

MIRROR SYMMETRIC CHICANE-TYPE EMITTANCE EXCHANGE BEAMLINE WITH TWO DEFLECTING CAVITIES

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Abstract

In this paper we present the conceptual design of a mirror symmetric chicane-type beamline with two dipole-mode cavities for transverse-to-longitudinal emittance exchange.

INTRODUCTION

Optical systems for transverse-to-longitudinal emittance exchange (EEXs) were initially proposed in application to the free electron lasers for transverse emittance reduction in an electron beam with a smaller longitudinal emittance [1]. Since then EEXs involving single transverse deflecting cavity (TDC) were in great details studied theoretically and experimentally, and many interesting applications of such beamlines were suggested [2–6].

Among all EEXs chicane-type beamlines are of keen interest, because they do not alter the beam propagation direction. To our knowledge, the first design of such EEX was presented in [6] and, in minimal configuration, can be seen in Fig. 1 (beamline one). Besides TDC and four dipoles, this beamline includes fundamental (accelerating) mode cavity (FMC) to cancel the longitudinal acceleration in the TDC (the so-called thick-lens effect) and two quadrupoles (green ellipses) to reverse the dispersion at the TDC location.

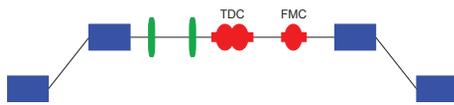


Figure 1: Schematic of non-symmetric chicane-type EEX.

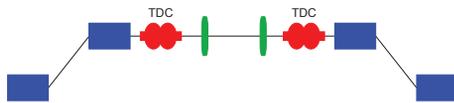


Figure 2: Schematic of symmetric chicane-type EEX.

The beamline one demonstrates two features which are common to all EEXs with a single TDC. First, such EEXs cannot be mirror symmetric with respect to the TDC center and, second, without involvement of at least one FMC the emittance exchange cannot be made exact even on the level of the linear beam dynamics. One of the ways to overcome these limitations was found in [7] where properties of EEXs utilizing two TDCs instead of one were investigated. In this paper we detail some results of [7] and present a mirror symmetric chicane-type EEX with two TDCs which does not require additional FMCs for compensation of the thick-lens effect. Schematic of this beamline (again, in minimal configuration) can be seen in Fig. 2 (beamline two), and there are the following similarities and differences in comparison with the beamline one:

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- Quadrupoles in the beamline one are used for the reversion of the entrance dogleg dispersion and both are horizontally focusing. Quadrupoles in the beamline two have similar purpose and also must provide dispersion sign change, but in this case between the TDC centers. Besides that, they have an additional duty and work for suppression of the thick-lens effect, which allows to avoid usage of FMCs.
- With equal dispersions generated by the entrance dogleg, the total transverse deflection required is equal for both beamlines. It means that the strength of each TDC in the beamline two is two times smaller than the strength of the single TDC in the beamline one.
- The mirror symmetry of the beamline two automatically cancels part of nonlinear aberrations in the beamline map.

MATRICES AND SYMMETRIES

In the deriving conditions for a beamline to be an EEX, we consider the linear symplectic dynamics in the horizontal and longitudinal degrees of freedom and ignore the vertical degree of freedom, which (on the linear level) is assumed to be decoupled from the two others. Still, for convenience, we index elements of the 4×4 horizontal-longitudinal transport matrices as if these matrices were extracted from the complete three degrees of freedom 6×6 matrices, where the first degree of freedom is horizontal, the second is vertical, and the longitudinal comes as the third.

Transport Matrices

From the assumptions made it follows that the horizontal-longitudinal matrix of a magnetostatic system has the form

$$M = \begin{bmatrix} m_{11} & m_{12} & 0 & m_{16} \\ m_{21} & m_{22} & 0 & m_{26} \\ m_{51} & m_{52} & 1 & m_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

and, as concerning the TDC matrix, we take it in the commonly used approximation

$$R(\kappa, l_c, q) = \begin{bmatrix} 1 & l_c & \kappa l_c/2 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & \kappa l_c/2 & q\kappa^2 l_c & 1 \end{bmatrix}, \quad (2)$$

where l_c is the cavity length, κ is the deflecting strength, and q is the energy gain factor. The particular value of q depends from the cavity design and, for example, for the n -cell pillbox resonator satisfies $1/6 < q \leq 1/4$.

Approximations made in the equations of motion in order to obtain matrix (2) include among others the neglect of the terms of the order $O(\gamma_0^{-2})$, where γ_0 is the Lorentz factor of the reference particle. To be consistent with this, we

assume that these terms were also neglected during derivation of all other matrices. With this convention matrices of straight drift-quadrupole systems have $m_{56} = 0$ and the TDC matrix can be decomposed into the product

$$R(\kappa, l_c, q) = D(l_c/2) C(\kappa, ql_c) D(l_c/2), \quad (3)$$

where $D(l)$ is the matrix of a drift space of the length l and

$$C(\kappa, w) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & 0 & w\kappa^2 & 1 \end{bmatrix} \quad (4)$$

is the ‘‘thin-lens image’’ of the matrix (2).

