

# EMITTANCE AND OPTICS MEASUREMENTS ON THE VERSATILE ELECTRON LINEAR ACCELERATOR AT DARESBUARY LABORATORY\*

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## Abstract

The Versatile Electron Linear Accelerator (VELA) is a facility designed to provide a high quality electron beam for accelerator systems development, as well as industrial and scientific applications. Currently, the RF gun can deliver short (of order a few ps) bunches with charge in excess of 250 pC at up to 5.0 MeV/c beam momentum. Measurement of the beam emittance and optics in the section immediately following the gun is a key step in tuning both the gun and the downstream beamlines for optimum beam quality. We report the results of measurements (taking account of coupling and space charge) indicating normalised emittances of order  $0.5 \mu\text{m}$  at low bunch charge.

## INTRODUCTION: VELA LAYOUT

The injection beamline of VELA [1] (Fig. 1) comprises a 2.5 cell S-band photocathode gun with copper photocathode. The gun is driven with the third harmonic of a short ( $<76$  fs rms) pulsed Ti:Sapphire laser with a typical pulse energy of 1 mJ. The size of the laser spot on the photocathode is typically below 0.5 mm. The gun is immersed in the magnetic field of a main gun solenoid which provides emittance compensation and focusing of the beam in the initial section of the injection line. A bucking coil located beside the gun zeroes the field on the photocathode. Further focusing is provided by four quadrupole magnets, each with a length of 0.1 m. These magnets are also used in the procedure for emittance measurement. Beam diagnostics include a wall current monitor for charge measurement, and three combined diagnostic stations containing YAG screens installed at  $45^\circ$  to the beam line. The emittance measurements presented in this paper are based on beam images observed on the YAG screens with high-sensitivity, high-resolution CCD cameras. Vertical and horizontal slits on YAG-02 and YAG-03 allow alternative methods for emittance characterization. A Transverse Deflecting Cavity (TDC) which is presently under commissioning will complete the diagnostic suite for 6D beam characterisation.

## EMITTANCE AND OPTICS MEASUREMENTS WITH COUPLING

In general, the solenoid fields around the VELA gun (and especially any uncompensated magnetic field on the photocathode) will introduce coupling in the beam. Further-

\*Work supported by the Science and Technology Facilities Council, UK.

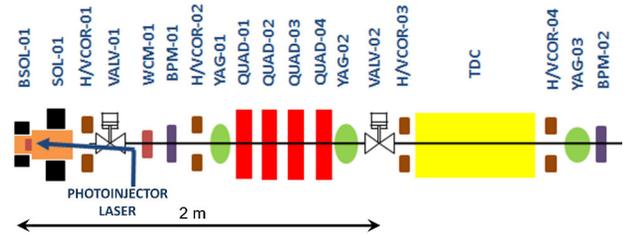


Figure 1: Layout of the VELA injection line.

more, space charge effects are expected to be significant, or even dominant, in many parameter regimes of interest for VELA. The techniques used for emittance and optics measurements therefore need to take into account both coupling and space charge. For the present, we assume that at low bunch charge (of order 10 pC) it is possible to include transverse space charge effects in a linear approximation: this will be discussed in more detail later.

Our goal is to determine the transverse emittances and Courant–Snyder parameters for the beam in the section of the VELA beam line immediately following the gun. Longitudinal effects will play some role, especially in the presence of space charge, and we plan to include the longitudinal dynamics in future work. For now, we assume that the relevant beam properties can be described by the  $4 \times 4$  transverse covariance matrix  $\Sigma$  with elements  $\Sigma_{ij} = \langle x_i x_j \rangle$ , where  $x_i$  is an element of the phase-space vector  $\vec{x} = (x, p_x, y, p_y)$  for a single particle, and the brackets  $\langle \cdot \rangle$  indicate an average over all particles in the bunch.  $x$  and  $y$  are respectively the horizontal and vertical co-ordinates of a particle, and  $p_x$  and  $p_y$  the horizontal and vertical momenta scaled by a reference momentum  $P_0$  ( $= 4.5$  MeV/c in the present case). The eigenemittances are constant for a given beam under linear symplectic transport, and can be obtained from the covariance matrix  $\Sigma$  using the fact that the eigenvalues of  $\Sigma S$  are  $\pm i\varepsilon_I$  and  $\pm i\varepsilon_{II}$ , where  $S$  is the  $4 \times 4$  antisymmetric matrix with block diagonals:

