

3D COMPUTER SIMULATIONS OF THE ULTRARELATIVISTIC BEAM DYNAMICS IN SUPER COLLIDERS*

M.A. Boronina, V.A. Vshivkov, ICM&MG SB RAS, Novosibirsk, Russia
 G.I. Dudnikova, ICT SB RAS, Novosibirsk, Russia

Abstract

The problem of numerical modeling of beam-beam interaction with high relativistic factor ($\sim 10^4$) is considered. We present a 3D self-consistent simulation model based on particle-in-cell method. The mixed Euler-Lagrangian decomposition is used in the parallel algorithm for achieving good load balancing and reducing communication costs. Stable regimes of beam dynamics, depending on the beams configuration (beta-function, emittance, energy, currents and relative offset) can be found on the base of the model. In the calculations we used grid $100 \times 100 \times 100$ with 10^8 macro-particles, the number of processors depends highly on the beam shape.

INTRODUCTION

We present a new PIC code for mathematical modeling of the collective ultrarelativistic beam dynamics with the aim to achieve large luminosities. The high relativistic factors of the beam particles ($\gamma \sim 10^3-10^5$) lead to the luminosity restriction, the negative effects can be reduced by optimization of the beam configuration.

The standard slice approach represents the slices rearrangement, each slice of one beam impacts on the counter beam particles by two-dimensional forces. The approach is widely used for cyclic accelerators, where the beam deformations are not strong, the computations of a single interaction are fast, number of slices 5-50 and number of macro-particles in the bunch 10^5 provide sufficient accuracy [1]. The single collision of ultrarelativistic beams of high densities may lead to a strong compression and even disruption of the bunch, and in this case longitudinal resolution must be adequate to the number of pinches. The longitudinal effects cannot be simulated by a quasi-3D model, but in the case of critical beam densities such as in ILC they may play significant role, and the redistribution of energy may cause ruinous consequences. The fine resolution requires an appropriate number of macro-particles in the beam (10^8-10^{10}). A monoprocessor machine allows performing 3D simulations with maximum $2 \cdot 10^6$ macro-particles, and new efficient parallel algorithms are needed to simulate the critical regimes of the ultrarelativistic beams interaction.

In our parallel 3D algorithm we apply the particle-in-cell (PIC) method and the leap-frog scheme [2,3] to solve Vlasov-Liouville equation the three-dimensional set of Maxwell's equations. The three-dimensional nature of interaction together with the new special methods for initial and boundary conditions computations [4] allow automatically account for above difficulties and demonstrate good results.

MATHEMATICAL MODEL

We consider the motion of counter beams in a rectangular domain in Cartesian coordinates. The positron/electron beams are focused by the external focusing field and move in the self-consistent electromagnetic fields. The boundaries of the computational domain are located very close to the beams, in the near wave zone, no radiation effects are considered [5]. We use Vlasov kinetic equation for the distribution function of the particles and the three-dimensional set of Maxwell's equations to describe the beam dynamics. In dimensionless variables, which are obtained from the characteristic beam length $L = 1$ cm and the characteristic particle speed $v=c=299792.458$ km/s, the equations may be rewritten in the following form:

$$\frac{\partial f_{+,-}}{\partial t} + \mathbf{v}_{+,-} \cdot \frac{\partial f_{+,-}}{\partial \mathbf{r}} + \mathbf{F}_{+,-} \cdot \frac{\partial f_{+,-}}{\partial \mathbf{p}} = 0$$

$$\text{rot} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t},$$

$$\text{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t},$$

$$\text{div} \mathbf{E} = 4\pi (n_- e^- + n_+ e^+),$$

$$\text{div} \mathbf{H} = 0.$$

The Lorentz's force can be calculated from the following equation for each particle:

$$\mathbf{F}_{+,-} = e^{+,-} (\mathbf{E} + \mathbf{v}_{+,-} \times \mathbf{H} / c)$$

and the particle moment:

