Intra-beam Scattering Study for Low Emittance of BAPS

S.K. Tian, J.Q. Wang, G. Xu, Y. Jiao (IHEP)
Outline

- Intra-beam scattering theory
- BAPS machine parameters (a temporary design lattice)
- Recent BAPS IBS calculations
- Conclusions
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Intra-Beam Scattering

Particles in a circulating bunch execute transverse betatron oscillations, the transverse velocities are statistically distributed, these particles can be scattered by collisions so transferring transverse momenta into longitudinal momenta, in general, one should distinguish between:

Large-angle Single Scattering ---TOUSCHEK EFFECT--- Lift Time

Multiple Small-Angle Scattering ---INTRA-BEAM SCATTERING--- Rise Time

\[
\text{collision } \Rightarrow \delta (x, xp, y, yp, z, \Delta p) \Rightarrow \delta \epsilon_i (j) \Rightarrow \delta \epsilon_i = \frac{1}{2N} \sum_{j'=1}^{N} \delta \epsilon_i (j)
\]
Conventional Theory of IBS

Bjorken-Mtingwa derive $T_i$ by the formula:

$$
\frac{1}{T_p} \equiv \frac{1}{\sigma_p} \frac{d\sigma_p}{dt}; \quad \frac{1}{T_v} \equiv \frac{1}{\varepsilon_v^{1/2}} \frac{d\varepsilon_v^{1/2}}{dt}; \quad \frac{1}{T_h} \equiv \frac{1}{\varepsilon_h^{1/2}} \frac{d\varepsilon_h^{1/2}}{dt}
$$

$$
\frac{1}{T_i} = 4\pi A(\log) \left\langle \int_0^\infty d\lambda \frac{\lambda^{1/2}}{[\det(L + \lambda I)]^{1/2}} \right\rangle \times \left\{ TrL^i Tr\left( \frac{1}{L + \lambda I} \right) \right\} ^{-3/2} 
$$

$$
\log = \ln \frac{b_{\text{min}}}{b_{\text{max}}} = \ln \frac{2}{\theta_{\min}} \quad \text{Coulomb log factor}
$$

$$
A = \frac{r_0^2 cN}{64\pi^2 \beta^3 \gamma^4 \varepsilon_i \varepsilon_s \sigma_s \sigma_p}
$$

$$
\frac{1}{T_i} = \sum_{m=1}^{M} \frac{S_{m+1} - S_m}{C \left( \frac{1}{T_i^m} \right)}
$$

$S_{M+1} = C, S_1 = 0$

$S$ : elements(drift, quadrupole, bend - magnet...)

<……> indicates that the integral is to be averaged around the accelerator lattice
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Ultimate Storage Rings (USR)

Diffraction limited operation at 1 Å

\[ \varepsilon < \frac{\lambda}{4\pi} \]

Both transverse planes emittance \(\sim 10\) pm

For BAPS, we firstly obtain the total emittance about 20pm locally-round beam method \(\Rightarrow \varepsilon_x = \varepsilon_y \approx 10\) pm
## Basic Parameters of BAPS (Bare Lattice)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol, unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>$E$, GeV</td>
<td>5</td>
</tr>
<tr>
<td>Circumference</td>
<td>$C$, m</td>
<td>1366.4</td>
</tr>
<tr>
<td>Current</td>
<td>$I_0$, mA</td>
<td>100</td>
</tr>
<tr>
<td>Bunch number</td>
<td>$n_b$</td>
<td>1836</td>
</tr>
<tr>
<td>Number of particles per bunch</td>
<td>$N_b$</td>
<td>$1.55\times10^9$</td>
</tr>
<tr>
<td>Natural bunch length</td>
<td>$\sigma_{l0}$, mm</td>
<td>1.2</td>
</tr>
<tr>
<td>RF frequency</td>
<td>$f_{rf}$, MHz</td>
<td>500</td>
</tr>
<tr>
<td>RF voltage</td>
<td>$V_{rf}$, MV</td>
<td>9</td>
</tr>
<tr>
<td>Harmonic number</td>
<td>$h$</td>
<td>2279</td>
</tr>
<tr>
<td>Natural energy spread</td>
<td>$\sigma_{e0}$</td>
<td>$7\times10^{-4}$</td>
</tr>
<tr>
<td>Momentum Compaction</td>
<td>$\alpha_p$</td>
<td>$4\times10^{-5}$</td>
</tr>
<tr>
<td>Emittance of bare lattice</td>
<td>$\varepsilon_x$ pm</td>
<td>51</td>
</tr>
<tr>
<td>Energy loss per turn</td>
<td>$U_0$, MeV</td>
<td>1.07</td>
</tr>
</tbody>
</table>
7BA*36 Linear Optics

