THE CORRECTION OF LINEAR LATTICE GRADIENT ERRORS USING AN AC DIPOLE*

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Abstract

Precise measurements of optics from coherent betatron oscillations driven by ac dipoles have been demonstrated at RHIC and the Tevatron. For RHIC, the observed rms beta-beat is about 10%. Reduction of beta-beating is an essential component of performance optimization at high energy colliders. A scheme of optics correction was developed and tested in the RHIC 2008 run, using ac dipole optics for measurement and a few adjustable trim quadrupoles for correction. In this scheme, we first calculate the phase response matrix from the measured phase advance, and then apply a singular value decomposition (SVD) algorithm [1,2] to the phase response matrix to find correction quadrupole strengths. We present both simulation and some preliminary experimental results of this correction.

INTRODUCTION

It has been demonstrated in RHIC and the Tevatron that an ac dipole, as a non-destructive diagnostic tool, can be used to precisely measure optics [3, 4]. One important application of this technique is to correct linear gradient errors. The general relation between quadrupole strength and betatron phase variations under the linear approximation is given by

$$\begin{pmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \cdots \\ \Delta\phi_{n_{bpm}} \end{pmatrix} = M \begin{pmatrix} \Delta kl_1 \\ \Delta kl_2 \\ \cdots \\ \Delta kl_{n_q} \end{pmatrix} \tag{1}$$

where $\Delta\phi_i$ is the phase variation at the location of the i th beam position monitor (bpm), Δkl_i is the gradient variation of the i th quadrupole, M is the phase response matrix defined as

$$\begin{aligned} M_{i,j} = \frac{\beta_j}{4\sin(2\pi\nu)} &\{\sin(2\pi\nu) + \sin(2\phi_j - 2\pi\nu) \\ &+ sign(\phi_i - \psi_j)[\sin(2\pi\nu) + \sin(2|\phi_i - \psi_j| - 2\pi\nu)]\} \end{aligned} \tag{2}$$

β_j and ψ_j are the betatron function and phase at the position of the j th quadrupole respectively, ϕ_i is the betatron phase at the position of the i th bpm and ν is the unperturbed betatron tune. Since the phase beat $\Delta\phi$ can be measured from the coherent oscillation excited by an ac dipole, the inversion of equation (1) can be used to find the proper strengths of a few adjustable quadrupoles such that the measured phase beat is reduced. The number of bpms is usually much larger than the number of adjustable quadrupoles, which makes equation (1) an over determined linear system. For such a system, the least χ^2 solution for the quadrupoles' strengths is given by a singular value decomposition (SVD) algorithm[3].

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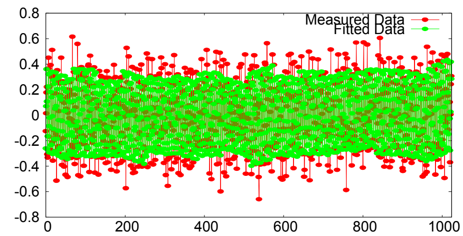
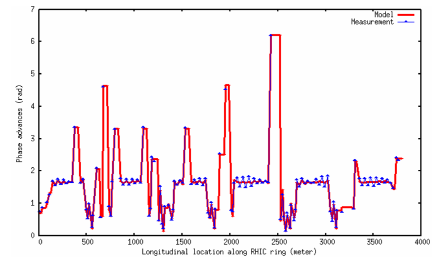


Figure 1: Example of excluded bpm data due to suspiciously large χ^2. The red dots are measured data and the green line is the fitting result.

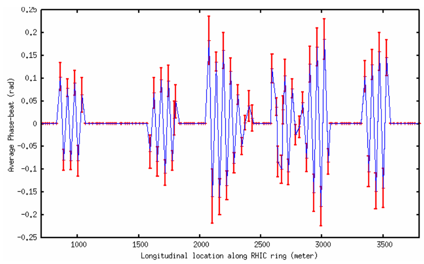
PHASE BEAT MEASUREMENT

For beam betatron oscillations driven by an ac dipole, the betatron phase at each bpm location is obtained by fitting the measured turn by turn data[1]. We used three filters to exclude unreliable bpm data from our analysis. The first filter is a status bit that arrives with the bpm data. The FFT of the turn-by-turn data provides the second filter - data with apparent driving tune errors are screened out. After fitting all bpms, the fitting χ^2 serves as the third filter. Bpms with suspiciously larger fitting χ^2 compared with other bpms are excluded from further analysis. Fig. 1 shows an example of bpm data with suspiciously large fitting χ^2.

Fig. 2 shows preliminary results of phase beat measurement in the RHIC 2008 run. As shown in Fig. 2 (b), the error bars are around 20%, as calculated from the variance of 5 measurements. Improving bpm data quality and the number of measurements are critical to reduce the statistical errors.