$$\mathbf{p}_{+,-} = \gamma_{+,-} m_e \mathbf{v}_{+,-}$$

$$\gamma_{+,-} = 1 / \sqrt{1 - |\mathbf{v}_{+,-}|^2 / c^2}$$

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We use particle-in-cell method with PIC form factor and the leap-frog scheme. All the components are calculated at the half-step shifted grids. In this case, all the derivatives involved in the equations are written with central differences, and this scheme provides second order by time and space. For example, we use the following scheme for the Maxwell's equations:

$$\frac{\mathbf{H}^{m+\frac{1}{2}} - \mathbf{H}^{m-\frac{1}{2}}}{\tau} = -rot_h \mathbf{E}^m$$

$$\frac{\mathbf{E}^{m+1} - \mathbf{E}^m}{\tau} = j^{m+\frac{1}{2}} + rot_h \mathbf{H}^{m+\frac{1}{2}}.$$

We use the Villancenor-Buneman scheme[6] for the computations of the currents, because this method satisfies exactly the Gauss's law in final differences and thus significantly reduces the approximation error and makes the algorithm more robust.

The three-dimensional Maxwell's equations allow calculating the beams movement regardless of the collective motion direction. However, as the boundaries are close to the beams, the accurate initial and boundary conditions must be set. The beam fields can be computed by summation of the particle density contributions, where the particles have shape of needles with the length h_z [4].

The stability condition of the leap-frog scheme requires a smaller spatial step to perform computations for flatter beams. The PIC method accuracy depends on the particle number in cell, so using a fine spatial grid necessitate using high number of particles [7]. The majority of the particles is concentrated in a small region of the domain due to the highly non-linear density distribution in space and time, and the particle data cannot be allocated in the memory of a single computer. The well-balanced 3D parallel code of high scalability was developed to overcome the difficulties [8].

SIMULATION RESULTS

The algorithm described has been tested on some characteristic examples of relativistic beam motion [9], the results showed good accuracy for the case of known analytical solution.

We present the results of numerical experiments in case of two equal counter beams of opposite charges: $Q_l = -Q_2 = 10^{12} e$. The particle density is Gaussian in the crossover plain:

$$\rho(X, Y, Z) = \frac{1}{(2\pi)^{\frac{3}{2}} (\sigma_x^* \sigma_y^* \sigma_z^*)^{\frac{1}{2}}} \times$$

$$\exp\left[-\frac{1}{2} \left(\frac{X-X_c^2}{\sigma_x^{*2}} + \frac{Y-Y_c^2}{\sigma_y^{*2}} + \frac{Z-Z_c^2}{\sigma_z^{*2}} \right)\right],$$

where $\varepsilon_x = 5 \cdot 10^{-5}$ is horizontal and $\varepsilon_y = 5 \cdot 10^{-7}$ is vertical beam emittances, $\beta_x^* = \beta_y^* = 0.1$ are the corresponding beta-function values, $\sigma_x^* = \sqrt{\beta_x^* \varepsilon_x}$, $\sigma_y^* = \sqrt{\beta_y^* \varepsilon_y}$ are the transversal beam sizes, the sign * denotes the values in the interaction point, $\sigma_z = 0.1$ is the beam size along z-axis. So, the beam size ratio is $\sim 10:1:100$. $(X_{1c}, Y_{1c}, Z_{1c}) = (0, 0, -0.35)$ is the center coordinates of one beam in the crossover plane, another beam is located symmetrically, $(X_{2c}, Y_{2c}, Z_{2c}) = (0, 0, 0.35)$. The interaction point coincides with the center $(x_c, y_c, z_c) = (0.05, 0.005, 0.75)$ of the domain $[0, 0.1] \times [0, 0.01] \times [0, 1.5]$.

The transversal momenta are distributed in the crossover plain according to the following law:

$$\rho(x', y') = \frac{1}{2\pi \sigma_{p_x}^* \sigma_{p_y}^*} \exp\left[-\frac{1}{2} \left(\frac{x'^2}{\sigma_{p_x}^{*2}} + \frac{y'^2}{\sigma_{p_y}^{*2}} \right)\right]$$

with $\sigma_{p_x}^* = \sqrt{\varepsilon_x / \beta_x^*}$, $\sigma_{p_y}^* = \sqrt{\varepsilon_y / \beta_y^*}$.

