONLINEAR RESONANCE MEASUREMENTS AND CORRECTION IN STORAGE RINGS

R. Bartolini<sup>1,2</sup>

<sup>1</sup>Diamond Light Source, Oxfordshire, UK, <sup>2</sup>John Adams Institute, University of Oxford, UK,

## Abstract

Several theoretical and experimental techniques have been developed in recent years to correct the detrimental effect of nonlinear resonances on dynamic aperture, beam lifetime, injection efficiency and beam loss distribution. These issues are equally important in synchrotron light sources and high energy colliders. We present a review of the theoretical and experimental results obtained at the Diamond light source on the characterization of the nonlinear resonances and on the comparison between the nonlinear model of the machine to the real accelerator.

## INTRODUCTION

The Diamond storage ring has been commissioned in January 2007 [1]. After the successful implementation of the linear optics of the storage ring, many machine development shifts were used to test several different techniques for the characterisation of the nonlinear beam dynamics. These studies have a clear operational drive in the improvement of the performance of the ring, by understanding the limiting factors of the dynamic and momentum apertures. The techniques used are based on the analysis of turn by turn data to detect and characterise the resonances excited in the storage ring. The information obtained can be used to correct the beam dynamics reducing the impact of the resonance on the stability of motion. In this way we can improve the injection efficiency, the Touschek lifetime and understand the beam losses distribution around the ring.

The main techniques tested at Diamond are based on the analysis of the frequency map [2] and of the resonance spectral lines [3, 4]. They offer a complementary view on the resonances excited in the ring by giving the global resonances structure in the frequency space and the local s-dependent variation of the resonance driving terms. Both these techniques have been used, not only to understand which resonances are excited and measure their strength, but to actively correct [5] their effect on the beam dynamics and to calibrate [6] the nonlinear model of the storage ring.

A necessary step of this investigation is the determination of a reliable model of the nonlinear beam dynamics. The nonlinear beam dynamics has been optimised for large dynamic and momentum aperture in the design phase of the ring and a correct implementation of the sextupole distribution is mandatory to achieve the best performance. It is clear that any further numerical optimisation of the beam dynamics has to be carried out on a model of the ring which reflects as much as possible the behaviour of the real machine. The effort therefore

has been threefold: measurement of the frequency map, measurement of the spectral lines and refinement of the model of the ring to be able to apply meaningful corrections guided by the model.

Since September 2007, Diamond is equipped with the necessary hardware to perform an advanced investigation of the nonlinear beam dynamics. Two pinger magnets can excite betatron oscillations of the stored beam to large amplitudes. A crucial hardware component is a well understood systems of highly performing turn-by-turn BPMs. In particular, a correct understanding of the Frequency Maps and of the resonant driving terms depends critically on the proper reconstruction of the betatron oscillation from the turn by turn BPMs data. Significant improvements in the modelling were made after a series of unsuccessful comparison pointed out a number of subtleties in the BPMs response to the betatron oscillations which required careful consideration and correction.

In this paper we review the main results of the campaign of measurements performed at Diamond in the last three years on the analysis of the nonlinear dynamics, the experimental characterisation of the nonlinear resonances and the issue met in the correct reconstruction of the betatron oscillations from the turn by turn signals acquired at all BPMs.

## MEASURED AND MODEL FM

Frequency Maps and detuning with momentum can be used to understand the resonance excited in the ring, to compare measured data with the prediction of the nonlinear ring model and ultimately to calibrate the model of the ring.

To this aim, all available measured magnetic components of each individual quadrupole and sextupole have been introduced into the model, albeit for the dipole field where we were forced to rely on the measurement of a single prototype. The betatron motion has been described with the exact Hamiltonian and fringe fields are added to both dipoles and quadrupoles. The measured longitudinal roll off of the magnetic field of the main dipole is also taken into account. The code used for comparing tracking and beam data is a modified version of AT [7] where new pass method were developed for the above mentioned items and have been benchmarked with MADX-PTC [8].

The dependence of the betatron tunes on the off-momentum deviation and on the amplitude of the oscillations was used as target vectors for a fit of the sextupole families values. We have built a vector whose

components are the betatron tunes at a selected number of off-momentum points and the betatron tunes at a selected number of amplitudes in x and y, from the frequency map. In this way we obtain a target vector that we can measure on the real machine and, at the same time, can be computed on the model. The differences between these two vectors are used to make a least square fit of the sextupole gradients. The systematic multipole components in the main dipole were also used as fit parameters. The result of the matching on the detuning with momentum and frequency maps is reported in Figure 1 and 2. The corresponding agreement of the measured and model dynamic aperture is reported in Fig. 3

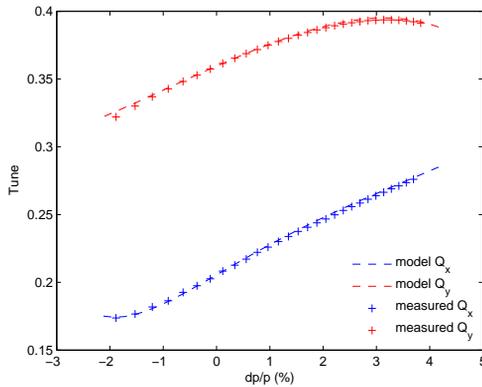


Figure 1: detuning with momentum: measured (crosses) model (continuous line barely visible below the crosses).