Optics Symmetries

The matrix of a beamline which is mirror symmetric about the $x - y$ plane to the beamline with the 4×4 horizontal-longitudinal transport matrix

$$M = \begin{bmatrix} M_{11} & M_{13} \\ M_{31} & M_{33} \end{bmatrix} \quad (5)$$

is given by the formula

$$M_R = T_R M^{-1} T_R = \begin{bmatrix} SM_{11}^T S & -SM_{31}^T S \\ -SM_{13}^T S & SM_{33}^T S \end{bmatrix}, \quad (6)$$

where $T_R = \text{diag}(1, -1, -1, 1)$ and

$$S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad (7)$$

and the matrix of the beamline which is mirror antisymmetric to the original beamline (reversed and then rotated by 180° around the longitudinal axis) can be calculated according to the rule

$$M_A = T_A M^{-1} T_A = \begin{bmatrix} SM_{11}^T S & SM_{31}^T S \\ SM_{13}^T S & SM_{33}^T S \end{bmatrix}, \quad (8)$$

where $T_A = \text{diag}(-1, 1, -1, 1)$.

With these definitions the TDC matrix (2) satisfies

$$[R(\kappa, l_c, q)]_R = R(-\kappa, l_c, q), \quad (9a)$$

$$[R(\kappa, l_c, q)]_A = R(\kappa, l_c, q), \quad (9b)$$

which means that the TDC matrix is mirror antisymmetric with respect to itself. It gives simple and natural explanation to the fact that EEX with a single TDC cannot be designed to be mirror symmetric with respect to the TDC center.

Mirror Symmetric and Antisymmetric EEXs

By definition, EEX is a beamline with the transfer matrix T which has zero 2×2 diagonal submatrices T_{11} and T_{33} . So the eight elements of the matrix T must be equal to zero, but it gives only four independent constraints because owing to the symplectic conditions equations $T_{11} = 0$ and $T_{33} = 0$ are equivalent. For mirror symmetric or antisymmetric beamline equations $T_{11} = 0$ and $T_{33} = 0$ can be re-expressed in terms of the matrix M of the first beamline half with the result that these equations become equivalent to the constraints

$$ST_{11} = M_{11}^T S M_{11} - M_{31}^T S M_{31} = 0, \quad (10a)$$

$$ST_{33} = M_{33}^T S M_{33} - M_{13}^T S M_{13} = 0, \quad (10b)$$

for the mirror symmetric EEX, and to the conditions

$$ST_{11} = M_{11}^T S M_{11} + M_{31}^T S M_{31} = 0, \quad (11a)$$

$$ST_{33} = M_{33}^T S M_{33} + M_{13}^T S M_{13} = 0, \quad (11b)$$

for the mirror antisymmetric case.

One sees that the transpose symmetry of the matrices ST_{11} and ST_{33} in Eqs. (10) and (11) reduces the number of constraints from four for the general beamline to three for the beamline which is mirror symmetric or antisymmetric.

Symmetric EEXs with Two TDCs Separated by Drift-Quadrupole Optics

Let us consider symmetric EEX with two TDCs separated by the drift-quadrupole optics, i.e. let us assume that the matrix of the first half of the beamline has the form

$$M = B C(\kappa, w) A, \quad (12)$$

where $w = ql_c$, B is the matrix of a drift-quadrupole system, A is the matrix of a general dispersive magnetostatic system, and drifts from the decomposition (3) are included into the optics blocks described by the matrices A and B . Then constraints (10) turn into requirements

$$2 a_{16} \kappa = -1, \quad (13a)$$

$$b_{11} a_{16} + b_{12} a_{26} = 0, \quad (13b)$$

$$b_{12} b_{22} = w, \quad (13c)$$

and constraints (11) become equivalent to the equations

$$2 a_{16} \kappa = -1, \quad (14a)$$

$$b_{21} a_{16} + b_{22} a_{26} = 0, \quad (14b)$$

$$b_{12} b_{22} = -w, \quad (14c)$$

where a_{16} and a_{26} are the horizontal dispersion and its slope at the center of the first TDC, respectively.

Let us discuss, for example, Eqs. (13) in more details. One sees, that Eq. (13a) requires cavity strength to be matched to the horizontal dispersion at the cavity center, Eq. (13c) works for the suppression of the thick-lens effect, and Eq. (13b) forces horizontal dispersion to cross zero in the beamline symmetry point and brings dispersion and its slope at the center of the second TDC to the values $-a_{16}$ and a_{26} , respectively. Note also that from Eqs. (13a) and (13b) it follows that $b_{12} \neq 0$.

