BEAM AND SPIN DYNAMICS IN AN ELECTRIC PROTON EDM RING

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- Question: Why is there more matter than anti-matter in the present universe?
- Electric dipole moment (**EDM**) measurements of protons (and other charged baryons) may help to answer.
- This will be possible in a ring in which protons are stored for at least fifteen minutes, with polarization "frozen" and with little depolarization.
- This paper discusses beam and spin dynamics in an **all-electric** lattice with these characteristics.

1 Theoretical Motivation for EDM Measurement

- Distant past: theoretical speculations concerning matter/anti-matter imbalance by Sakarov and (null) neutron EDM measurement by Ramsay
- 1981: Ellis et al.: "we deduce an order-of-magnitude lower bound on the **neutron electric dipole moment**: $d_n \approx 3 \times 10^{-28}$ e cm."
- 1992: Weinberg, conference summary: " …electric dipole measurements seem to me to offer one of the most exciting possibilities for progress in particle physics."
- 2007: Nuclear Science Advisory Committee (NSAC) emphasized the importance of electric dipole moment (EDM) measurements for addressing the matter/anti-matter imbalance.

- Recent insight: stored for many minutes in a storage ring, **proton EDM's** should be be more accurately measurable than **neutron EDM's**.
- 2011: Arkani-Hamed, at a Conference on Fundamental Physics, identified **EDM's** (along with quark and lepton flavor physics) as the **areas of greatest promise.**
- This paper discusses **experimental** practicalities of **measuring the proton EDM**.

2 Symmetry Violations for a Particle with both MDM and EDM

- Magnetic dipole (**MD**) is a **pseudo-vector** aligned with some axis.
- Electric dipole (**ED**) is a **vector** aligned with the **same axis**.
- The ED and MD of the same particle **cannot be said to be "parallel" without violating parity P**—viewed in a mirror ED and MD would be anti-parallel.
- **It would also violate time reversal T—run backwards, MD would reverse, ED would not.**
- Certainly a **proton has an MDM**. For it **to also have an EDM** implies **violation of T symmetry**.
- With CPT symmetry assumed, this **also implies the violation of CP symmetry**. which is a **necessary condition for the cosmic evolution to unbalanced fractions of matter and anti-matter**. (Sakarov)

3 Estimate of EDM-Induced Spin Precession

- Optimistically **an EDM of** 10^{-29} e**-cm can be persuasively distinguished from zero in one year of running**. In S/I units

$$d_{\rm nom} = 10^{-29} \cdot (1.602 \times 10^{-19}) \cdot (0.01) = (1.602 \times 10^{-50})\,[\mathrm{SI}]. \quad (1)$$

- Ratio to nuclear magneton:

$$\frac{d_{\rm nom}}{\mu_B} = \frac{(1.602 \times 10^{-50})}{(5.05 \times 10^{-27})} = 3.127 \times 10^{-24}, \ \text{S.I. units} \qquad (2)$$

- This ratio is not dimensionless. The missing factor is E/B. For our configurations, in SI units, this ratio is typically $10^7/0.1 \approx 10^8\,$m/s.
- After multiplying by this factor, the relative-effectiveness ratio of EDM to MDM has a numerical value of about 3×10^{-16}.

- **Relative precession task:** Distinguish EDM-induced vertical precession from spurious, wrong-plane, MDM-induced, precession.
- **Absolute precession task:** For a pure Dirac particle in a magnetic field the precession is 2π per turn. At one microsecond per turn, this is of order 10^7 radians/s.
- Applying the E/B factor mentioned above, we therefore plan to measure a "nominal" EDM-induced precession of order 10^{-9} r/s.
- This is about $0.1\,$mr/day.