$$S_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (1)$$

and  $\varepsilon_I, \varepsilon_{II}$  are the eigenemittances [2]. The Courant–Snyder parameters can be obtained from the eigenvectors of  $\Sigma S$ . To determine the eigenemittances and Courant–Snyder parameters at a given point in the beamline, we therefore need to determine the elements of the covariance matrix  $\Sigma$  at that point in the beamline. This can be done using quadrupole scans, as follows.

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Consider two points in the beamline:  $s_0$  (the reconstruction point) where we wish to determine the elements of  $\Sigma$ , and  $s_1$  (the observation point) where there is a screen allowing us to observe the transverse distribution of the beam in co-ordinate space. We assume that it is possible to change the transfer matrix  $R(s_1, s_0)$  from  $s_0$  to  $s_1$ , for example by changing the strengths of one or more quadrupole magnets between  $s_0$  and  $s_1$ . For a given transfer matrix  $R = R(s_1, s_0)$ , the covariance matrices  $\Sigma(s_1)$  and  $\Sigma(s_0)$  at  $s_0$  and  $s_1$  respectively, are related by:

$$\Sigma(s_1) = R\Sigma(s_0)R^T, \quad (2)$$

where  $R^T$  is the transpose of  $R$ . Hence, we can write:

$$u(s_1) = Cv(s_0), \quad (3)$$

where  $u(s_1)$  is a (column) vector with components  $(\langle x^2 \rangle, \langle xy \rangle, \langle y^2 \rangle)$  at  $s_1$ ,  $C$  is a  $3 \times 10$  matrix constructed from combinations of the elements of the transfer matrix  $R$ , and  $v(s_0)$  is a (column) vector with 10 components corresponding to the independent elements of the (symmetric) matrix  $\Sigma(s_0)$ . We do not give explicit expressions for the elements of  $C$  in this paper because of space constraints; but it is not difficult to construct the required expressions.

The elements of the matrix  $u(s_1)$  can be obtained from the images on the screen at the observation point,  $s_1$ . Suppose that we make  $N$  observations corresponding to  $N$  different transfer matrices  $R(s_1, s_0)$  (for example, with  $N$  different strengths of a given quadrupole between  $s_0$  and  $s_1$ ). We can assemble the observed beam distributions into a single vector  $u(s_1)$  with  $3N$  components, and at the same time extend the matrix  $C$  (using the corresponding transfer matrices) into a  $3N \times 10$  matrix. The vector  $v(s_0)$ , however, remains unchanged since the covariance matrix at  $s_0$  is unaffected by beamline components between  $s_0$  and  $s_1$ . Finally, we can invert Eq. (3) to obtain  $v(s_0)$  in terms of the inverse of the known matrix  $C$ , and the known vector  $u(s_1)$ . The vector  $v(s_0)$  contains all the elements of the covariance matrix  $\Sigma(s_0)$  at the reconstruction point: we can then obtain the eigenemittances and Courant–Snyder functions from the eigenvalues and eigenvectors of  $\Sigma(s_0)S$  as described above.

This technique is based on a well-known procedure for measuring the emittance in one degree of freedom (see, for example [3]), but extended to take account of coupling. Space charge effects are a little more difficult, since even in a linear approximation the effects depend on the beam size, which is not known *a priori*, but is determined as a result of the analysis. Space charge is discussed in the following section; but first, we complete the present section by considering some specific requirements for the measurements.