The focusing condition is described by the transformation between the laboratory system coordinate and the accelerator coordinate system:

$$x = X + (z - z_c) X',$$

$$y = Y + (z - z_c) Y',$$

$$z = Z + z_c,$$

$$p_x = p_z X',$$

$$p_y = p_z Y'.$$

We assumed that the beams are monoenergetic with $\mathbf{p}^2 = \gamma^2 - 1$ in dimensionless variables, $\gamma = 6.85 \cdot 10^3$, the longitudinal component of the moment $p_z = \sqrt{\mathbf{p}^2 - p_x^2 - p_y^2}$. These parameters describe pre-critical interaction regime.

In the numerical experiments we used $100 \times 100 \times 100$, number of macro-particles $J=10^8$, $h_x=10^{-3}$, $h_y=10^{-4}$, $h_z=1.5 \cdot 10^{-2}$, $\tau=5 \cdot 10^{-5}$, 14000 time steps.

Figures 1 and 2 demonstrate the beam coordinates (Z, X) and (Z, Y) respectively in the accelerator coordinate system in the time moments $t=0, T, 2T..5T$, where $T=3.33$ ps, the beam interaction begins at $t=T$ and finishes at $t=6T$. The beam changes before the interaction are minor. At $t > T$ the beam deformation along both transversal axes due to the field of the counter beam is observed. In the y direction the beam size is smaller, and the beam undergoes the overfocussing with the further spread. At $t=3.5T$ the focusing is maximal.

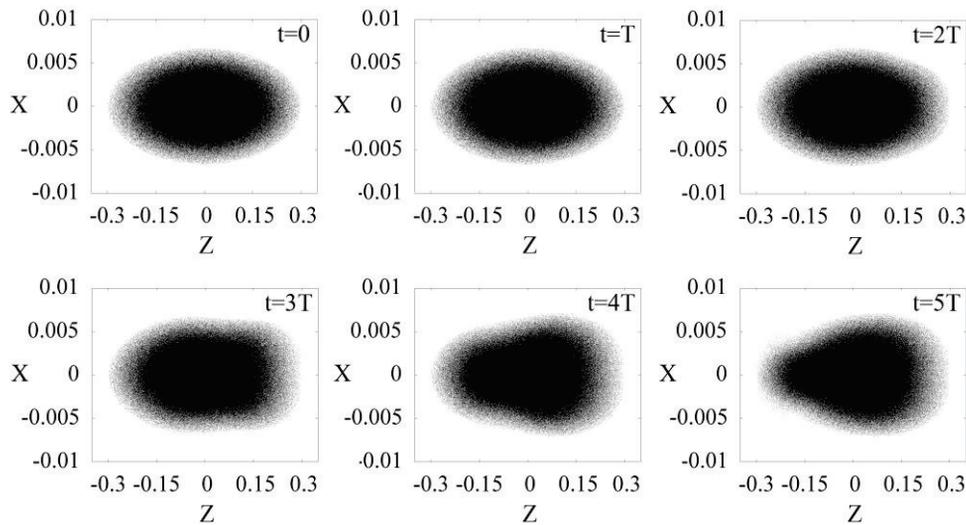


Figure 1: Beam coordinates (Z, X) in the accelerator coordinate system in different time moments.

