NOVEL TECHNIQUE OF SUPPRESSING TBBU IN HIGH-ENERGY ERLS*

Vladimir N. Litvinenko #, Brookhaven National Laboratory, Upton, NY, USA
Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY, USA

Abstract

Energy recovery linacs (ERLs) is an emerging generation of accelerators that promise to revolutionize the fields of high-energy physics and photon sciences. These accelerators combine the advantages of linear accelerators with that of storage rings, and augur the delivery of electron beams of unprecedented power and quality. However, one potential weakness of these devices is transverse beam break-up instability that could severely limit the available beam current. In this paper, I propose a novel method of suppressing these dangerous effects using the chromaticity of the transverse motion.

In this short paper I am able only to touch the surface of the method and a complete description of the method with all relevant derivations can be found in [1].

INTRODUCTION

Energy-recovery linacs (ERLs, see Fig.1) belong to a family of recirculating linacs (RLs) that accelerate a beam of charged particles multiple times in the same linear accelerator, accumulating the beam's energy on each pass.

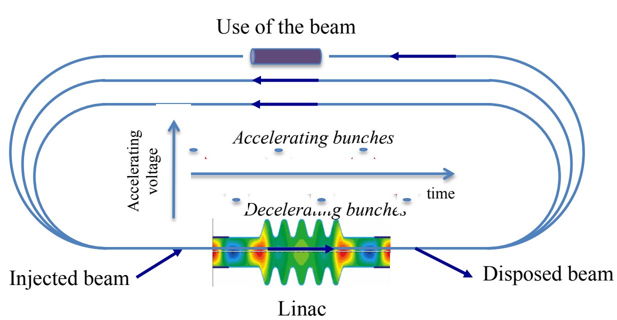

Figure 1: A sketch of a three-pass ERL.

One main challenge for these accelerators is the transverse beam break-up instability (TBBU) that especially is severe for SRF recirculating linacs. Early experiments with recirculating SRF accelerators at Stanford [2] and Illinois [3], where this instability occurred at a few microamperes of the average beam current, highlighted this problem. Dipole high-order modes (HOMs) of the SRF cavities were identified as the culprits driving this instability [4-5], and several remedies were developed for raising this threshold [6].

Detailed theoretical approaches and TBBU simulation programs were developed in the late 80s [7-9]. The renewed interest also stimulated refinements of the TBBU theory, simulation programs [10-12] and their experimental verification [13] all are driven by the need for high current ERLs, and also by the rewards of resolving this complex problem. Nevertheless, strong damping of HOMs in SRF linacs while maintaining high accelerating gradients remains one of the major unsolved issues. In the general ERL case, we can write a complete matrix of dispersion relations [7-12], but an analytical solution of the TBBU's instability threshold can be derived only for a single HOM mode, and a single-pass ERL, and as detailed in [5]:

$$I_{th} = \frac{2c^2}{e\,R_g \cdot Q \cdot \omega} \cdot \frac{1}{T_{12}\sin(\omega t_r)} \qquad (1)$$

where c is the speed of the light, e is the elementary charge, $R_g \cdot Q$ is the HOM's impedance (measured in Ω), ω is its frequency, and t_r is the beam's travel time through the returning loop. Since there are many HOM modes in the cavities encompassing a large range of the spectrum, there can be modes with $|\sin(\omega t_r)| \sim 1$; then, the only meaningful way of increasing the threshold is by reducing the values of Q and the $|T_{12}|$. As shown in [10], eq. (66), in an ERL with N passes through its linac, we can estimate the TBBU threshold by

$$I_{th} = \frac{2c^2}{e\,R_g \cdot Q \cdot \omega} \cdot \frac{1}{\sum_{J=1}^{2N}\sum_{I=J+1}^{2N} T^{IJ} \sin\omega(t_I - t_J)}, \qquad (2)$$

where $T^{IJ} \equiv T_{12}(s_J|s_I)$ is the element of transport matrix between the J^{th} and I^{th} pass through the linac. Analyzing eq. (3) the authors of ref. [10] offered their natural conjecture that the TBBU threshold scales as follows:

$$I_{th} \propto \left(R_g \cdot Q \cdot N^2 \left\langle \left|T^{IJ}\right| \right\rangle\right)^{-1}. \qquad (3)$$

The unfavorable scaling discussed above may have major implications on the cost of a high-energy ERL. Since an SRF linac usually more expensive than magnetic elements, cost-effective solutions [14-16] lead to three- to six-pass ERLs. If the current in such ERL designs suffer from severe beam current limitation, then the extent of their use and their energy reach will be restricted.