4 All-Electric Storage Ring Design

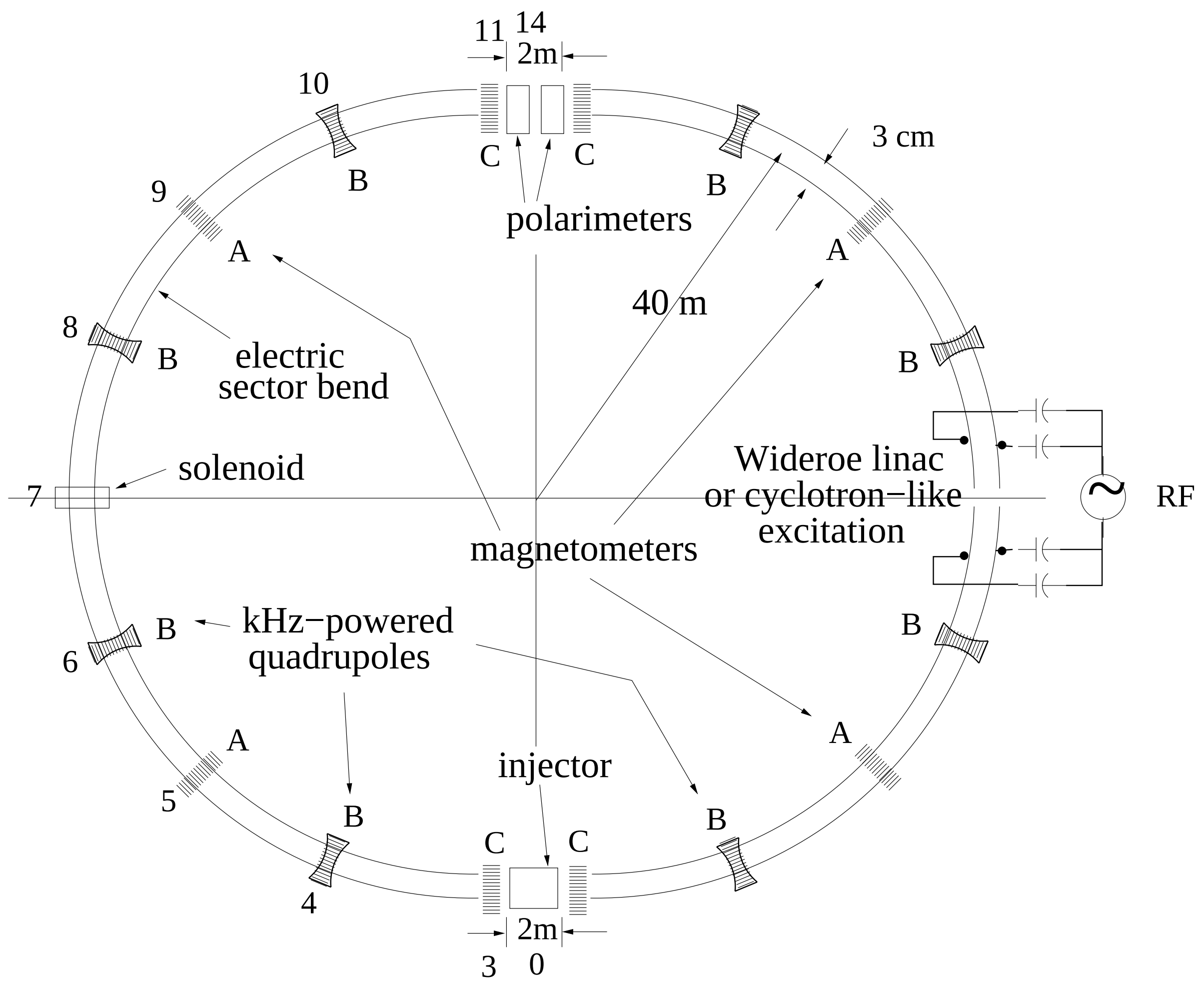

Figure 1: A (very) weak focusing all-electric lattice (BNL proposal) for measuring the electric dipole moment of the proton. There are counter-circulating proton beams.