In the simplest case, we can consider a single quadrupole between  $s_0$  and  $s_1$ : then, in the thin-lens approximation, the quantities  $\langle x^2 \rangle$ ,  $\langle xy \rangle$  and  $\langle y^2 \rangle$  vary quadratically with the focusing strength  $k_1L$  of the quadrupole. Fitting a parabola to a curve of  $\langle x^2 \rangle$  vs  $k_1L$  yields three parameters, which are related to the elements of the covariance

matrix  $\Sigma(s_0)$ . From the curves of  $\langle xy \rangle$  and  $\langle y^2 \rangle$  vs  $k_1L$  we obtain a further six parameters, hence nine parameters in total. However, the  $4 \times 4$  symmetric covariance matrix  $\Sigma(s_0)$  has ten independent elements: therefore, by scanning a single quadrupole, we lack sufficient constraints to determine uniquely the elements of  $\Sigma(s_0)$ . In the analysis procedure described above, this will be evident by one of the singular values of  $C$  being vanishingly small. However, by scanning two quadrupoles we obtain six parabolas and 18 constraints: the fitting of  $\Sigma(s_0)$  to the data then becomes over-constrained, which allows any discrepancy between the model and the data to be assessed from the quality of the fit to the measurements. Scanning three quadrupoles provides 27 constraints, which over-constrains the fit even more strongly.

## SPACE CHARGE EFFECTS

The evolution of the transverse beam size in a DC beam in the presence of space charge (and neglecting coupling) is described by the envelope equation:

$$\frac{d^2\sigma_x}{ds^2} + k_1\sigma_x - \frac{\varepsilon_x}{\sigma_x^3} - \frac{K}{2(\sigma_x + \sigma_y)} = 0, \quad (4)$$

where  $\sigma_x = \sqrt{\langle x^2 \rangle}$  is the rms beam size,  $k_1$  is the local focusing strength (e.g. from quadrupole magnets),  $\varepsilon_x$  is the horizontal emittance, and  $K = 2I/\beta^3\gamma^3I_c$  is the perveance. Here,  $I$  is the beam current,  $\beta$  and  $\gamma$  the scaled velocity and relativistic factor for particles in the beam, and  $I_c$  the critical current (or Alfvén current  $\approx 17.045$  kA in the case of an electron beam). The evolution of the vertical beam size is described by a similar equation. For a beam with a Kapchinsky–Vladimirsky (KV) distribution, the envelope equation (4) gives an exact description of the beam size evolution (with constant emittance). For non-KV distributions, the emittance is no longer constant, but the envelope equation is still, in many cases, a good approximation for the beam behaviour. For bunched beams, the situation is even more complicated since the current is a function of longitudinal position within the bunch, and hence the space charge forces will also vary with longitudinal position. Nevertheless, we may assume that if space charge forces are not too strong (i.e. the bunch charge is not too large) then the evolution of the beam size can be approximated by the envelope equation with some “average” value  $\bar{K}$  for the perveance. Space charge forces may then be taken into account in the analysis described in the previous section (for measurement of the beam covariance matrix) by including a linear defocusing term:

$$k_{1,sc} = -\frac{\bar{K}}{2\sigma_x(\sigma_x + \sigma_y)} \quad (5)$$

at each point along the beamline for the horizontal motion, and a similar (also defocusing) term for the vertical motion. The difficulty is that the beam sizes  $\sigma_x$  and  $\sigma_y$  are not known at each point along the beamline from  $s_0$  to  $s_1$ .

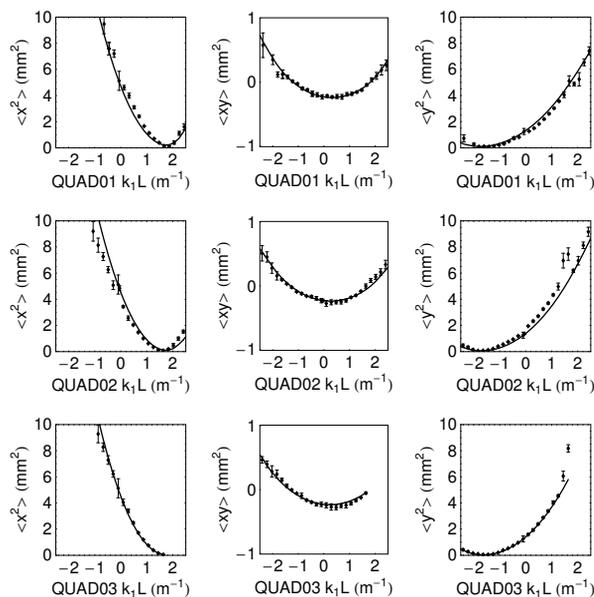


Figure 2: Typical measurements of beam size vs quadrupole strength for three quadrupoles on VELA.