One way of resolving these confines lies in reducing the Q of all dangerous HOMs by developing complex HOM-damping schemes, and in addition, circumscribing the number of cells per linac module so to avoid trapping high-Q HOMs.

The other way of increasing the threshold current is by lowering $\left\langle \left|T^{IJ}\right| \right\rangle$. The latter is the topic of this paper.

*Work supported by Brookhaven Science Associates under Contract No. DE-AC02-98CH10886 with the U.S. DOE

#vl@bnl.gov

ISBN 978-3-95450-122-9

SUPPRESSION OF THE BEAM'S RESPONSE USING CHROMATICITY

Since the bunch is an ensemble of particles, its transverse response to the kick will be the average if the responses of the individual particles are

$$\langle x_r \rangle = \langle T_{12} \rangle \cdot x', \tag{4}$$

where $\langle a \rangle$ signifies the average value of a parameter, a. I suggest using the chromaticity of the returning loops and an energy spread in the beam (induced by an RF chirp, if needed) beam to reduce $\langle T_{12} \rangle \to 0$.

For a particle with a small energy-deviation $p = p_o(1+\delta)$; $|\delta| << 1$, the envelope function $\beta_o \equiv \mathrm{w}_o^2$ and the phase advance can be extended as

$$\begin{aligned} \mathrm{w}(s,\delta) &\cong \mathrm{w}_o(s)\left(1+\delta \cdot \upsilon(s)\right); \\ \psi(s,\delta) &\cong \psi_o(s) + \delta \cdot \phi(s), \end{aligned} \tag{5}$$

where the subscript "o" indicates the values for particles with the designed energy. The straightforward perturbation theory (see [1]) yields the following expression:

$$\begin{aligned} \varphi(s) &= -\frac{1}{2}\int_0^s K(z)\mathrm{w}_o^2(z)\left(1+\cos 2\left(\psi_o(z)+\phi_o\right)\right)dz; \\ \upsilon(s) &= -\frac{1}{2}\int_0^s K(z)\mathrm{w}_o^2(z)\cdot \sin 2\left(\psi_o(z)+\phi_o\right)dz. \end{aligned} \tag{6}$$

One important (and well known) feature of this solution is that the phase deviation (i.e., the chromaticity of oscillations) is defined by a monotonic function with average rate of $-\langle K(z)\mathrm{w}^2{}_o(z)\rangle_z/2$, while the envelope deviation is a fast oscillating function with double-betatron frequency. Hence, in a large accelerator system, chromaticity grows steadily, and can reach significant values $C(s) = \phi(s)/2\pi >> 1$, and, even for a modest deviation in relative energy, phase variation can be large, while the relative variation in the envelope function remains small $\delta \cdot \upsilon(s) << 1$.

Naturally, the level of suppression of the beam's response (transfer function, $\langle T_{12} \rangle$) depends on the energy-distribution function; Table 1 gives a few examples. Smooth bell-shape energy-distribution functions (Gaussian, Lorentzian, $\kappa-2$) assure the strong suppression of $\langle T_{12} \rangle$ whose value falls fast with the increase of $|X| \equiv 2\pi|C|\sigma_\delta$; sharp-edged distribution functions (such as rectangular- or triangular-ones) cause the oscillating behavior of $\langle T_{12} \rangle$ which declines more slowly with the growth of $|X|$. In all cases, a highly chromatic lattice resulting in $|X| >> 1$ effectively suppresses the beam's response to transverse kicks.

