CHROMATICITY CORRECTION FOR A MUON COLLIDER OPTICS *

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Abstract

Muon Collider (MC) is a promising candidate for the next energy frontier machine. However, in order to obtain peak luminosity in the 10^{34} cm^{2}s^{-1} range the collider lattice design must satisfy a number of stringent requirements.

In particular the expected large momentum spread of the muon beam and the very small β^* call for a careful correction of the chromatic effects.

Here we present a particular solution for the interaction region (IR) optics whose distinctive feature is a three-sextupole local chromatic correction scheme. The scheme may be applied to other future machines where chromatic effects are expected to be large.

INTRODUCTION

The expected large muon energy spread requires the optics to be stable over a wide range of momenta whereas the required luminosity calls for β^* in the mm range. To avoid luminosity degradation due to hour-glass effect, the bunch length must be comparatively small. To keep the needed RF voltage within feasible limits the momentum compaction factor must be small over the wide range of momenta. A low β^* means high sensitivity to alignment and field errors of the Interaction Region (IR) quadrupoles and large chromatic effects which limit the momentum range of optics stability and require strong correction sextupoles, which eventually limit the Dynamic Aperture (DA).

Finally, the ring circumference should be as small as possible, luminosity being inversely proportional to the collider length.

A promising solution for a 1.5 TeV center of mass energy MC with β^*=1 m in both planes has been proposed [1]. This β^* value has been chosen as a compromise between luminosity and feasibility based on the magnet design and energy deposition considerations [2], [3].

The proposed solution for the IR optics together with a new flexible momentum compaction arc cell design allows to satisfy all requirements and is relatively insensitive to the beam-beam effect.

CHROMATIC CORRECTION

To obtain large momentum acceptance it is necessary to correct the dependance on momentum of β-functions and tunes. The former is described in terms of the Montague chromatic functions [4]

$$B_z \equiv \frac{1}{\beta_z^{(0)}}\frac{\partial \beta_z}{\partial \delta} \qquad A_z \equiv \frac{\partial \alpha_z}{\partial \delta} - \alpha_z^{(0)} B_z$$

($z = x/y$ and $\delta \equiv \Delta p/p$) which obey the equations

$$\frac{dB_z}{ds} = -2A_z\frac{d\mu_z}{ds} \quad \text{and} \quad \frac{dA_z}{ds} = 2B_z\frac{d\mu_z}{ds} - \beta_z^{(0)} k \quad (1)$$

where $k \equiv \pm(K_1 - D_x K_2)$ (the positive and negative sign holding for the x and y plane respectively), K_1 and K_2 being the quadrupole and sextupole strengths respectively.

The dependence of tunes on momentum is described in terms of chromaticity. Second order chromaticity may be written in the form [5]

$$\xi_z^{(2)} = \frac{1}{8\pi}\int_0^C ds\,\left(-kB_z \pm 2K_2\frac{dD_x}{d\delta}\right)\beta_z^{(0)} - \xi_z^{(1)}$$

($\xi_z^{(1)}$ is the linear chromaticity) which shows that chromatic function B_z and second order dispersion should be both compensated.

"Classical" approach

The classical approach consists in correcting the chromatic IR beta-wave with sextupole families in the arcs where the dispersion is naturally large. Different families of sextupoles are used to correct the linear and second order chromaticity and possibly the first order dependence of $\alpha(s)$ on momentum; non-orthogonality of such corrections result in an increase of the needed sextupole strenght.

Constraints on the phase advance between sextupoles of the same family allow to make the lowest order driving terms of the 3th order resonances vanish.

This scheme, used in successfully operating colliders as Tevatron and LHC, has been tried for some earlier MC versions but led to extremely small values of the momentum stability range.

"Special sections" approach

Owing to the necessarily high chromaticity of a MC IR, the failure of the classical approach is not surprising.

As an alternative, it has been proposed the use of "special sections", with large beta functions and dispersion, next to the low-β region: after the first sextupole located at a knot of the IR chromatic wave, a pseudo $-I$[1], section is inserted between it and a "*twin*" sextupole compensating the nonlinear kick. In practice such kind of schemes may be prone to focusing errors.

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[1] $\mu = \mu_0 + (2n+1)\pi$ and $\beta = \beta_0$, no requirements on α

Eventually, if sextupoles correcting horizontal and vertical chromaticity are not orthogonal, a *non-interleaved* correction scheme may be considered.

Some 3 mm β^* MC optics using this concept are described for instance in [6], [7], [8]. These designs however have insufficient DA and/or small momentum acceptance.

A similar approach for the IR chromatic correction of a 3 mm β^* 2×2 TeV MC optics has been applied by K. Oide[9]. It uses non-interleaved sextupoles for both IR and arcs. This design has large DA and momentum acceptance, but owing to the extremely large peak value of β_y (900 km) and the very strong sextupoles is likely unrealistic.

