USING THE POWER SPECTRAL DENSITY METHOD TO CHARACTERIZE AND EVALUATE THE X-RAY MIRRORS SURFACES*

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Abstract

Rapid progress in synchrotron X-ray beams' coherence and X-ray optics performance places a high demand on characterization and evaluation of optical surface figure and slope errors and roughness on meter-long optics over spatial frequencies as short as 0.1mm. In this paper, the propagation model of hard X-ray beams through reflecting mirror surface is proposed based on wave-front propagation, and numerical simulations are performed for predicting the hard X-ray focusing performance of different imperfect mirrors using a Fresnel diffraction calculation. The imperfect mirror surface height maps synthesized from power spectral density (PSD) functions are used to analyze and evaluate the influences of different mirror surface errors on the reflected hard X-ray beam properties.

INTRODUCTION

Owing to the development of third generation synchrotron radiation and the wide employment and incessant improvement of undulator, the emittance of synchrotron sources was gradually decreased and the available coherent output has been growing rapidly, consequently some coherent X-rays experiments have been carried out [1, 2]. And the high-quality hard X-ray beams impose rigid requirements on the high quality of the optical components that comprise current coherent X-ray experimental set-ups [3-5]. The height profile of the reflecting mirror and the compositional and structural deficiencies of some auxiliary components can introduce additional distorted wave-fronts and can produce unavoidable artefact images in the form of speckle pattern which could affect the accuracy of experimental results [6-9].

In this paper, we investigate the influence of the imperfect reflecting mirror surface on the focused hard X-ray beam properties. Firstly, we synthesized imperfect mirror surface height maps using PSD functions. Secondly, a simple model for calculating the intensity profiles of X-rays focused by reflecting mirrors is proposed, based on the numerical calculations of Fresnel diffraction integral. Finally, we perform wave-optical simulations with synthesized mirrors satisfying different fractal distributions.

SYNTHESIZING SURFACE HEIGHT MAPS FORM PSD FUNCTIONS

The rapid evolutions taking place in synchrotron sources and the coherent X-ray experiments are producing an urgent need for X-ray optical components with increasing size and precision. To obtain well focused and spatial coherent X-ray beams, reflective mirrors are often used to transport the X-ray beam to sample position while preserving the beam brilliance and coherence. In reality, reflecting mirror surface can not be perfect, and deviations of the mirror surface from the ideal ellipse will introduce aberrations in the reflected wave which affect the focal spot and images measured downstream [10-12].

Mirror surfaces are commonly specified with a peak to valley (PV) value and root-mean-square (RMS) value. In recent years, PSD is considered to be a more powerful way to characterize the super-smooth mirror surface [4, 13]. PSD is a mathematical quantity that defines the spectral content of the mirror surface. And a PSD analysis transforms a surface map into a spatial frequency representation. The PSD of the mirror surface can be expressed as

$$\begin{aligned} PSD(f) &= \frac{|H(f,L)|^2}{L} \\ &= \frac{1}{L}\left|\int_{-L/2}^{L/2} h(x)\exp(-j2\pi fx)dx\right|^2 \end{aligned} \quad (1)$$

The parameter L is the width of the mirror surface height map. We assume that PSD(f)=bf-α, which mean that the finish of the highly polished mirror surface is frequently fractal like [14, 15]. The parameter α is the slope of the power spectrum on a log-log scale and it's determined by the fractal dimension D satisfying 1<D<2, D=(5-α)/2. To obtain a spatial mirror surface height map with a fixed fractal distribution in PSD functions, we add the random phase distribution ϕ(f) to the fixed distribution of the Fourier spectrum amplitudes. By applying the standard procedure of the inverse Fourier transform, we can calculate spatial distribution of mirror surface h'(x) from the fixed PSD(f) by the following equation [15, 16]

$$h'(x) = \frac{M}{L}F^{-1}\left\{\sqrt{L\cdot PSD(f)}\cdot\exp\left[j\phi(f)\right]\right\}. \quad (2)$$

Where F^{-1} represents an inverse-FFT algorithm, ϕ(f) is an randomly valued phase map satisfying –π< ϕ(f)<π, and M is the pixel number of h'(x) along x axes. In this simulation, we set the mirror surface length L=10cm and the pixel size is 1μm, so the pixel number M is 10^5.

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WAVE-OPTICAL EVALUATION OF SYNTHESIZED MIRROR SURFACES

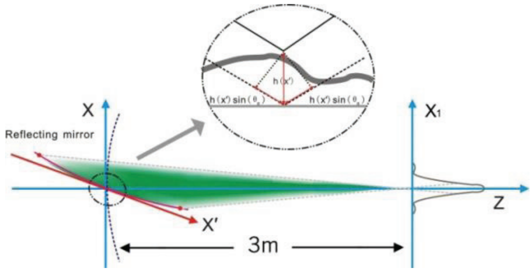

Figure 1: Simulation geometry for grazing incidence reflection by an elliptical mirror.