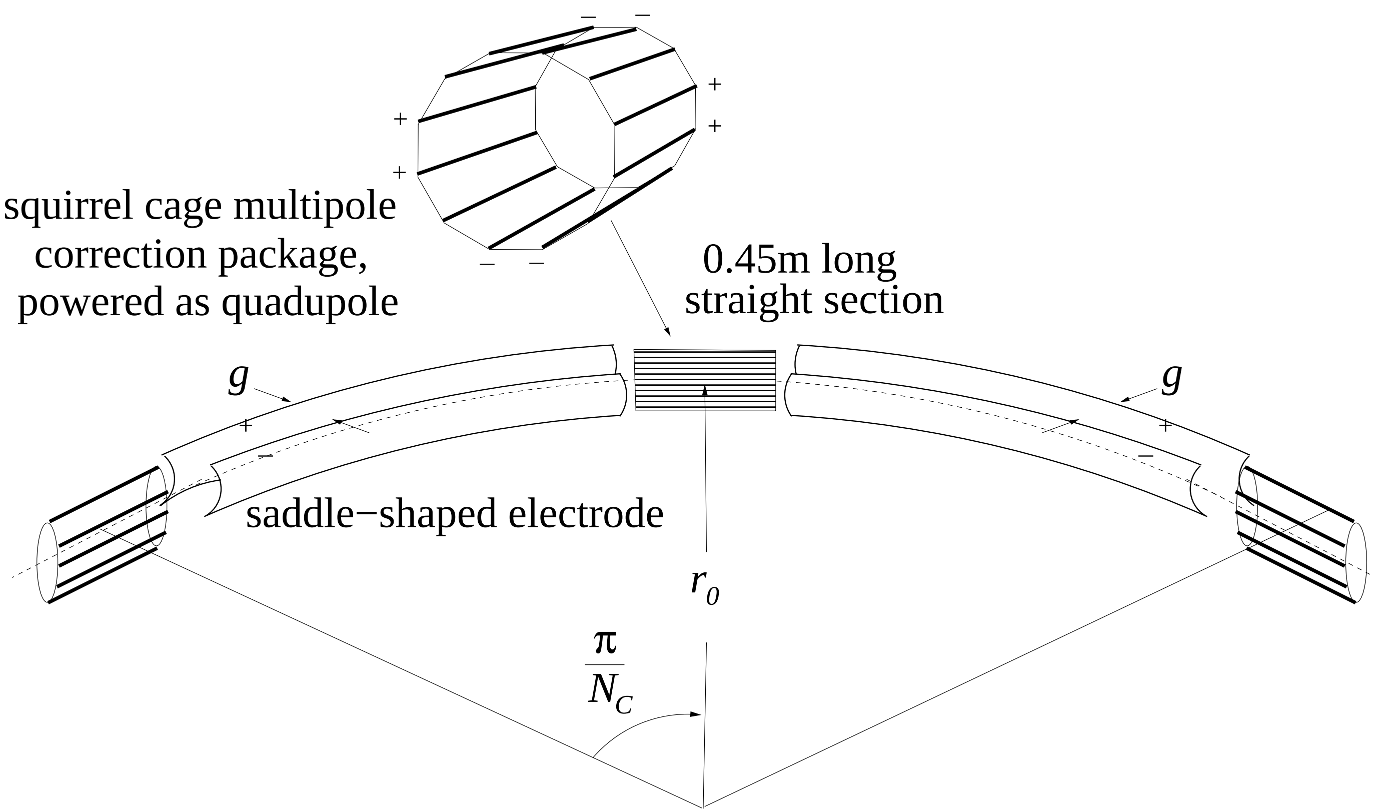

Figure 2: One cell of all-electric proton EDM lattice, with electrodes shaped for design field index m. $g = 3\,\mathrm{cm}$, $r_0 = 40\,\mathrm{m}$

