ANALYSIS OF ELECTRONIC DAMPING OF MICROPHONICS IN SUPERCONDUCTING CAVITIES*

S. U. De Silva[#1] and J. R. Delayen[†1,2]
[1]Old Dominion University, Norfolk, VA 23529, U.S.A.
[2]Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, U.S.A.

Abstract

In low current applications superconducting cavities have a high susceptibility to microphonics induced by external vibrations and pressure fluctuations. Due to the narrow bandwidth of the cavities, the amount of rf power required to stabilize the phase and amplitude of the cavity field is dictated by the amount of microphonics that need to be compensated. Electronic damping of microphonics is investigated as a method to reduce the level of microphonics and of the amount of rf power required. The current work presents a detailed analysis of electronic damping and of the residual cavity field amplitude and phase errors due to the fluctuations of cavity frequency and beam current.

INTRODUCTION

In superconducting cavities microphonics and ponderomotive effects are the major causes of the fluctuations in the cavity fields [1]. Microphonics are the changes in the cavity frequency caused by the connections to the external world, such as external vibrations, pressure fluctuations, etc., and the ponderomotive effects are changes in the cavity frequency caused by the electromagnetic field (radiation pressure). The amount of rf power required in controlling the cavity field–to maintain the amplitude and phase stabilized–is dominated in low-current superconducting cavities by the presence of microphonics.

Electronic damping may be an effective way of reducing microphonics; it requires an active modulation of the cavity field to induce ponderomotive effects that counteract the effects of external sources. The model developed to control the cavity field is to operate the superconducting cavity in a self excited loop with amplitude and phase feedback. This paper analyses the method of electronic damping of microphonics and presents a generalized analytical solution for the residual changes of the cavity field amplitude and phase with and without the presence of beam.

MODEL AND EQUATIONS

The model, described in detail in [2-4], operates the cavity in a self excited loop. The transfer function diagram for the model described in [2-4] is given in Figure 1.

*Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.
#pdesilva@odu.edu, †delayen@jlab.org

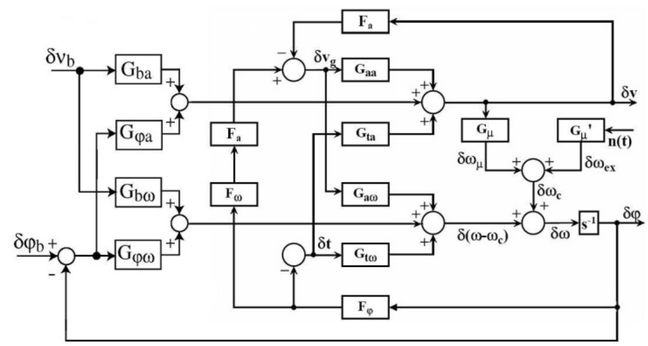

Figure 1: Transfer function representation of the system.

The corresponding transfer functions are as follows.

$$G_{aa}=\frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\frac{1}{1+\tau s},\ G_{ta}=-\frac{\sin(\theta_l+\theta_f)}{\cos\theta_l}\frac{1}{1+\tau s},$$

$$G_{a\omega}=\frac{1}{\tau}\frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\left[y-\frac{y_r}{1+\tau s}\right],\ G'_{\mu}=\frac{\Omega_\mu^2}{s^2+\frac{2}{\tau_\mu}s+\Omega_\mu^2},$$

$$G_{t\omega}=\frac{1}{\tau}\frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\left[1+\frac{y\,y_r}{1+\tau s}\right],\ G_{\mu}=-\frac{2\Omega_\mu^2\ k_\mu V_0^2}{s^2+\frac{2}{\tau_\mu}s+\Omega_\mu^2},$$

$$G_{ba}=-\frac{m}{1+\tau s},\qquad G_{\varphi a}=-\frac{m\,y_0}{1+\tau s},$$

$$G_{b\omega}=\frac{m}{\tau}\left[-y_0+\frac{y_r}{1+\tau s}\right],\qquad G_{\varphi\omega}=-\frac{m}{\tau}\left[1+\frac{y_0 y_r}{1+\tau s}\right],$$

$$y=\tan(\theta_l+\theta_f),\ y_r=\frac{\tau_0(\omega_r-\omega_c)}{(1+\beta)}=\tau(\omega_r-\omega_c),\ y_0=\tan\theta_0,$$

F_a, F_φ, F_ω : Amplitude, Phase and Frequency feedbacks,

$\tau=\frac{\tau_0}{1+\beta}$: Loaded amplitude decay time,

$b=\frac{V_b\cos\varphi_0}{V_0}$: Ratio of power absorbed by the beam to power dissipated in the cavity,

$m=\frac{b}{1+\beta}$: Beam matching coefficient,

φ_0 : Nominal phase between the beam and the rf field,

τ_0 : Intrinsic amplitude decay time,

β : Coupling coefficient,

Ω_μ : Frequency of the mechanical mode of the cavity,

τ_μ : Decay time of the mechanical mode of the cavity,
k_μ : Lorentz coefficient of the mechanical mode,
θ_l : Loop phase shift, θ_f : Feedback phase shift,
V_0: Cavity field amplitude.