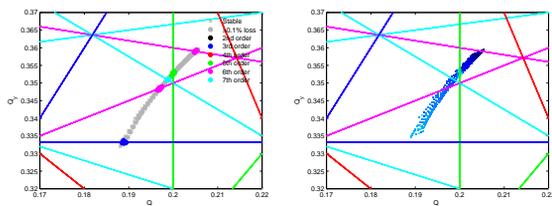


Figure 2: measured (left) and model (right) frequency map for the Diamond storage ring

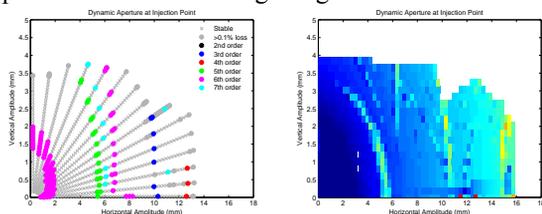


Figure 3: Dynamic aperture and resonance lines plotted in the (x, y) plane: measured (left) and model (right).

This matching was achieved with a variation of the sextupole gradients which are within 2 percent. This value appears to be high with respect to the measurement errors of the sextupole gradient and a careful re-analysis of the calibration tables of the sextupole gradient is in progress. Another possible source of discrepancy is the lack of agreement with natural chromaticity in the V plane which is then redistributed among the gradients of the various sextupole families. The origin of this discrepancy is still under investigation. The fit allowed the identification of an excessive contribution to the normal octupole

component in the main dipole which is responsible for significant variation in the extension of the frequency map. The normal octupole component initially used in the model comes from the main dipole measurement of a single prototype and was initially assumed as a systematic error in all the dipoles. This assumption produced an unrealistically large detuning with amplitude and a wide frequency map. Using the normal octupole component of the main dipole as a fit parameter, we finally achieve a good agreement between machine and model as shown in Figures 1-3.

### MEASURED AND MODEL SPECTRAL LINES

Nonlinear resonance driving terms can be directly measured from the Fourier analysis of the betatron oscillations of the beam in the storage ring [3, 4]. The basic idea is to connect the amplitude and phase of the Fourier coefficients of the spectral lines with the amplitude and phase of the driving terms of a given resonance. This relation allows building a fit algorithm for the reconstruction of the nonlinear machine model based entirely on the comparison of the amplitude and phase of the spectral lines. The model reconstruction has been demonstrated in tracking data and a first experimental investigation has shown that it is indeed possible to correct simultaneously several resonance driving terms [5]. The procedure introduced in [5] mimics closely the approach that LOCO [9] takes to correct the linear optics of the ring where the role of the orbit response matrix is taken by the Fourier coefficients of the spectral lines excited by nonlinear resonance driving terms, measured at all BPMs, and the role of the quadrupoles is now taken by the sextupoles.

Several experiments with pinged beams performed at Diamond showed that the amplitude of the spectral lines related to nonlinear resonances can be measured with very good precision and corrected to restore the original pattern of the amplitude along the ring. In the experiment we targeted the amplitude of the  $Q_x - Q_y$  spectral line measured in the vertical plane and the amplitude of the  $-2Q_x$  spectral line measured in the horizontal plane. These are related to the driving terms of the resonances  $Q_x \pm 2Q_y$  and  $3Q_x$  respectively. We have verified experimentally that targeting a single spectral line can produce a good correction of the driving term as shown in Fig. 4. However this does not necessarily improve the DA and momentum aperture of the ring and can produce unrealistic sextupole gradients (red line in Fig. 5) If two spectral lines are taken into account the correction can have beneficial effects on the performance of the ring and an improvement of the Touschek lifetime of 10% was measured.

This technique has some limits: firstly it is based on the assumption that the first order perturbative theory adequately describes the nonlinear beam dynamics, secondly it relies on very precise measurements of the turn-by-turn data. Decoherence of the excited oscillations

reduces the number of turns available and the machine tune stability has also to be controlled carefully if meaningful results are to be extracted. Nevertheless the indication provided by the experiment shows that this technique holds great potential for the characterisation of the nonlinear beam dynamics in storage rings.