Local chromatic correction

More recently a *truly local* chromatic correction has been proposed[1].

Let us look at the solutions of Eqs.(1) with starting conditions B_z=A_z=0 at the IP. Moving away from the IP, A_z becomes non zero when the low-β quadrupoles are encountered, but as long as the phase advance does not change, the amplitude function, B_z, is unchanged too. At the low-β quadrupoles, where β_z is large, the phase advance changes slowly and there is a possibility of correcting the chromatic perturbation before $\beta_z(\delta)$ and $\mu_z(\delta)$ start differing from the unperturbed values.

Such local correction with sextupoles is possible if the IR dispersion is made non-vanishing. If D_x=D'_x=0 at the IP, insertion of relatively strong bending magnets in the IR region is necessary. They can be beneficial for avoiding neutrinos hot spots too.

IR OPTICS OPTIMIZATION FOR CHROMATIC CORRECTION

Fig. 1 shows the IR layout and optics of the proposed MC.

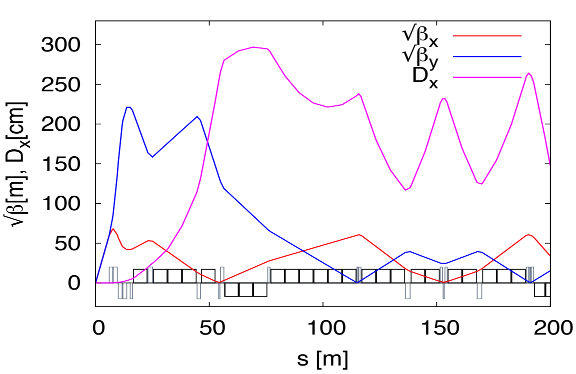

Figure 1: IR layout and Twiss functions.

The peak beta asymmetry of the [9] design has ben kept. In particular $\hat{\beta}_x$ and the IR horizontal chromaticity are much smaller than in the vertical plane. The larger IR vertical chromatic wave is corrected first, with a single sextupole, S1 (see Fig. 2 where MAD-X IR chromatic functions and sextupole locations are shown), at $\Delta\mu_z \simeq 0$ from the low beta quadrupoles. The choice $\hat{\beta}_y > \hat{\beta}_x$ (rather than the opposite) is very convenient. Indeed, by constraining β_x at the location of this strong sextupole to a very small value (1 m in this design) it is possible to keep the detuning with amplitude coefficients [10], which for a *normal* sextupole are proportional to β_x, small. This is more in general, a consequence of the fact that the hamiltonian of a normal sextupole has the form $H = ax^3 - 3axz^2$. Thus, if ϵ_x=ϵ_y as for muons, non-linear kick is weak due to smallness of $|x_\beta|$ and does not need compensation by a second sextupole. 5 dipoles, with the help of quadrupole offsets, create 2.6 m dispersion at S1 location. The integrated strength of the sextupole is 0.142 m^{-2}.

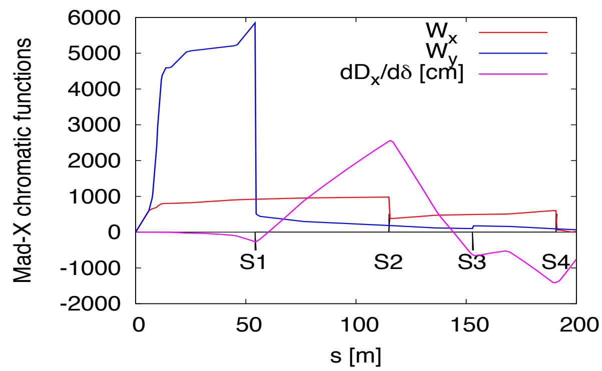

Figure 2: IR MAD-X chromatic functions, $W_z = \sqrt{(A_z^2 + B_z^2)}$, and sextupole locations. The sextupole S3 at 150 m corrects $dD_x/d\delta$.

A local correction of the horizontal chromatic wave is not possible, as it would require to keep a large value of β_x until the corrector location. Thus a second sextupole (S2), at $\Delta\mu_x$=π from the low-β quadrupoles, corrects the IR horizontal chromaticity. A pseudo $-I$ section is inserted between this sextupole and a "twin" sextupole (S4) which compensates S2 aberrations [2]. The dispersion function at the center of the I section can be made rather large while β_x is small (about 1 m in the present design) making it a convenient place for one more sextupole (S3) which provides control of the second order dispersion. Multipoles for higher order vertical chromaticity correction can be installed there as well. They introduce non-linear detuning which is compensated with additional octupoles in D_x=0 location. The absolute value of MAD-8 STATIC detuning coefficients after correction are smaller than 1×10^4 m^{-1}.