(a)



(b)

Figure 2: Phase beat measurement from the RHIC 2008 run. The abscissa is the location of the bpms in units of meters and the ordinate is the phase in units of radians. (a) shows the result of one measurement. The red line is calculated from model and the blue dots are measured data. (b) shows the averaged phase beat of 5 measurements.

SVD

RHIC has 36 trim quadrupoles with separate power supplies. We plan to use them as knobs to correct linear gradient errors. The phase response matrix M is written:

$$M = UWV^T$$

with $U^TU = I$, $V^TV = I$ and W being diagonal matrices. The strengths of the trim quadrupoles are obtained by inverting equation (1):

$$\begin{pmatrix} \Delta kl_1 \\ \Delta kl_2 \\ \ldots \\ \Delta kl_{n_{tq}} \end{pmatrix} = V \begin{pmatrix} 1/w_1 & 0 & 0 & 0 \\ 0 & 1/w_2 & 0 & 0 \\ 0 & 0 & \ldots & \ldots \\ 0 & 0 & \ldots & 1/w_{n_{tq}} \end{pmatrix} U^T \begin{pmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \ldots \\ \Delta\phi_{n_{bpm}} \end{pmatrix}. \quad (3)$$

Except for giving the least χ^2 solution for quadrupole correction strengths, the SVD algorithm also provides information about the correcting range of the trim quadrupoles and their vulnerabilities to noise in measured phase beat. We rewrite equation (5) as

$$\begin{pmatrix} \Delta kl_1' \\ \Delta kl_2' \\ \ldots \\ \Delta kl_{n_{tq}}' \end{pmatrix} = \begin{pmatrix} 1/w_1 & 0 & 0 & 0 \\ 0 & 1/w_2 & 0 & 0 \\ 0 & 0 & \ldots & \ldots \\ 0 & 0 & \ldots & 1/w_{n_{tq}} \end{pmatrix} \begin{pmatrix} \Delta\phi_1' \\ \Delta\phi_2' \\ \ldots \\ \Delta\phi_{n_{tq}}' \end{pmatrix} \quad (4)$$

where

$$\Delta kl_i' = \sum_j V^T{}_{ij} \Delta kl_j \,,$$
$$\Delta\phi_i' = \sum_j U^T{}_{ij} \Delta\phi_j$$

and we arrange the system such that $w_1 > w_2 > \ldots > w_{tq}$. As shown in equation (4), the phase beat, as a n_{bpm} dimension vector, is mapped into a vector in a subspace with n_{tq} dimension. Components normal to this subspace are out of the correcting range of the trim quadrupoles. Furthermore, equation (4) also grouped the phase beat and the correction trim quadrupoles into n_{tq} modes. Modes with larger eigenvalues are relatively easier to correct for two reasons. First, for a given phase beat amplitude, smaller correcting quadrupole strengths are required for a larger eigenvalue mode. Second, for given noise levels in phase beat measurements, the resultant noise in the strengths of trim quadrupoles is smaller for modes with larger eigenvalues than those with smaller eigenvalues.

PROOF OF PRINCIPLE

In the RHIC 2007 and 2008 runs, experiments were peformed to verify these algorithms. In these experiments, gradient errors were intentionally applid to trim quadrupoles. The correction algorithm was then used to find a correction. The resulting error strengths of the trim quadrupoles should then reproduce the preset gradient errors with opposite signs. Fig. 3 shows that the SVD algorithm successfully reconstructed the preset gradient error in the RHIC 2007 run. Data analysis for the RHIC 2008 run is still in progress. Analysis of RHIC 2008 run data presents two challenges: noise in the measured phase beat, and trim quadrupole range limits. As described in aprevious section, depending on its eigenvalue, each mode has different sensitivity to phase beat noises.

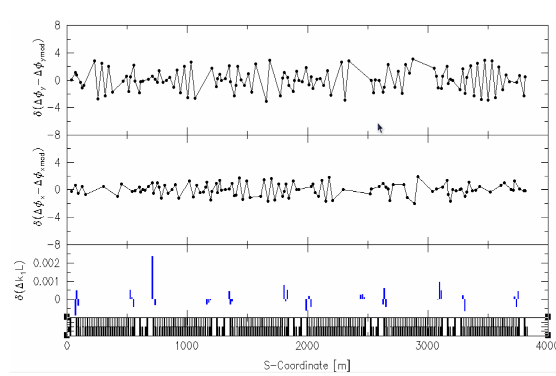


Figure 3: Experiment result in RHIC 2007 run. The top two graphs show the measured phase beat due to the preset trim quadrupole error and the bottom graph show the strengths of trim quadrupoles required to correct the phase beat as calculated from the SVD algorithm.