Circumference: 1366.4 m, 2 superperiods, 36 supercells
Working point: 111.39, 39.30
Natural chromaticity: -184, -181
Natural emittance: 51 pm.rad
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Two high-beta 10-m straight sections for injection in the middle $\beta_{x/y} = (80, 21)$ m
Lattice Layout in One Superperiod

High-beta 10-m Straight section, $\beta_{x/y} = (80, 21)$ m
Low-beta 7-m Straight section, $\beta_{x/y} = (5, 1.11)$ m
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Impact of Energy

\[ \varepsilon_{x0} = C_q \frac{\gamma^2 \langle H \rangle}{J_x \rho} \quad \Rightarrow \gamma \uparrow \Rightarrow \varepsilon_o \uparrow \]

\[ \frac{1}{T_i} \propto A = \frac{r_0^2 cN}{64\pi^2 \beta^3 \gamma^4 \varepsilon_h \varepsilon_v \sigma_s \sigma_p} \quad \Rightarrow \gamma \uparrow \Rightarrow \frac{1}{T_i} \downarrow \]

"the optimal energy for our lattice?"

Conditions for calculation:
Bunch length=constant
Coupling factor=0.01

Other cases:
Over Voltage Factor \( \frac{V_{RF}}{U_0} \)=constant
RF voltage \( V_{RF} \)=constant

The emittance minimum is near the nominal energy in any case!

Emittance vs. energy for different currents
Impact of Initial (Input) Emittance

Optimized Radiation Integral $\rightarrow$ Decrease The Nature Emittance

Phase Space Manipulations $\rightarrow$ Change The Vertical and The Horizontal Emittance

Damping Wiggler

Round Beam with Solenoid Field (at Special Straights)
Damping Wiggler

\[
\frac{\varepsilon_{w0}}{\varepsilon_0} = \left( \frac{J_{xw}}{J_x} \right) \frac{4C_q}{15\pi J_x} N_w \left\langle \beta_x \right\rangle y^2 \frac{\rho_0}{\rho_w} \theta^3_w
\]

\[
\theta_w = \frac{\lambda_w}{2\pi \rho_w}
\]

\[
L_w = N_w \lambda_w
\]

The emittance reduction depends on the wiggler period length, the wiggler peak field, and the total wiggler length.

Relative emittance reduction vs. wiggler length (left) and vs. wiggler field (middle) for different wiggler period.
Steady-state emittances for different bunch current in BAPS with long damping wigglers

Damping wiggler parameters: $B_w=2.3\,T$, $\lambda_w=3.1\,cm$, $L_w=62\,m$
Impact of Coupling Factor

For simplicity and for the purpose of IBS calculations, we assume that the vertical emittance is primarily generated by the coupling, and the effects of the vertical dispersion can be ignored.

\[ \frac{1}{T_i} \propto A = \frac{r_0^2 cN}{64\pi^2 \beta^3 \gamma^4 \varepsilon_h \varepsilon_v \sigma_s \sigma_p} \]

\[ \kappa = \frac{\varepsilon_y}{\varepsilon_x} \quad \varepsilon_{nat} = \varepsilon_x + \varepsilon_y \]

\[ \varepsilon_x = \frac{1}{1+\kappa} \varepsilon_{nat} \quad \varepsilon_y = \frac{\kappa}{1+\kappa} \varepsilon_{nat} \]
locally-round beam has no effect on control the IBS effect. The solenoid-ID-anti-solenoid section in a long straight section a dispersion-free region the dispersion invariant $\mathcal{H}$ is zero.