G_μ represents the transfer function for ponderomotive effects that couples the cavity field amplitude and the cavity frequency. The transfer function G'_μ represents the coupling between the external sources and the cavity frequency. Fluctuations in the cavity frequency ($\delta\omega_c$) are a direct result of the frequency fluctuations due to ponderomotive effects ($\delta\omega_\mu$), and the frequency fluctuations due to external sources ($\delta\omega_{ex}$). Fluctuations in the cavity frequency are $\delta\omega_c = \delta\omega_\mu + \delta\omega_{ex}$ with $\delta\omega_\mu = G_\mu \delta v$ and $\delta\omega_{ex} = G'_\mu n(t)$. δv is the residual amplitude error and $n(t)$ is the external source of vibration. The ponderomotive effects are used to counteract the effects of microphonics. The residual amplitude and phase errors due to the effects of microphonics ($\delta\omega_{ex}$) with absence of the beam are

$$\delta v = \frac{F_\varphi\left(F_a F_\omega G_{aa} - G_{ta}\right)}{D}\delta\omega_{ex}, \quad \delta\varphi = \frac{\left(1+F_a G_{aa}\right)}{D}\delta\omega_{ex},$$

with

$$D = (1+F_a G_{aa})(s + F_\varphi G_{t\omega} - F_a F_\varphi F_\omega G_{a\omega}) + F_\varphi(F_a F_\omega G_{aa} - G_{ta})(F_a G_{a\omega} - G_\mu).$$

DAMPING BY FEEDBACK PHASE SHIFT

Electronic damping relies on the generation of ponderomotive effects that counteract the effects of externally-induced microphonics [4]. In this model, the feedbacks applied are simple proportional amplitude and phase feedback gains of F_a=k_a and F_φ=k_φ. The system is analyzed on resonance (θ_l) with a feedback phase shift (θ_f) [3-5] producing a coupling between phase and amplitude feedback that acts as a damping mechanism for the mechanical mode. It is assumed that the cavity is operated with no frequency feedback, no beam loading $(G_{ba} = G_{\varphi a} = G_{b\omega} + G_{\varphi\omega} = 0)$, small feedback angle $(\theta_f \ll 1)$, large feedback gains $(k_a, k_\varphi \gg 1, \tau\Omega_\mu)$ and $\tau/\tau_\mu \ll 1$. The resultant residual amplitude and phase errors are

$$\delta v = \frac{\Omega_\mu^2 k_\varphi \theta_f}{A(s)} n(s), \quad \delta\varphi = \frac{\Omega_\mu^2 (1 + k_a + \tau s)}{A(s)} n(s),$$

with

$$A(s) = a_4\left(\frac{s}{\Omega_\mu}\right)^4 + a_3\left(\frac{s}{\Omega_\mu}\right)^3 + a_2\left(\frac{s}{\Omega_\mu}\right)^2 + a_1\left(\frac{s}{\Omega_\mu}\right)^1 + a_0$$

where the coefficients are

$$a_4 = \tau\Omega_\mu^4, \qquad a_3 = (1+k_a+k_\varphi)\Omega_\mu^3 + \frac{2\tau\Omega_\mu^3}{\tau_\mu},$$

$$a_2 = \frac{(1+k_a)k_\varphi\Omega_\mu^2}{\tau} + \frac{k_a k_\varphi \Omega_\mu^2 \theta_f^2}{\tau} + \frac{2(1+k_a+k_\varphi)\Omega_\mu^2}{\tau_\mu} + \tau\Omega_\mu^4,$$

$$a_1 = \frac{2(1+k_a)k_\varphi\Omega_\mu}{\tau\tau_\mu} + \frac{2k_a k_\varphi \Omega_\mu \theta_f^2}{\tau\tau_\mu} + (1+k_a+k_\varphi)\Omega_\mu^3,$$

$$a_0 = \frac{(1+k_a)k_\varphi\Omega_\mu^2}{\tau} + \frac{k_a k_\varphi \Omega_\mu^2 \theta_f^2}{\tau} + 2k_\varphi\Omega_\mu^2 k_\mu V_0^2 \theta_f.$$