ARCS

The IR in this design has a large positive contribution to α_p which must be compensated in the arcs so to get $|\alpha_p| \leq 10^{-4}$. The proposed arc cell is described in [1].

[2] Although the horizontal chromaticity is relatively small the use of a twin sextupole in the horizontal plane turned out to be necessary for achieving a large DA.

It allows almost orthogonal correction of chromaticity in the two planes and control of α_p and its derivative with relative momentum deviation. The arc chromaticity is corrected with just one (interleaved) family per plane. 12 cells are needed for closing the ring. Phase advance per cell is 300 degrees which gives cancellation over 6 cells of the lowest 3th order resonance driving terms.

A *dispersion free* tuning section was introduced between IR and arcs. Compared with other designs, the ring is compact ($\mathcal{L}$=2727 m, two IPs). The resulting momentum compaction is -1.2$\times10^{-5}$.

LATTICE PERFORMANCE

The energy range in which the optics is stable is $\pm1.2\%$, sufficient for the various expected scenarios. Tracking performed over 1000 turns, the time needed for the beam current to decay by about a factor 2, shows that the on-energy DA is 5.7 σ for ϵ_N=25 μm. More details may be found in [1].

Beam-beam effect

Dynamic beta considerations recommend choosing phase advances between IPs to be just above multiples of π. However, while this reduces β^*, it increases the β functions at the low-β quadrupoles making this choice dangerous for a MC where, owing to high gradient requirements and necessity of protecting the magnets from muon decay products, the low-β quadrupole aperture is tight.

For this reason it has been chosen the working point Q_x=20.56 and Q_y=16.58 which is beneficial for minimizing detuning with amplitude and orbit sensitivity to misalignments, and provides space for the beam-beam tune shift. With $\mu_x(S1) - \mu_x(IP) = \mu_y(S2) - \mu_y(IP) \simeq \pi$ and this choice of the working point, for small beam-beam parameter value, ξ_0, where analytical expressions may be used, it can be seen that the beam-beam effect does not compromise the orthogonality of the correction. This has been confirmed by simulations with MAD-X for ξ_0 $\simeq$0.1/IP. Simulations by using Mathematica [11] have shown also that, with ξ_0 $\simeq$0.1/IP, the beam-beam effect reduces β^* for long bunches, $\sigma_\ell \simeq \beta^*$, almost completely compensating for the hour-glass effect, similarly to beam disruption in linear colliders. Beam-beam studies done on a previous optics version (w/o tuning section) indicate a DA reduction of about 20% in presence of a beam-beam parameter of 0.1/IP.

Fringe fields effect

Magnet imperfections, such as multipole errors and fringe fields, can compromise the dynamic aperture, which for a muon collider should be at least 3σ. Preliminary studies [11] showed that effects of multipole errors can be almost completely compensated with higher order non-linear correctors.

The impact of the fringe field of the low-β quadrupoles on DA is showed in Fig. 3 for an earlier version of the lattice, without IR quadrupole displacements, having a somewhat smaller nominal DA (4.4σ). Dynamic aperture is computed in terms of starting coordinates at IP, with x'_0 =y'_0 =0, using MAD-X PTC which treats fringe fields in "hard-edge" approximation. The beam transverse σ at the IP is 6 μm for ϵ_N=25 μm. A significant reduction in DA is observed so that special correction is necessary.

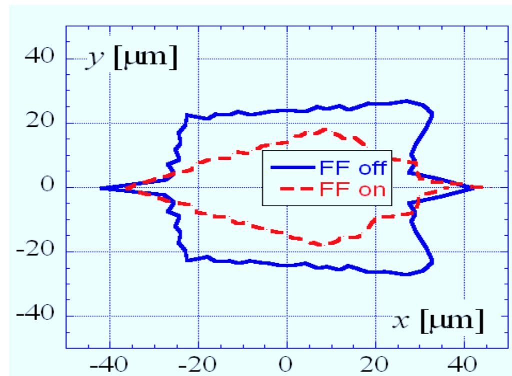

Figure 3: 1000 turns DA w/o (blue contour) and with (red) fringe fields.

SUMMARY

Owing to the large chromaticity, the IR optics of a high luminosity Muon Collider must be designed having non-linear corrections in mind. In the scheme here presented the larger vertical chromatic wave is corrected "in loco", while a pseudo $-I$ insertion provide convenient locations for the correction of the horizontal chromatic wave. The energy range, within the machine is stable, is about $\pm1.2\%$. The dynamic aperture for the nominal optics is sufficiently large ($\geq 5\sigma$), however special attention should be paid to compensation of multipole errors and fringe fields effects.

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