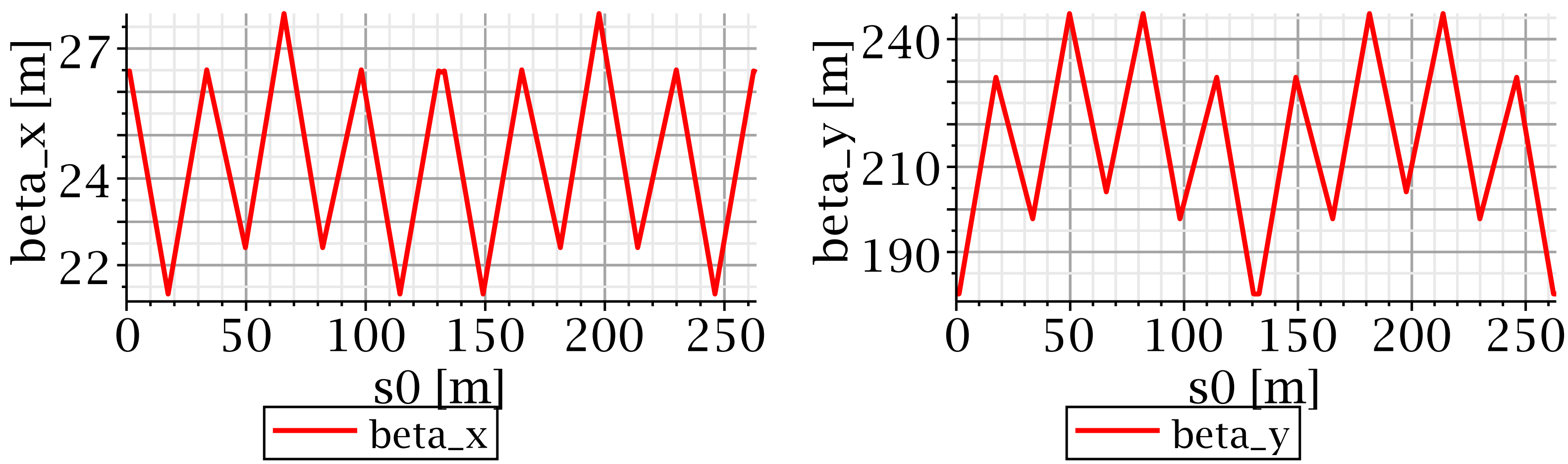

Figure 3: Plots of β functions of proton EDM lattice. β_y is *necessarily* very large, since Q_y has to be small.

5 Essential Experimental Features

- Ideally the focusing is weak, with vertical tune $Q_y < 0.1$, with no straight sections, and with field index (defined below) $m \approx 0$. These have to be relaxed.
- About 10^{10} protons need to be stored in multiple, low emittance, low energy spread, highly polarized bunches, for at least a quarter of an hour and preferably a day.
- The statistical precision with which the polarization can be measured is limited by the number of protons.
- To maximize spin coherence time (SCT) **beam emittances will be minimized**: with pre-run electron cooling and (probably) **stochastic cooling during the run.**

- The radial electric field E_r has to be maximized. The design calls for E_r=10 MV/m.
- There is a "magic" velocity β=0.6 for which the spin can be "frozen", parallel or anti-parallel to the proton velocity. Any EDM-induced spin precession will then accumulate monotonically.

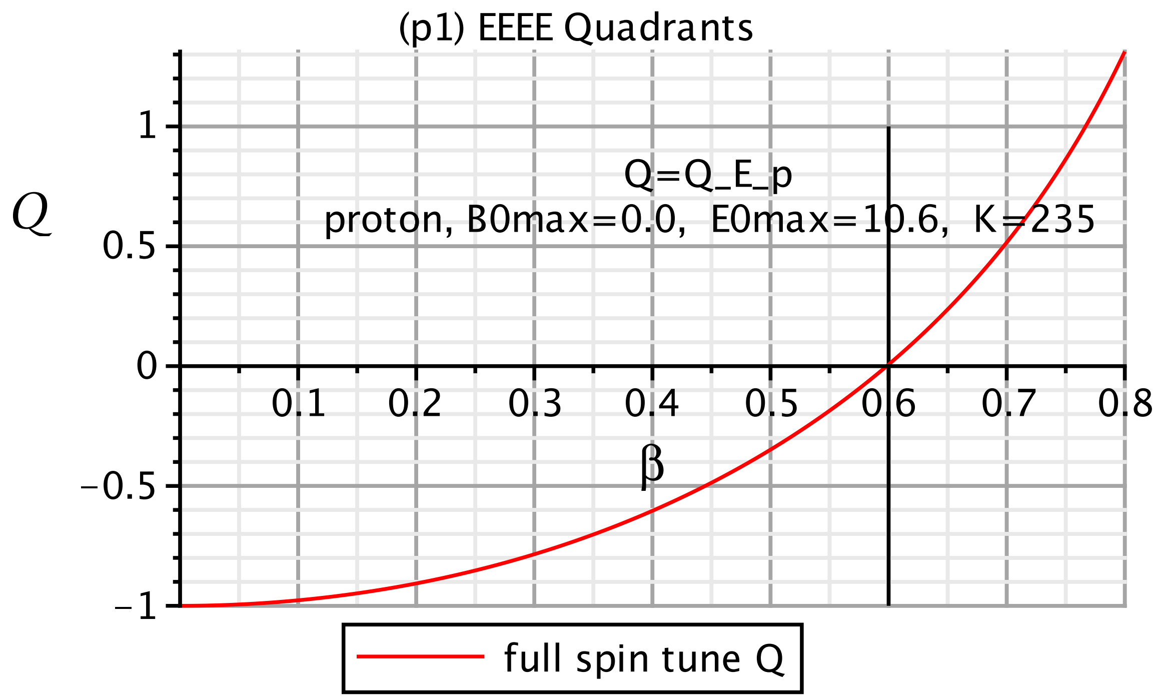

Figure 4: Velocity dependence of (magnetic) spin tune Q for protons in all-electric lattice. The spin is "globally frozen" for β=0.6. The spin is locally, but not globally, frozen for β=0.76, where Q is an integer other than zero.