The above system is analyzed by performing a frequency sweep of ω with $s = i\omega$, for several feedback phase shifts. The maximum response is found at

$$\Omega_r = \Omega_\mu\sqrt{1 + \frac{2\tau k_\mu V_0^2}{1 + k_a(1+\theta_f^2)}\theta_f}.$$

This result is shown in Fig. 2 and reproduces the simulation of [5] where the same assumptions were used: $\tau = 6.4\times10^{-3}$ s, $k_a = 10^2$, $k_\mu = 3.69$ Hz/(MV/m)^2, $V_0 = 20$ MV/m.

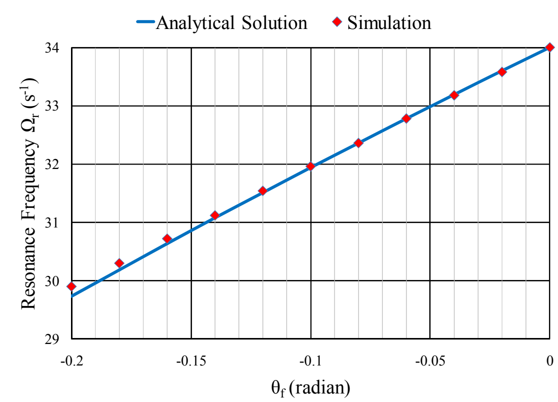

Figure 2: Resonance frequency (Ω_r) as a function of feedback phase shift (θ_f).

At maximum response, the residual phase and amplitude errors are

$$\delta v(\theta_f) = \tau\delta\omega\frac{\Omega_\mu}{\Omega_r}\frac{(1+k_a)k_\varphi\theta_f}{\left[(1+k_a+k_\varphi)\tau^2\tau_\mu\Omega_\mu^2 k_\mu V_0^2\theta_f - (1+k_a)^2 k_\varphi\right]},$$

$$\delta\varphi(\theta_f) = \tau\delta\omega\frac{\Omega_\mu}{\Omega_r}\frac{(1+k_a)\left[(1+k_a)^2 + \tau^2\Omega_r^2\right]^{1/2}}{\left[(1+k_a)^2 k_\varphi - (1+k_a+k_\varphi)\tau^2\tau_\mu\Omega_\mu^2 k_\mu V_0^2\theta_f\right]}.$$

As found analytically above and from simulations in [5] damping by amplitude to phase feedback coupling through the feedback phase shift is not very effective at damping broadband microphonics. However, since it is effective at shifting the resonant frequency, it might be useful in situations where the microphonics are due to a single frequency source close to a mechanical mode of the cavity.

DAMPING BY FREQUENCY FEEDBACK

Another method is through frequency feedback where the signal driving the resonator is intentionally modulated to counteract the externally-induced microphonics [4].

The amount of modulation supplied at the generator is, $\delta\nu_\omega = F_a F_\varphi F_\omega \delta\varphi$ with $F_\omega = -\frac{k_\omega \Omega_\mu}{s}$ where k_ω is the frequency feedback gain. The resultant amplitude and phase errors are

$$\delta\nu = -\frac{k_a k_\varphi k_\omega \Omega_\mu^3}{B(s)} n(s), \quad \delta\varphi = \Omega_\mu^2 \frac{(1 + k_a + \tau s)s}{B(s)} n(s),$$

with

$$B(s) = b_5\left(\frac{s}{\Omega_\mu}\right)^5 + b_4\left(\frac{s}{\Omega_\mu}\right)^4 + b_3\left(\frac{s}{\Omega_\mu}\right)^3 + b_2\left(\frac{s}{\Omega_\mu}\right)^2 + b_1\left(\frac{s}{\Omega_\mu}\right) + b_0$$

where the coefficients are

$$b_5 = \tau\Omega_\mu^5, \quad b_4 = \frac{2\tau\Omega_\mu^4}{\tau_\mu} + \left(1 + k_a + k_\varphi\right)\Omega_\mu^4,$$

$$b_3 = \tau\Omega_\mu^5 + \frac{2(1 + k_a + k_\varphi)\Omega_\mu^3}{\tau_\mu} + \frac{(1 + k_a)k_\varphi\Omega_\mu^3}{\tau},$$

$$b_2 = (1 + k_a + k_\varphi)\Omega_\mu^4 + \frac{2(1 + k_a)k_\varphi\Omega_\mu^2}{\tau\tau_\mu},$$

$$b_1 = \frac{(1 + k_a)k_\varphi\Omega_\mu^3}{\tau}, \quad b_0 = -2k_a k_\varphi k_\mu V_0^2 \Omega_\mu^3 k_\omega .$$