In the TBBU theory [7-12], the low-energy passes through the linac are considered to be the ones most vulnerable to TBBU instability. Specifically, the conclusions are that the TBBU threshold is proportional to the lowest energy in the recirculating path, and also inversely proportional to number of the recirculating passes. The examples described in this paper as well in ref. [1] show that this rule is not applicable to an ERL wherein the accumulated chromatic phase spread is significantly larger than unity.

Examples

eRHIC's low-emittance ERL lattice has chromaticities $C_{x\_t} \cong -175$; $C_{y\_t} \cong -125$ per turn. The LHeC low emittance lattice has $C_{x\_t} \cong -510$; $C_{y\_t} \cong -460$ per turn. Hence, an induced correlated Gaussian energy spread of $\sigma_\delta = 5\cdot 10^{-3}$ will suppress $\langle T_{12}(s|s+C\rangle$ 3,000 –fold in vertical direction and by astronomic $3.7\cdot 10^6$ in horizontal direction. In LHeC ERL $\sigma_\delta \cong 1.3\cdot 10^{-3}$ will suffice for suppressing a single-turn response 1,000-fold.

Table 1. Suppression of the Beam's Response on a Transverse Kick by the Chromaticity: $X = \phi\sigma_\delta$; $Y = \upsilon\sigma_\delta$.

$f(\delta)$	Suppression factor $\langle T_{12} \rangle / \mathrm{w}_{io}\mathrm{w}_o$	Type
$\frac{1}{\sqrt{2\pi}\sigma_\delta}\exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right)$	$e^{-X^2/2}\cdot\left(\sin\psi + X\cdot Y\cos\psi\right)$	Gaussian
$\frac{1}{\pi\sigma_\delta}\left(1+\frac{\delta^2}{\sigma_\delta^2}\right)^{-1}$	$e^{-\lvert X\rvert}\cdot\left(\sin\psi + Y\cdot sign(X)\cos\psi\right)$	Lorentzian
$\frac{2}{\pi\sigma_\delta}\left(1+\frac{\delta^2}{\sigma_\delta^2}\right)^{-2}$	$e^{-\lvert X\rvert}\cdot\left((1+\lvert X\rvert)\sin\psi + X\cdot Y\cos\psi\right)$	$\kappa-2$
$\frac{1}{2\sigma_\delta}\left(\theta(\delta-\sigma_\delta)-\theta(\delta+\sigma_\delta)\right)$	$\frac{\sin X}{X}\sin\psi + Y\frac{\sin X - X\cos X}{X^2}\cos\psi$	Rectangular

Numeric example

Some higher order effects can, in principle, reduce or at least, significantly modify the predictions given in Table 1. To address this concern, I tested the concept by directly calculating the response of the beam with a Gaussian energy-distribution propagating through a beam-line encompassing 1024 FODO cells. Each FODO cell comprises one focusing- and one defocusing-quadrupole. Hence, I use the exact analytical expression for the transport matrix of the FODO cell for an ultra-relativistic particle with arbitrary momentum $p = p_o(1+\delta)$: $T_{FODO}(\delta)$. The matrix is multiplied by the necessary number of times

$$T_{total}(\delta) = \left[T_{FODO}(\delta)\right]^{1024} \tag{7}$$

to form the exact matrix of the beam-line. The beam's displacement at the end of the beam-line to an angular kick, θ , at its entrance entails a simple convolution:

$$\langle x \rangle = \langle T_{12\ total} \rangle \cdot \theta; \quad \langle T_{12\ total} \rangle = \int_{-\infty}^{\infty} T_{12\ total}(\delta) f(\delta) d\delta$$

where $T_{12\ total}(\delta)$ is an exact, analytical, nonlinear function of δ. For comparison, I define the suppression factor of the transverse response as

$$S = \max\left(\left|T_{12\ total}(\delta)\right|\right) / \left|\int_{-\infty}^{\infty} T_{12\ total}(\delta) f(\delta) d\delta\right|.$$