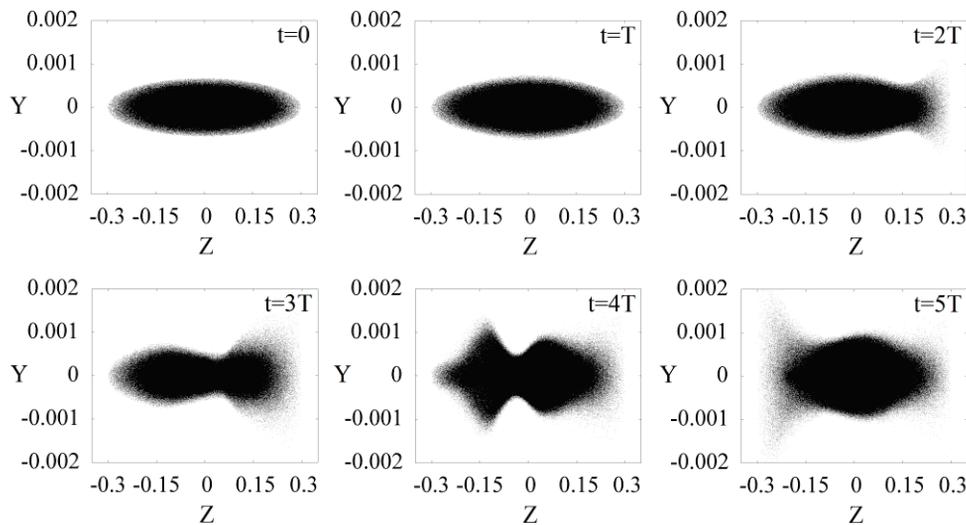


Figure 2: Beam coordinates (Z, Y) in the accelerator coordinate system in different time moments.

All the computations were performed on the Siberian Supercomputer Center cluster (ICM&MG SB RAS, Novosibirsk), 576 4-core processors Intel Xeon X5570/X5670 and the supercomputer Lomonosov (MSU, Moscow), 12346 4-core processors Intel Xeon X5570/X5670. The computations with for 10^7 macro-particles take 4 hours of 48 cores, in the case of 10^8 macro-particles 240 cores work 6 hours, for 10^9 macro-particles 1400 cores are needed.

CONCLUSION

A new fully three-dimensional model for the beam-beam simulation in super-colliders is presented. The model is based on the solution of Vlasov equation with the PIC method and takes into account the three-

dimensional nature of the beam interactions in cases of high relativistic factors ($\gamma \sim 10^3-10^5$). The mixed Euler-Lagrangian decomposition used in the parallel algorithm for achieving good load balancing allowed performing 3D numerical experiments with spatial grid $100 \times 100 \times 100$ and number of macro-particles 10^8 . The results of the simulation for two counter ultrarelativistic beams with size ratio 10:1:100 are presented.

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REFERENCES

- [1] Y. Zhang, J. Qiang. Application of BeamBeam3D to ELIC Studies. ICAP09, San Francisco. 2009.
- [2] Hockney, R. W. & Eastwood, J. W. Computer Simulation Using Particles. McGrawHill. (1981)
- [3] A.B. Langdon and B.F. Lasinski// Meth. Comput. Phys., v.16. (1976).
- [4] V.A. Vshivkov, M.A. Boronina. Three-dimensional simulation of ultrarelativistic charged beams dynamics: study of initial and boundary conditions. Matem. Mod, V. 24, 2, pp. 67-83 (2012).
- [5] M. Boronina, V. Vshivkov, E. Levichev, S. Nikitin, V. Snytnikov, "3D PIC Method Development For Simulation Of Beam-Beam Effects In Supercolliders", Proceedings of Particle Accelerator Physics (2007), pp. 3366-3368.
- [6] J.Villanenor, O. Buneman. Rigorous charge conservation for local electromagnetic field solvers, Computer Physics Communications, Volume 69, Issue 2-3, p. 306-316 (1992).
- [7] K.V. Lotov, I.V. Timofeev, E.A. Mesyats, A.V. Snytnikov, V.A. Vshivkov. Note on quantitatively correct simulations of the kinetic beam-plasma instability. Physics of Plasmas (1994-present) 22 (2) (2015).
- [8] M.A. Boronina, V.D. Korneev. The parallel three-dimensional PIC code for the numerical modeling of ultrarelativistic beams. Bulletin of the NCC: Numerical Analysis 16, 15–22 (2013).
- [9] M.A. Boronina et al., "Parallel three-dimensional PIC code for beam-beam simulation in linear colliders", MOPME027, Proc. of IPAC'14, Dresden, Germany (2014).