The above system is analyzed with the same frequency sweep as in the previous section for several frequency feedback gains (k_ω). In this case the maximum response always occurs at the frequency of the mechanical mode. The maximum amplitude residual error and the normalized maximum residual phase error are

$$\delta\nu(k_\omega) = \frac{\tau k_\omega \delta\omega}{1 + k_\mu V_0^2 \Omega_\mu k_\omega}, \quad \delta\varphi(k_\omega) = \frac{\delta\varphi(0)}{1 + \tau\tau_\mu \Omega_\mu k_\mu V_0^2 k_\omega}.$$

and are shown in Fig. 4 with the values mentioned above and $\tau_\mu = 0.3$ s.

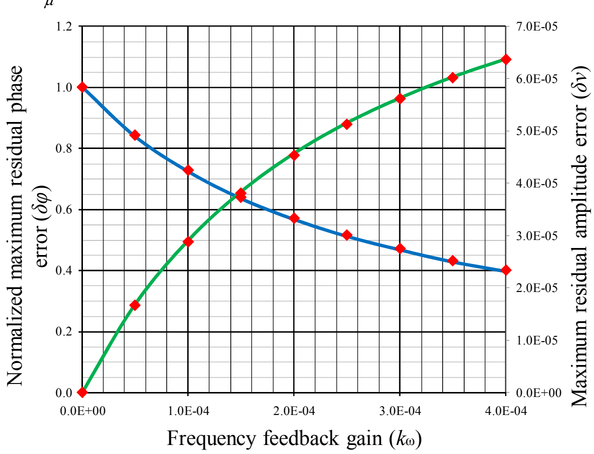

Figure 3: Maximum amplitude (green) and phase (blue) residual errors as a function of k_ω. Simulation results are in red.

Frequency feedback with beam

The frequency feedback method is further analyzed with light beam loading neglecting the fluctuations in the beam current ($\delta\nu_b = 0$) and beam phase ($\delta\varphi_b = 0$). The transfer function diagram is shown in Figure 1 with the corresponding transfer functions. The resultant residual amplitude and phase errors are

$$\delta\nu(k_\omega) = \frac{k_\varphi \tau\delta\omega k_\omega}{k_\varphi - m + k_\varphi \tau\tau_\mu \Omega_\mu k_\mu V_0^2 k_\omega},$$

$$\delta\varphi(k_\omega) = \delta\varphi(0)\frac{k_\varphi - m}{k_\varphi - m + k_\varphi \tau\tau_\mu \Omega_\mu k_\mu V_0^2 k_\omega}.$$

In the presence of beam and with well matched coupling, the beam matching coefficient (m) varies in the range of (0,1) and does not produce a significant effect on residual errors. However, with the beam, the loaded amplitude decay time (τ) reduces leading the term τk_ω in a decisive fashion in residual amplitude and phase errors. Subsequently τk_ω needs to be optimized to reduce the residual errors.

CONCLUSIONS

Electronic damping can be effectively used to damp the effects of microphonics on the fluctuations of the cavity frequency. The method is analyzed in detail and analytical results are obtained for the residual amplitude and phase errors that are in confirmation with the simulations in [5]. In amplitude and phase feedback coupling method the feedback phase shift needs to be reduced in order to operate the system on resonance. Frequency feedback method is introduced as an effective way of damping the effects of microphonics. The analytical results show that with the increase in the frequency feedback gain, the residual amplitude error increases while the residual phase error decreases; hence feedback gain needs to be optimized to achieve optimal residual errors. Further analysis with the beam confirms that the frequency feedback gain requires controlling the factor τk_ω to reduce the residual errors.

REFERENCES

[1] J. R. Delayen, "Ponderomotive Instabilities and Microphonics ", Physica C 441 (2006) p. 1.
[2] J. R. Delayen, "Phase and Amplitude Stabilization of Superconducting Resonators", Ph.D. thesis, California Institute of Technology, 1978.
[3] J. R. Delayen, "Phase and Amplitude Stabilization of Beam-Loaded Superconducting Resonators", Proc. LINAC92, p. 371.
[4] J. R. Delayen, "Electronic Damping of Microphonics in Superconducting Cavities", Proc. PAC 01, p. 1146.
[5] A. Hofler and J. R. Delayen, "Simulation Study of Electronic Damping of Microphonic Vibrations in Superconducting Cavities", Proc. PAC 05, p. 3916.