AM-PM CONVERSION INDUCED INSTABILITY IN AN I/Q FEEDBACK CONTROL LOOP

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Abstract

Most RF feedback control systems today use the I/Q demodulation and modulation scheme because of its simplicity. Its performance, however, depends on the alignment of the feedback loops. If the loop contains elements that have a high AM-PM conversion factor such as a class C amplifier or a high power klystron, then the misalignment is dynamic and power dependent. In most systems the phase noise is increased, and in some cases the I/Q loops become unstable and the system settles into a limit-cycle oscillation.

INTRODUCTION

The 92MHz RF Booster has been in operation in the TRIUMF cyclotron for more than 15 years[1]. The original system was an analogue system that used amplitude/phase modulation. When the control system was replaced in 2005 with a digital signal processor system, the control algorithm was converted into I/Q modulation. After the conversion the feedback system would operate satisfactory at 90% nominal power, but at 100% nominal power, on some occasions, the I/Q loops and the tuner loop would jump into limit cycle oscillations. It was also determined that the booster RF amplifier chain has a large phase shift that is power dependent. In an I/Q system, detuning of the resonant cavity will introduce cross-talk between the I and the Q channels. Normally the detuning will be suppressed by the tuning feedback loop but in the TRIUMF RF Booster the tuning motor movement is relatively slow and is unable to correct the detuning within a few seconds. During this crucial time the cross-talk is enhanced by the dynamic phase shift of the amplifier chain. Its growth rate, depending on the initial detuning, can sometimes exceed the damping rate provided by the tuner loop and cause the collapse of this RF system.

BOOSTER RF POWER AMPLIFIER

The Booster RF power amplifier chain consists of a three-stage amplifier: a 10W solid state driver amplifier, a 10kW tube pre-amplifier and a 100kW final tube power amplifier. The final stage amplifier is a grounded grid power amplifier for the booster cavity of the TRIUMF cyclotron, operating at the frequency 92MHz. The configuration is shown in Figure 1.

A huge phase shift in the Booster PA was detected which was dependent on RF output power, possibly due to a thermal effect. Phase changes from 0 to -15 degrees with power going from 0 to 15 kW and then from -15 to 40 degrees when output ramping up to full power (40kW) as indicated in Figure 2. Bipolar phase shift response is most likely caused by alternate behaviour of the FM transmitter and final PA, and is difficult to suppress with a phase compensation circuit inside the Booster RF power amplifier.

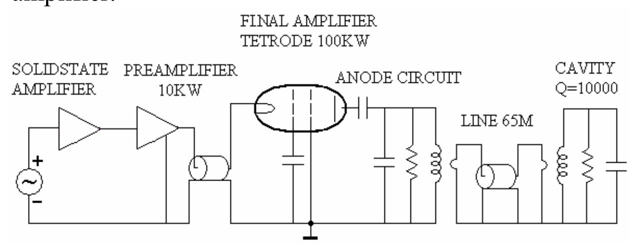

Figure 1: TRIUMF RF booster amplifiers.

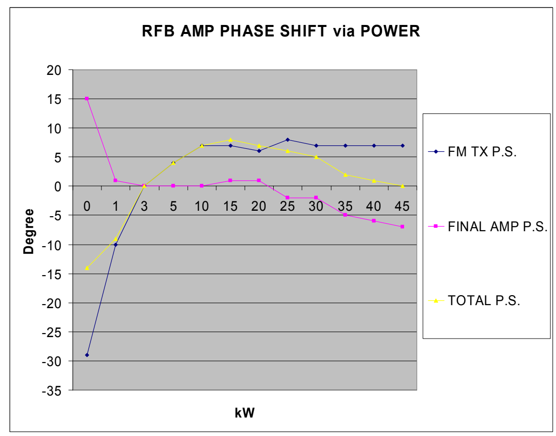

Figure 2: Phase shift vs. power.

This high AM-PM conversion phase shift enhances I/Q loop cross-talk, eventually making the Booster RF system operation unstable.