CHICANE-TYPE SYMMETRIC EEX

Let us return to the beamline two and look at the relations between its parameters in the framework of the thin-lens formalism. Because of the need of dispersion reversion between the TDC centers, both quadrupoles in this beamline are horizontally focusing. So, in order to enable at least some possibilities for the vertical focusing control, let us add to the minimal configuration one more quadrupole placing it in

the system symmetry point. With this assumption the matrix B in Eq. (12) takes on the form

$$B = Q(k_2/2) D(l_2) Q(k_1) D(l_1), \quad (15)$$

where

$$Q(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ k & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

is the matrix of a quadrupole thin-lens, and $l_1 > l_c/2$ and $l_2 > 0$ are the distances from the center of the first TDC to the first thin-lens and from the first thin-lens to the system symmetry point, respectively.

With the dispersion slope $a_{26} = 0$, Eqs. (13) can easily be solved with respect to the unknowns k_1 and k_2 . The solution

$$k_1 = -1/l_2, \quad k_2 = 2(l_1 + w - l_2)/l_2^2 \quad (17)$$

is unique and completely fixes the horizontal focusing block of the triplet matrix $B_R B$ to the form

$$(B_R B)_{11} = \begin{bmatrix} -1 & 2w \\ 0 & -1 \end{bmatrix}. \quad (18)$$

One sees that the only possibility left by Eqs. (17) for the vertical focusing control is the proper choice of the distances l_1 and l_2 . It is not much and, in general, more quadrupoles need to be added to the system, but still there are some positive opportunities. For example, for $l_2 = 2(l_1 + w)/3$ we have $k_2 = -k_1$ and the complete triplet acts on the vertical motion as a drift of the length $2(4l_1 + w)/3$, which is shorter than the triplet length $2(5l_1 + 2w)/3$.

Dispersion Boosting by Means of Quadrupoles

Due to the reciprocal relationship (13a) between dispersion at the TDC center and the TDC strength, increasing of dispersion allows to reduce excitation of TDCs, which is important for high energy applications. In the case when the dispersion boosting can be done by means of quadrupoles, it becomes possible to use not only weaker cavities, but also weaker dipoles, which is helpful for suppression of the effects of coherent synchrotron radiation.

Dispersion boosting in our system can be done, for example, by placing two more quadrupoles (four in total, due to the system symmetry) somewhere between the exit of the first chicane dipole and the entrance of the first TDC. In this situation one can not only increase the dispersion, but also preserve the condition $a_{26} = 0$, so that the parameters of the inner triplet still will be given by Eqs. (17). But, because Eqs. (13) do not require dispersion slope at the TDC center to be equal to zero, it could make sense to consider also the simpler configuration in which only one horizontally defocusing quadrupole is added to each system half as it is shown in Fig. 3. Solving for this beamline Eqs. (13) with respect to the unknowns k_1 and k_2 , one obtains

$$k_1 = -1/l_2 - u/l_1, \quad (19a)$$

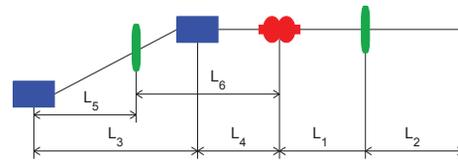


Figure 3: First half of dispersion boosted EEX.

$$k_2 = 2 \left[w + (1-u)l_1 - (1-u)^2 l_2 \right] / \left[(1-u)^2 l_2^2 \right], \quad (19b)$$

where

$$u = (l_1 a_{26} / a_{16}) (1 + l_1 a_{26} / a_{16})^{-1}. \quad (20)$$

As concerning position l_5 and strength k_0 of the additional lens, let us first introduce dispersion boosting factor z (the ratio of the boosted and the original dispersions) and, second, let us assume that $l_4 < l_3$. Taking now $l_5 = l_6 = (l_3 + l_4)/2$, which in the small angle approximation for the dipole deflection is the position providing the largest z for the fixed k_0 value, one obtains

$$k_0 = \frac{l_3}{l_5^2} (z - 1), \quad l_1 \frac{a_{26}}{a_{16}} = \frac{l_1 z - 1}{l_5 z}. \quad (21)$$

Eqs. (19) and (21) tell us that when the factor z is fixed, then the possibility left for the control of the vertical focusing is again, as in the system without dispersion boosting, only the choice of the distances l_m . Unfortunately, the question is whether exists a satisfactory choice of these distances or not cannot be answered in the general form. It depends on particular vertical focusing requirements and it is, eventually, a designer choice either use minimal or next-to-minimal EEX configuration, or add more quadrupoles to the system.

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