- The **EDM signal is a vertical tipping** of the polarization. The MDM-induced tipping is horizontal.
- To reduce spurious MDM-induced precession several orders of magnitude **suppression of magnetic field** using both passive magnetic shielding and active B_r correction coil will be provided.
- To reduce polarimeter bias the **polarizations of circulating bunches alternate, forward and back.**
- **Counter-circulating beams** will be stored, and the **difference of their vertical polarizations** measured. EDM-induced precessions will sum in this difference, while MDM-induced precessions cancel.

- Also any average radial magnetic field will produce vertical separation between the counter-circulating beams. **Feedback from vertical beam position (BPM) monitors to B_r compensation coils to null the vertical beam separation** will force the average value of B_r to zero.
- Squid magnetometers will be used for this nulling.
- Q_y will oscillate about its nominal value, **parametrically pumping the beam separation** at a frequency in the kilohertz range, for which noise is minimal.
- **Synchronous, lock-in detection** will permit high vertical beam separation accuracy

6 "Exact" ETEAPOT Tracking

- We need to account for the **potential energy variation accompanying transverse oscillations**, an effect which is absent in magnetic elements.
- The **conventional Courant-Snyder formalism is not valid in general**, but the standard formalism can be consistently maintained **outside electric elements,** and then interpolated.
- Within the Unified Accelerator Library (UAL) modeling framework we have developed a code, **ETEAPOT**, patterned after TEAPOT, capable of simulating an all-electric ring.

- An electric field with "field index" m power law dependence on radius r for y=0 is

$$\mathbf{E}(r,0) = -E_0 \frac{r_0^{1+m}}{r^{1+m}} \hat{\mathbf{r}}, \tag{3}$$

- The electric potential $V(r)$, adjusted to vanish on the central orbit at $r = r_0$, is

$$V(r) = -\frac{E_0 r_0}{m} \left(\frac{r_0^m}{r^m} - 1 \right). \tag{4}$$

- The "cleanest" case has m=1, which is the well-known **Kepler** or **hydrogen atom** case, but we must use **relativistic mechanics**. The Lorentz force equation is

$$\frac{d\mathbf{p}}{dt} = -k \frac{\hat{\mathbf{r}}}{r^2}. \tag{5}$$

- **(Only for this case) the exact 2D relativistic solution can be expressed in closed form for arbitrary amplitude.**
- Muñoz/Pavic formulation is especially appropriate for our relativistic accelerator application. Their “generalized”-Hamilton vector

$$\mathbf{h} = h_r\,\hat{\mathbf{r}} + h_\theta\,\hat{\boldsymbol{\theta}} \tag{6}$$

  is especially convenient for describing relativistic accelerator orbits.
- $\mathbf{h}$ is conserved if and only if the orbit is circular, as it is on the central orbit of our proton EDM lattice.
- For long term tracking, we use this exact (and hence symplectic) m=1 evolution.

- But the actual storage ring field index value will have $m \neq 1$.
- To compensate for this incorrect focusing effect we "**kick correct**" to the actual m value, a process which also preserves symplecticity.
- In contrast to "approximate tracking in an exact lattice" this is "exact tracking in an approximate lattice";
- this kick compensation becomes **fully accurate in the limit of fine slicing.**

- Total energy is **conserved** (not counting RF);

$$\mathcal{E} = eV(\mathbf{r}) + \gamma(\mathbf{r}) m_p c^2, \tag{7}$$

- Angular momentum is **conserved** (not counting RF)

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}. \tag{8}$$

- But, unlike magnetic bending, β and γ are **not conserved**.

- The equations of motion for the generalized Hamilton vector components are

$$\boxed{\begin{aligned}\frac{dh_r}{d\theta} &= h_\theta,\\ \frac{dh_\theta}{d\theta} &= -\kappa^2\, h_r,\end{aligned}} \tag{9}$$

where **tune-like parameter** κ is given by

$$\kappa^2 = 1 - \left(\frac{k}{Lc}\right)^2. \tag{10}$$

- **These are the equations that justify the approach.**
- Their general solution, **valid at all amplitudes**:

$$\begin{aligned} h_\theta &= \mathcal{C}\ \cos\kappa(\theta-\theta_0)\\ h_r &= \frac{\mathcal{C}}{\kappa}\ \sin\kappa(\theta-\theta_0). \end{aligned} \tag{11}$$

- θ_0 and $\mathcal{C}$, are fixed by initial conditions.

- For transverse orbit description, we replace Courant-Snyder 4D phase space description (x, x', y, y') by the wobbling plane description illustrated in Fig. 5.