THE BOOSTER RF TUNING SYSTEM

The automatic frequency tuning system keeps the RF cavity at its resonant frequency. This achieves the desired RF voltage with minimum RF power input and minimum reflected RF power. The TRIUMF booster tuning unit uses two DC motors to move the upper and lower shorting plates in the root region of the cavity to accomplish the Booster cavity frequency tuning. To prevent wearing out of mechanical contacts due to constant movement of the tuner, a bang-bang controller is used for the booster tuning control. The motors are also heavily geared down in order to have enough force to move the tuning plate. All these factors contribute to the slow response of the tuning control system.

LINEAR THEORY OF AM-PM CONVERSION INDUCED INSTABILITY

In this section we will develop the linear theory for instability caused by AM to PM conversion. Figure 3 shows a typical I/Q driven mode feedback control system.

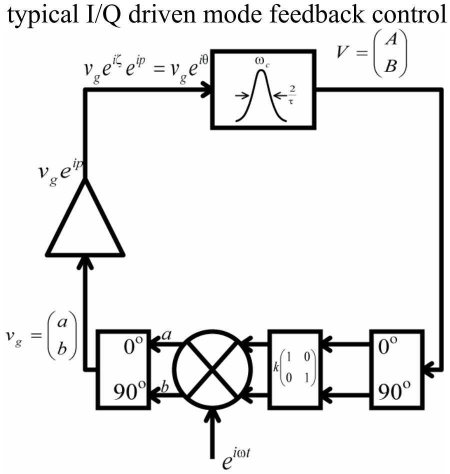

Figure 3: Basic I/Q driven mode with proportional feedback system.

The equation for the voltage of the cavity is [2]

$$\ddot{v} + 2\frac{1}{\tau}\dot{v} + \omega_c^2 v = 2\frac{\gamma}{\tau}\dot{V}_g \qquad (1)$$

Using the I and Q components of the input voltage V_g as the independent variables and the I and Q component of the output voltage v as the dependent variables,

$$v = Ve^{i\alpha} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix},$$

$$V_g = v_g e^{i(\theta+\omega t)} = \begin{pmatrix} \cos(\theta+\omega t) & -\sin(\theta+\omega t) \\ \sin(\theta+\omega t) & \cos(\theta+\omega t) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

where ω_c is the resonant frequency of the cavity and γ is the coupling coefficient into the cavity.

The phase shift ϕ of the RF cavity is

$$\phi = \tan^{-1}(\omega_c - \omega)\tau . \qquad (2)$$

By eliminating slowly varying terms and applying the Laplace transform, Equation 1 becomes

$$(1 + s\tau)V + i\tan\phi V = \gamma e^{i\theta} v_g \qquad (3)$$

where θ is the total phase shift from the control output to the cavity input. It can be separated into static phase shift ζ and an amplitude dependent phase shift ρ such that

$$(1 + s\tau)V + i\tan\phi V = \gamma e^{i\zeta} e^{i\rho} v_g \qquad (4)$$

where ρ is a function of v_g.

The equation is non-linear due to the dependency of ρ on v_g. A good approximation of ρ is

$$\rho = \eta v_g v_g^* = \eta P \qquad (5)$$

where P is the output power.

To linearize Equation 3, take the variations of both the dependent and independent variables

$$(1 + s\tau)\delta V + i\tan\phi\delta V = \gamma e^{i\theta}\left(iv_g\delta\rho + \delta v_g\right) \qquad (6)$$

With $\mathrm{P} \equiv \frac{d\rho}{dv_g} v_g = 2\eta a^2$ and $\varphi \equiv \frac{b}{a}$. Therefore

$$\left(iv_g\delta\rho + \delta v_g\right) = \begin{bmatrix} 1-\mathrm{P}\varphi & -\mathrm{P}\varphi^2 \\ \mathrm{P} & 1+\mathrm{P}\varphi \end{bmatrix} \begin{bmatrix} \delta a \\ \delta b \end{bmatrix} \qquad (7)$$

Equation 6 becomes

$$\begin{bmatrix} \delta A \\ \delta B \end{bmatrix} = \frac{\gamma}{\sigma^2 + \tan^2\phi} \begin{bmatrix} \sigma & -\tan\phi \\ \tan\phi & \sigma \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1-\mathrm{P}\varphi & -\mathrm{P}\varphi^2 \\ \mathrm{P} & 1+\mathrm{P}\varphi \end{bmatrix} \begin{bmatrix} \delta a \\ \delta b \end{bmatrix} \qquad (8)$$

where $\sigma = 1 + s\tau$.