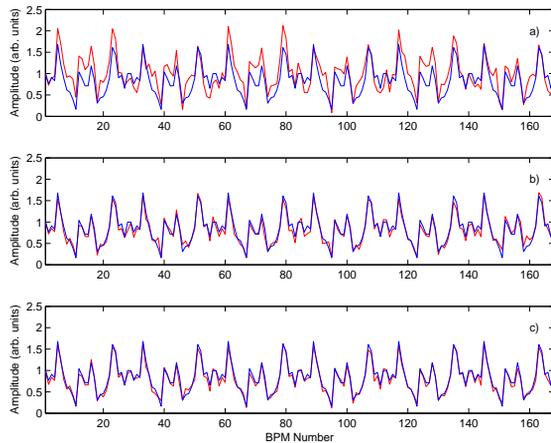


Figure 4: Measurement and correction of the spectral line  $Q_x - Q_y$  measured in the vertical plane: red – measured, blue – model, before correction (top) after one iteration (middle) after two iterations (bottom).

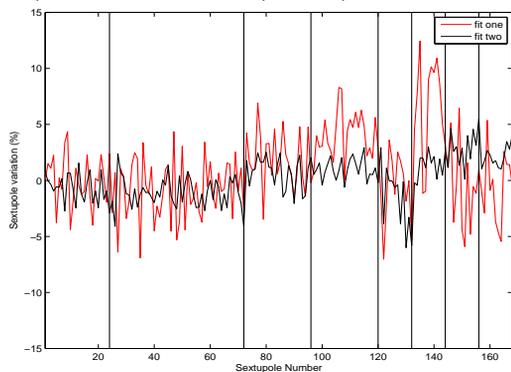


Figure 5: Sextupoles gradient variation required by the fit procedure outlined in the text; red for the case where one resonance was targeted; black for the case where two resonances were targeted.

## BETATRON OSCILLATIONS FROM TURN BY TURN BPMS DATA

In order to reconstruct correctly the betatron oscillation from the turn-by-turn data signal, a number of corrections to the beam data have to be applied. When investigating large amplitude oscillations the geometric nonlinearities of the BPMS have to be taken into account. To this aim we used a numerical procedure, proposed in [10], for the inversion of the nonlinear response of the four BPM buttons of the BPM to the  $(x, y)$  position of the beam inside the BPM block. Further investigation of this method at Diamond has put in evidence how this approach is superior to the Taylor expansion in the  $(x, y)$  plane of the BPM response based on the four button signal [11].

Another important aspect of the response of the BPMS to an excited betatron oscillation concerns the time filter

used to generate the BPM signal corresponding to a given turn from the ADC samples. The filter used is a characteristic of the BPM electronics hardware and firmware. The signal at a given turn is computed from the digitised samples provided by the BPM which produce 220 points within a turn (17 MHz sampling). To reduce this higher sampling rate to the turn by turn rate while fulfilling the Nyquist criteria and minimising out of band noise, the filter has to extend over the neighbouring turns so that the actual signal ends up containing some information on the position of the beam in the neighbouring turns. Figure 6 (left) shows the time filter (black dotted line) used, extending over more than 6 turns. In this way the filter will mix the contribution from the neighbouring turns. Since this process is entirely numerical as part of the digital signal processing of the BPMS, it can be simulated for a given fill pattern and its alignment relative to the filter. The resulting frequency response can be precisely calculated as reported in Figure 6 (right) and it shows clearly that the BPM response to an excitation with fixed amplitude depends significantly on the tune value [6].

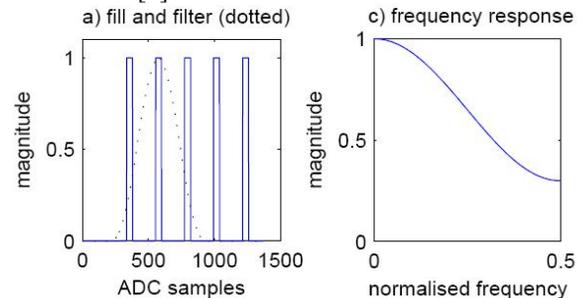


Figure 6: Time filter (left) and corresponding frequency response of the BPM (right).

This aspect was crucial in assessing correctly the amplitude of the betatron oscillations and it ultimately allowed a successful fit of the detuning with momentum and detuning with amplitude as described. Lastly we notice that the alignment of the time filter depends on the location of the BPM in the ring to take into account the time of flight of the beam between the various BPMS. It also depends slightly on the fill pattern used in the experiments. These effects, however, can be deconvoluted from the original BPM signal to provide the actual betatron oscillations of the beam.

## REFERENCES

- [1] R.P. Walker, pg. 66, in APAC07, (2007).
- [2] H. S. Dumas et al., PRL 70, pg. 2975, (1993)
- [3] J. Bengtsson, CERN 88-05, (1988).
- [4] R. Bartolini, Part. Acc. **59**, pg. 93, (1998).
- [5] R. Bartolini PRSTAB **11**, 104002, (2008).
- [6] R. Bartolini et al., in preparation
- [7] A. Terebilo, pg. 3203, in PAC01, (2001).
- [8] F. Schmidt, pg. 1272, in PAC05, (2005).
- [9] J. Safranek, NIM **A388**, 27, (1997).
- [10] R.W. Helms et al., PRSTAB **8**, 062802, (2005).
- [11] R. Bartolini et al, DLS AP-SR-REP-0174, (2010).