By G. Xu, Y. Jiao
Impact of Bunch Length

\[ A = \frac{r_0^2 cN}{64\pi^2 \beta^3 \gamma^4 \varepsilon_h \varepsilon_v \sigma_s \sigma_p} \]

From storage ring longitudinal beam dynamics theory \( \sigma_\Omega = \alpha_p \sigma_p = \text{constant} \)

Where \( \Omega^2 = \frac{\alpha e V_0}{T_0 E_0} \) Synchrotron Frequency

We can control on the longitudinal bunch size by changing the voltage slope seen by the bunch.

- Lowering the peak accelerating voltage reduce the bucket height decrease of beam lifetime
- Adding a harmonic cavity
Voltage and RF Acceptance

Voltage

\[ V_{m1} = 3.5 \]
\[ V_{m2} = 6 \]
\[ U_s = 1.14 \]

Potential

\[ \Phi_{m1} \]
\[ \Phi_{m2} \]

\[ \delta_{\text{acc1}} \]
\[ \delta_{\text{acc2}} \]

Charge density

\[ \text{cavity 1} \]
\[ \text{cavity 2} \]

Time [ps]

Time [ns]
Double RF System (Landau Cavity)

\[ V(t) = V_{RF} \left[ \sin (\omega_{RF} t + \phi_s) + k \sin (n \omega_{RF} t + n \phi_h) \right] \]

\[ V(0) = \frac{U_s}{e} = V_s \]
\[ \frac{\partial V}{\partial t} \bigg|_{t=0} = 0 \]
\[ \frac{\partial^2 V}{\partial t^2} \bigg|_{t=0} = 0 \]

\[ k = \sqrt{\frac{1}{n^2} - \frac{(U_0 / V_{RF})}{n^2 - 1}} \]
\[ \phi_h = \frac{1}{n} \arctan \left( \frac{1}{n} \tan(\phi_s) \right) \]
\[ \sin \phi_s = \frac{n^2 U_0}{n^2 - 1 V_{RF}} \]

- The voltage gain per turn of the synchronous particle is equal to the total loss per turn.
- The first derivative of voltage should vanish at the center of the bunch.
- The second derivative of voltage should vanish in order to avoid having a second region of phase stability close by.
Each bunch sees a flat voltage

\[ E_0 \]

\[ \sigma_t = 4 \text{ps} \]
\[ \sigma_s = 1.2 \text{mm} \]

\[ \sigma_t = 27 \text{ps} \]
\[ \sigma_s = 8.2 \text{mm} \]
Equilibrium Values for Emittance

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_x + \varepsilon_y$</th>
<th>$\varepsilon_x + \varepsilon_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare Lattice</td>
<td>51+1</td>
<td>75+1.47</td>
</tr>
<tr>
<td>Bare Lattice with LC</td>
<td>51+1</td>
<td>60+1.18</td>
</tr>
<tr>
<td>Lattice with DW(62) and LC</td>
<td>14+1</td>
<td>16.1+1.14</td>
</tr>
<tr>
<td>Lattice with DW(49.6) and LC</td>
<td>16+1</td>
<td>18.6+1.15</td>
</tr>
</tbody>
</table>

Equilibrium values for the emittances calculated with (right) and without (left) IBS
Without bunch lengthening, emittance growth from IBS is significantly larger and becomes even stronger for ultra-low emittance configurations.
Conclusions

Intra-beam scattering is an effect which becomes important in future low emittance ring-base light sources and becomes a limiting factor for reaching ultimate storage ring.

Introduce Harmonic Cavity to lengthen the bunch to further reduce the effect of intra-beam scattering.

Round beams could help to control the IBS effect, we need new efficient and robust solutions to obtain round beams in electron storage rings.
Reference:


Karl Bane, Kirk Bertsche, Yunhai Cai, etc A Design Report of the Baseline for PEP-X: an Ultra-Low Emittance Storage Ring


Thank you!