Equation 8 shows that the phase shift of the transfer function of the system is the result of the three "rotation" matrices. The left-hand most matrix is the phase shift due to cavity detuning, the second matrix is the phase shift due to total cable length. These two matrices can be reduced to rotation matrices and represent actual rotations of the input phasor, albeit time-dependent due to the dependence of σ on s. The last matrix $\begin{bmatrix} 1-\mathrm{P}\varphi & -\mathrm{P}\varphi^2 \\ \mathrm{P} & 1+\mathrm{P}\varphi \end{bmatrix}$ is due to amplitude-dependent phase shift of the amplifiers. Since it is not an orthogonal matrix, the stability of a feedback system involving power dependent phase shift is not guaranteed.

For small misalignment and detuning and keeping only first order terms

$$\begin{bmatrix} \delta A \\ \delta B \end{bmatrix} = \frac{\gamma}{\sigma} \begin{bmatrix} 1-\mathrm{P}\varphi-\mathrm{P}\theta-\mathrm{P}\frac{\phi}{\sigma} & -\theta-\frac{\phi}{\sigma} \\ \theta+\frac{\phi}{\sigma}+\mathrm{P} & 1+\mathrm{P}\varphi \end{bmatrix} \begin{bmatrix} \delta a \\ \delta b \end{bmatrix} \qquad (9)$$

Consider symmetric P controllers, the controller gain matrix is

$$F = fI \qquad (10)$$

where I is the identity matrix. The feedback transmission is simply

$$H = hI \qquad (11)$$

A new variable $\gamma' \equiv \gamma fh$ incorporates the controller gain f and the feedback gain h into the coupling coefficient γ of the cavity. Then the characteristic equation of the system is the determinant of the return difference

$$\left| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{\gamma'}{\sigma} \begin{bmatrix} 1-\mathrm{P}\varphi-\mathrm{P}\theta-\mathrm{P}\frac{\phi}{\sigma} & -\theta-\frac{\phi}{\sigma} \\ \theta+\frac{\phi}{\sigma}+\mathrm{P} & 1+\mathrm{P}\varphi \end{bmatrix} \right| = 0 \qquad (12)$$

or

$$\frac{s^2\tau^2}{\gamma'^2} + \frac{s\tau}{\gamma'}\left(\frac{2}{\gamma'} + 2 - \mathrm{P}\theta\right) + \frac{1}{\gamma'^2} + \frac{2}{\gamma'} - \frac{\mathrm{P}\theta}{\gamma'} + 1 - \frac{\mathrm{P}\phi}{\gamma'} = 0 \qquad (13)$$

under negative feedback the controller will adjust φ such that

$$\theta + \phi = 0 . \qquad (14)$$

Therefore

$$s\tau = \gamma' \frac{\Delta - 2 \pm \sqrt{\Delta^2 - 4\Delta}}{2} \text{ with } \Delta \equiv \mathrm{P}\theta \qquad (15)$$

where Δ can be viewed as the damping factor.

The following plot is the root loci of Equation 15.