Deviations of the reflecting mirror surface from the ideal ellipse can cause wave-fields aberration. To evaluate the synthesized mirror surfaces with different PSD curves, the computer simulations of X-ray beams focusing by elliptical mirrors are presented based on Fresnel diffraction integral. And the simulation geometry is shown in Fig. 1. The phase error of the focusing X-ray induced at the reflection on an imperfect mirror surface is given by $\psi_{error}(x)=2kh(x')\sin(\theta_0)$ [17], as shown in Fig. 1, where θ_0 and k is the grazing-incident angle and the wave-number of the X-ray, h(x') is the height deviations of the reflecting mirror surface from the ideal ellipse on position x'.

Here, we substitute the synchrotron source by an ideal point source, the hard x-ray wavelength is 0.1nm, the grazing-incident angle is 2mrad, and the focusing mirror is positioned 30m downstream from the source.

According to Eq. (2), the synthesized mirror surface height profile is determined by α, b and ϕ. Firstly, we fix the value of the parameters α and b, change the phase distribution ϕ. As is shown in Fig. 2(a), when the phase distribution ϕ is changed, the synthesized mirror surface height profiles are totally different with each other. But the difference between the X-ray beams' intensity profiles in the focal plane is quite modest when focused by the synthesized mirrors as shown in Fig. 2(b). In this paper, FWHM, FR and strehl ratio(SR) are used to assess the synthesized mirrors' surface error. Here, FR is the ratio between the FWHM based on the synthesized mirror and the ideal mirror. SR is the ratio between the maximum intensity of the synthesized mirror and the ideal mirror. Table 1 summarizes the numerical results for different synthesized mirrors and the small differences in FWHM, FR and SR represents that the phase distributions have little effect on the focused X-ray beam's intensity distributions.

Table 1: The SR, FWHM and FR of the X-ray Beam's Intensity Profiles in Fig. 2(b)

	SR	FWHM, μm	FR
Ideal mirror surface	1	1.3280	1
Mirror surface 1	0.7733	1.3160	0.9910
Mirror surface 2	0.7953	1.3600	1.0241
Mirror surface 3	0.8053	1.3860	1.0437

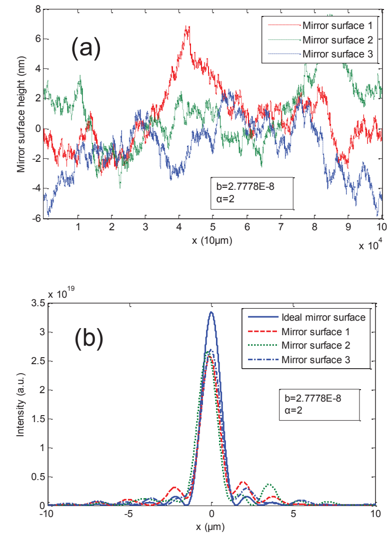

Figure 2: (a) The synthesized mirror surface height profiles with the same PSD but different phase distribution; (b) The X-ray beam's intensity profile in the optimal focal plane for the synthesized mirrors and the ideal mirror.

Secondly, the value of the parameter α and the phase distribution ϕ is fixed, and the value of the parameter b gradually increases and the PSD curves correspondingly shift upward with the constant slopes on a log-log scale as shown in Fig. 3. Fig. 4 shows the X-ray beam's intensity patterns for the synthesized mirrors with the PSD curves in Fig. 3(b), and we can find that when the parameter b increase, the focused X-ray beam's intensity is distorted much more severely by the synthesized mirror surface error. To evaluate the focusing performance of the synthesized mirrors, we also analyze the variation of the coordinate values of optimal focal point in the intensity patterns and the SR, FWHM and FR of the intensity profile in the optimal focal plane as shown in Fig. 5.

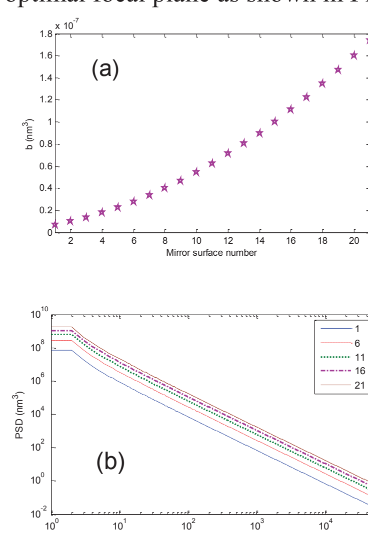

Figure 3: (a) The increasing value of the parameter b; (b) Corresponding PSD curves.