However, we can make reasonable initial estimates based on the beam sizes observed at  $s_1$  (and at  $s_0$ , if we choose a reconstruction point where it is possible to insert a YAG screen); the estimates can be improved based on analysis of the quadrupole scan measurements. The covariance matrix  $\Sigma(s_0)$  can then be determined by an iterative procedure, where the goal is to find a beam distribution consistent with the beam sizes assumed in calculating the space charge forces.

## EMITTANCE MEASUREMENT RESULTS

Data were collected on VELA from separate scans of three quadrupoles between YAG-01 and YAG-03, with 10 pC bunch charge and 4.5 MeV/c momentum. Although it was not possible to measure the bunch length, it is expected that in this regime space charge effects are significant, but not dominant (the space-charge term in the envelope equation is comparable in size to the emittance term).

Results of the analysis (including space charge) are shown in Fig. 2. The data points are obtained by fitting 2D Gaussian functions to the image intensity observed on YAG-03. Error bars represent the standard deviation over 10 machine pulses. The fitting procedure represented by inverting Eq. (3) is modified to give increased weight to points with smaller error bars; the errors can be propagated to give error estimates on the emittances. Given that the fit is highly over-constrained, there is reasonably good agreement between the data points and the results of the fits (represented by continuous lines in Fig. 2).

Figure 3 shows the measured eigenemittances for different currents in the bucking coil. Although there is some deviation from the theoretical behaviour (shown by the solid

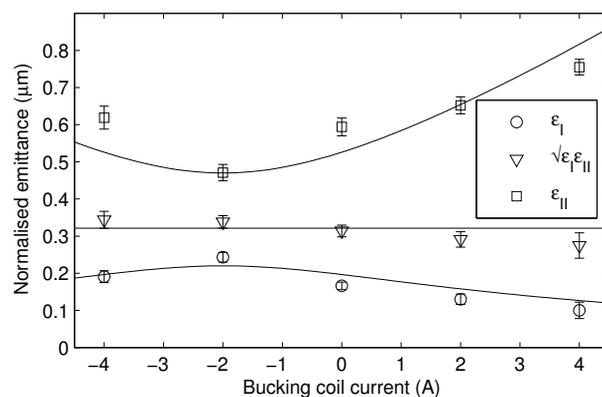


Figure 3: Variation of eigenemittances with bucking coil current on VELA.

curves in Fig. 3) at larger values of the current, there is reasonable qualitative agreement between measurement and theory. In particular, the product of the eigenemittances is roughly constant, with the values approaching each other most closely at a current in the bucking coil of -2 A, when the solenoid field on the cathode is expected to be zero. It should be noted that the theoretical expectation for the behaviour of the eigenemittances is based on a highly simplified model [4] that assumes (for example) constant particle energy from the cathode (i.e. neglecting acceleration within the gun), and ignores space charge effects.

Finally, by choosing a reconstruction point at a location in the beamline where a YAG screen can be inserted, we can validate the results of the quadrupole scan analysis by comparing the reconstructed beam sizes with the directly observed values. We find that although the beam sizes show the same trend with bucking coil current, the reconstructed values are systematically smaller than the directly observed values by about 10%. This could be for a variety of reasons, including nonlinear (or longitudinal) space charge effects, or systematic errors in quadrupole or screen image calibration. Nevertheless, we feel that the level of agreement at this stage provides some support for the validity of the analysis technique that we have developed.

## CONCLUSIONS AND FURTHER WORK

Using an analysis technique that includes the effects of coupling and (in a simple linear approximation) space charge, quadrupole scan measurements on VELA show normalised eigenemittances of order  $0.5 \mu\text{m}$  at 10 pC bunch charge. The eigenemittances vary with the bucking coil current in a way that is broadly in line with expectations from a simplified theoretical model. There remains some discrepancy between the observed beam sizes and the beam sizes expected from the results of the quadrupole scan analysis. We plan to carry out a more thorough investigation of systematic errors, and will also study the impact of nonlinear and longitudinal effects of space charge, including measurements at higher bunch charge.

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