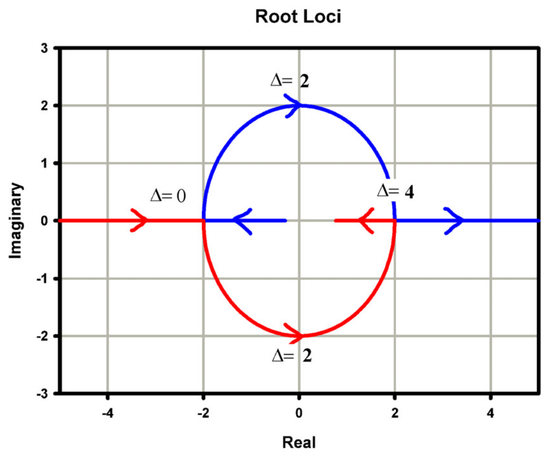

Figure 4: Root loci showing regions of stability.

The root loci plot indicates the system becomes unstable for $\eta a^2\theta > 2$ or $\left(\frac{d\rho}{dP}P\right)\theta > 2$. The system actually becomes under-damped for $\frac{d\rho}{dP}\theta > 0$. The system quickly becomes non-linear and the stability margin decreases further.

This instability has a simple explanation. Since in an I/Q driven system quadrature power is used to achieve a change in θ, the amplitude-dependent phase shift at one side of the misalignment causes θ to change in the opposition direction to that required. At higher power levels, or when misaligned, more quadrature power is required to create the same phase shift, so the feedback system becomes more susceptible to instability. Although the proportional gain does not affect the instability regime, it does affect the growth rate.

The above calculations were developed using small-signal variations, a proportional controller and a very simple amplitude dependent phase shift. The stability requirement will be more stringent as misalignment increases, system Q increases, amplitude dependent phase shift becomes more non-linear or more complicated poles and zeros are included in the feedback path. For the TRIUMF RF booster, using the data from Figure 2, $\frac{dp}{dP}P \approx 0.8$. The stability is marginal and instability did occur occasionally, particularly when the tuner dead band caused ϕ to become more negative and thus θ to become more positive. There are many ways to eliminate or suppress this form of instability. We can reduce $\frac{dp}{dP}$ by operating the amplifier in the linear region. We can try to operate the system in the region where $\frac{d\rho}{dP}\theta < 0$ by purposing misaligning the tuner, or we can operate using amplitude and phase modulation.

MODIFIED BOOSTER RF OPERATION

As the mechanism of the instability of the booster operation became evident due to AM-PM conversion, the RF control system was modified to eliminate this instability. Method 1 mentioned above is expensive since it involved rebuilding the amplifier chain. The operating frequency of the booster is phase-locked to the frequency of the cyclotron, which changes with its tune. This eliminates Method 2. An ISAC-2 controller which uses phase-locked self-excited mode of operation [3], was modified from I/Q modulation to Amplitude/Phase modulation by adding an electronic phase shifter. This modified controller changes the control algorithm from driven operation with I/Q modulation and demodulation, to phase-locked self-excited operation with amplitude/phase modulation and demodulation. By doing so the AM-PM conversion instability is completely eliminated. Powering up the cavity also becomes much easier. Previously, the tuner position had to be adjusted to match the cyclotron frequency before RF power could be applied. Now the self-excited loop automatically adjusts its frequency to match the resonant frequency of the booster, regardless of the tuner position. The problem of loss of synchronization with the main RF cyclotron has also been eliminated, since the phase locking can be turned off and the RF booster can operate in self-excited mode whenever the synchronization signal is lost.

CONCLUSION

Three conditions are required to AM-PM conversion instability to happen: amplitude dependent phase shift, operation at $\frac{d\rho}{dP}\theta > 0$, and the feedback system uses I/Q modulation. The RF booster happened to have these three conditions after the conversion from an amplitude/phase modulator to I/Q modulator, and its operation became unstable. After the the mechanism of the instability was understood, the control system was changed from I/Q driven mode to phase-locked, self-excited mode with Amplitude/Phase modulation. This eliminated the last condition for AM-PM instability and the RF booster has since been running satisfactorily.

REFERENCES

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[2] J. Delayen, "Self Excited Loop", LLRF Workshop, 2001.

[3] K. Fong, M. Laverty and S. Fang, "RF Control System for ISAC II Superconducting Cavities", Proc. EPAC 2002, Paris, p. 2226.