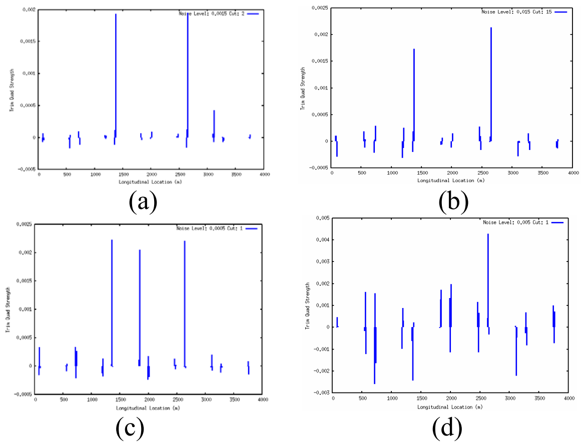


Figure 4: Simulation results of reconstructing gradient errors in presence of phase beat noises. The abscissa is the longitudinal location along the ring and the coordinate is the quadrupole strength. (a) Reconstruction of gradient errors set to trim quadrupoles without large components in modes with small eigenvalues. The noise level is 2%; (b) Same as (a) with noise level of 20%; (c) Reconstruction of gradient errors set to trim quadrupoles with large components in modes with small eigenvalues. The noise level is 1%. (d) Same as (c) with 10% noise.

Modes with substantially small eigenvalues compared with other modes are most vulnerable to phase beat noises and are typically cut to avoid ill-conditioned equations. However, if the phase beat due to the preset gradient errors has large components in modes with very small eigenvalues, it is difficult to reconstruct these errors by SVD as the cut-off process results in major information loss. Fig. 4 shows simulation results for two set of preset gradient errors reconstructed by the SVD algorithm. As shown in Fig. 4 (d), if a trim quadrupole with preset gradient errors has large components in noise sensitive modes, SVD does not reconstruct the phase errors well.

To verify the correction algorithm and test the range of the 36 RHIC trim quadrupoles, we performed simulations for the RHIC 2009 run optics. In these simulations, random gradient errors were assigned to quadrupoles throughout the ring and the SVD algorithm was applied to find the proper correcting strengths for the 36 trim quadrupoles.

The phase beats before and after the corrections are shown in Fig. 5. Fig. 5 (a) shows the effective correction of gradient errors randomly assigned to all 'QF' quadrupoles with peak relative amplitude of 2.5%. The rms phase beat reduced from 7.6% to 2.8% as a result of correction which indicates that the randomly generated phase beat has major components falling into the correction range of the trim quadrupoles. On the contrary, Fig. 5 (b) shows the correction result for another set of gradient errors which fall out of the correction range. Fig. 5 (c) and (d) shows similar results with the gradient errors assigned to all 'QD' quadrupoles.

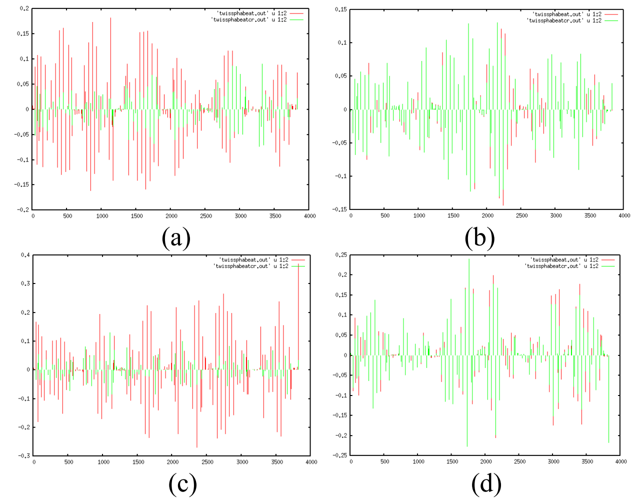


Figure 5: Simulations of gradient errors correction with RHIC 2009 run optics. The abscissa is the longitudinal location along the ring and the coordinate is the phase beat before (red) and after (green) the correction.

SUMMARY

The algorithm described here for linear gradient errors correction has been verified by simulation and experiment in the RHIC 2007 run. In the process of analyzing RHIC 2008 run data, bpm noise is a challenge for reconstructing preset gradient errors, especially when the gradient errors have large components in modes with small eigenvalues. Simulations to study the noise effects have been done Results show that modes with smaller eigenvalues are more vulnerable to noise and harder to reconstructed. Simulations have also indicated a reasonable correction range for the 36 trim quadrupoles. Within this range, the SVD correction is very effective in simulations.

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