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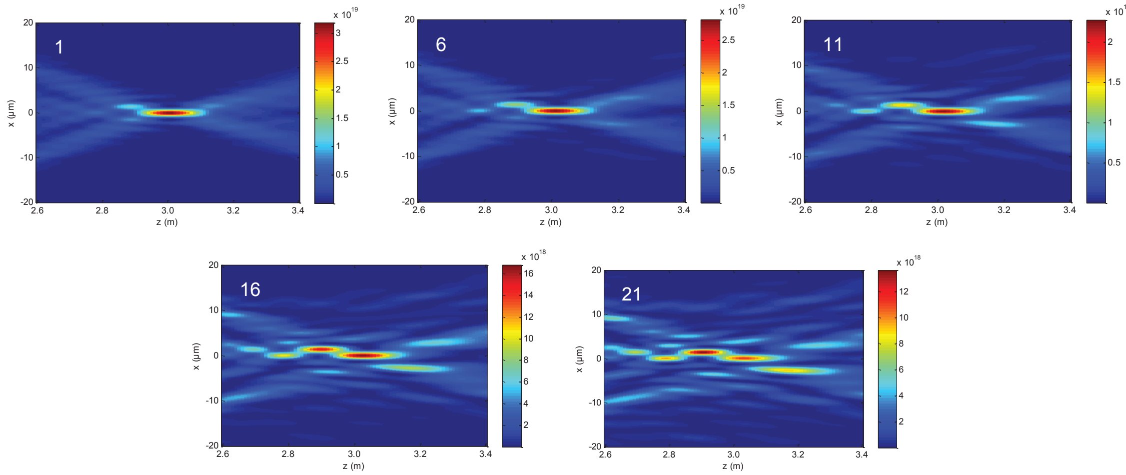

Figure 4: The X-ray beam's intensity patterns for the synthesized mirrors with PSD curves in Fig. 3(b).

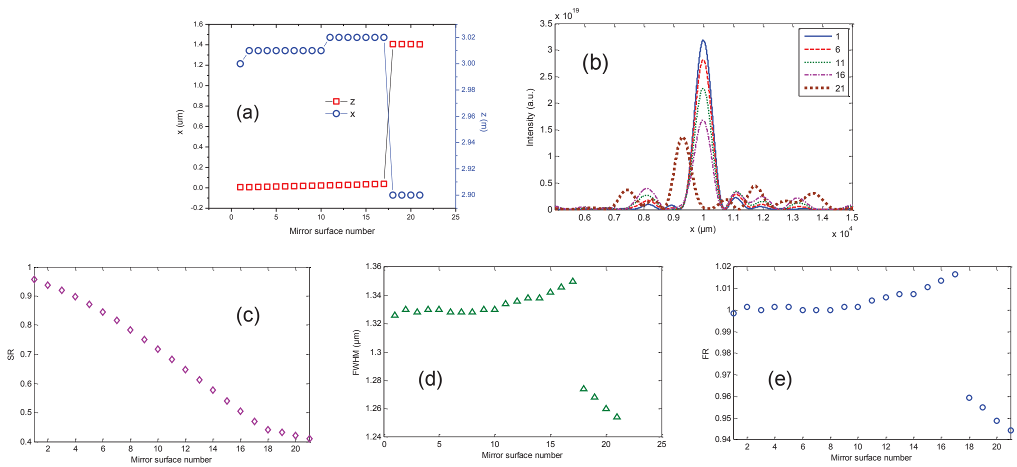

Figure 5: (a) The coordinate values of optimal focal point in the X-ray beam's intensity pattern for the synthesized mirrors with PSD curves in Fig. 3; (b) The X-ray beam's intensity profile in the optimal focal plane for the synthesized mirrors; The SR (c), FWHM (d) and FR (e) of the intensity profile in Fig. 5(b).

CONCLUSIONS

In this paper, we analyze the effects of reflecting mirror surface's imperfection on the reflected coherent X-ray wave-fronts. Firstly, various fractal mirror surface height maps are synthesized according to power spectral density functions and fractal theory. Secondly, the propagation properties of hard X-ray beams through imperfect reflecting mirror surface is calculated, and here we assume the synchrotron source is a totally coherent point source. Our numerical simulations demonstrate that the phase distributions for synthesizing mirrors surfaces have little effect on the focused X-ray beam's intensity distributions. And when the parameter b in the power spectral density function increase, the focused X-ray beam's intensity distort severely by the synthesized mirror surface error. In our future works, we would like to predict the value ranges of the parameters in the power spectral density function with which the synthesized imperfect mirror surfaces' focusing performance can meet the requirements and the physical conditions of coherent experiments.

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