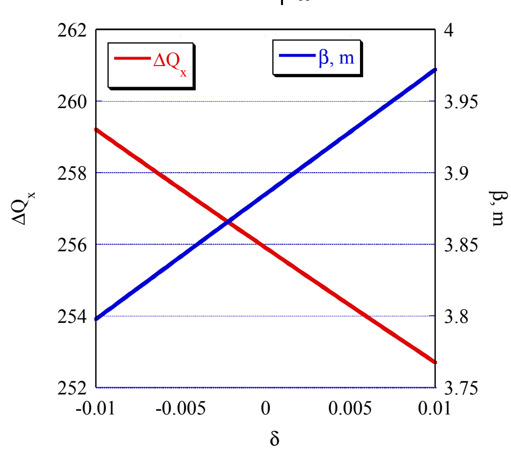

Figure 2. The FODO beam-line optics parameters as function of the particle momentum.

The exact matrix function was used to convolve it with Gaussian energy-distribution with $\sigma_\delta = 2 \cdot 10^{-3}$. I did not observe any significant deviations from the expected theoretical behavior either at a small values (as expected), or at large values of the of total beam-line's chromaticity. For example, the beam-line with tune advance of 0.22285 per cell has chromaticity of $C = -276.68$, and the calculated suppression factor of 428-fold that compares favorably with the theoretical estimation of 422-fold. Increasing the chromaticity further to $C = -328.51$ by increasing the tune advance per cell to 0.2499 increased the calculated suppression factor to $5.93 \cdot 10^3$. This also is reasonably close to the theoretical estimation of $5.02\ 10^3$.

CONCLUSIONS

I have shown in this paper that using the natural chromaticity of the arcs in ERLs affords an opportunity of suppressing the beam's transfer function (its response to transverse kicks), and either suppressing TBBU instability or significantly increasing allowable operating currents.

In an ERL comprising of a sequence of linacs and arcs, with the product of the chromaticity and the energy spread in the arcs $|\phi\sigma_\delta| >> 1$, the electron beam forgets the kick it received in the linacs while traveling through an arc. Then, the traditional ERL TBBU excitation scheme falls apart, and each linac effectively sees fresh electron beams.

ACKNOWLEDGMENT

The author would like to thank Ilan Ben-Zvi, Yue Hao, Dmitry Kayran (BNL) and Eduard Pozdeyev (FRIB) for fruitful, in-depth discussions of the methods, and Georg Hoffstaetter (Cornel University) and Frank Zimmermann (CERN) for their interest in the prospects of suppressing TBBU in high-energy ERLs.

REFERENCES

[1] V.N. Litvinenko, Physics Review Special Topics – Accelerators and Beams, Volume **15**, 074401 (2012)
[2] C M. Lyneis et al.,Trans. Nucl. Sci. **B**, 1979,
[3] P. Axel et al,., Trans. Nucl. Sci. **26**, 1979
[4] C.M. Lyneis et al., NIM 204 (1983) 269-28
[5] R.E. Rand, Recirculating electron accelerators, Harwood Acad. Publ., 1984
[6] R.E. Rand and T.I. Smith, Part Acc. **11,** p. 1 (1980)
[7] G.A. Krafft, J.J. Bisognano, S. Laubach, Tech-Note CEBAF-PR-90-39
[8] J.J. Bisognano, R.L. Gluckstern, Proc. PAC'87, Washington, DC, p. 1078.
[9] G.A. Krafft, J.J. Bisognano, Proc. PAC'87, Washington, DC, p. 1356.
[10] G.H. Hoffstaetter, I.V. Bazarov, PR ST-AB 7, 054401 (2004).
[11] E. Pozdeyev, PR ST-AB 8, 054401 (2005)
[12] G.H. Hoffstaetter, I.V. Bazarov and C. Song, PR ST-AB 10, (2007) 044401
[13] C.D. Tennant et al., PR ST-AB 8, 074403 (2005)
[14] C.W. Leemann, D.R. Douglas, G.A. Krafft, Ann. Rev. Nucl. Part. Sci. **51** (2001) 413
[15] V.N. Litvinenko et al., (2011), 438 http://arxiv.org/pdf/1108.1713v1
[16] F. Zimmermann et al., Proc. EPAC08, Genoa, Italy (2008) 2847

ISBN 978-3-95450-122-9