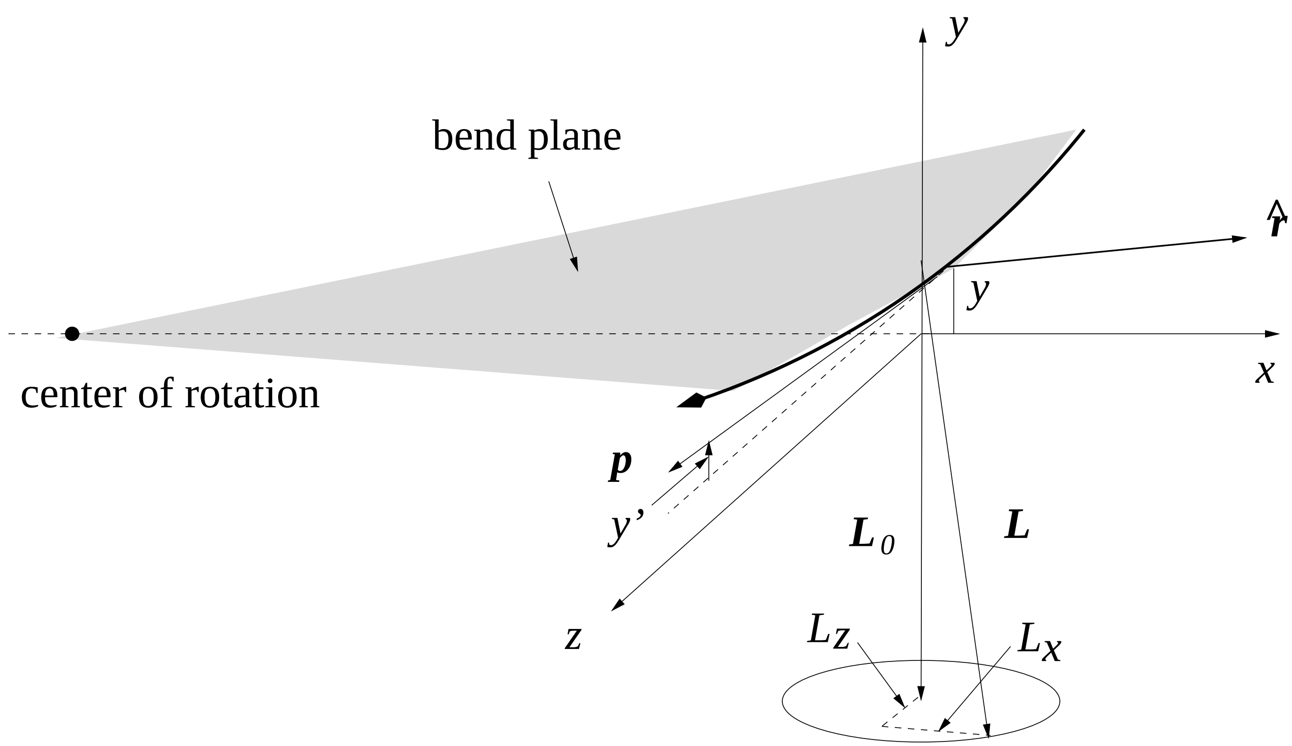

Figure 5: Wobbling-plane orbit coordinate definitions.

- Evolve the angular momentum pair (L_x, L_z), normal to the bend plane, rather than the pair (y, y').

- The ETEAPOT evolution formalism has been checked against a conventional linearized transfer matrix formalism

Table 1: Comparisons between (linearized) transfer matrix formalism and the arbitrary-amplitude UAL/ETEAPOT formalism. Bend slice thicknesses are about 0.5 m.)

file name	unit	linearized	ETEAPOT
cells/arc		20	
bend radius	m	40.0	
half drift length	m	1.0	
half bend per cell	r	0.078539816	
half bend length	m	3.141592	
circumference	m	331.327	
quadrupole inverse focal length	1/m	-0.00005960	
field index		1.0e-10	
horizontal beta	m	**36.1018**	**36.0962**
vertical beta	m	**263.6201**	**263.0767**
horizontal tune		**1.4578**	**1.4579**
vertical tune		**0.2000**	**0.2005**

7 Spin Coherence Time Estimate

- For simplicity restrict the discussion to a uniform, weak focusing lattice with no drift regions.
- We have to consider both coasting beams and bunched beams.
- Figure 6 shows the spin vector **s** in relation to the design orbit.

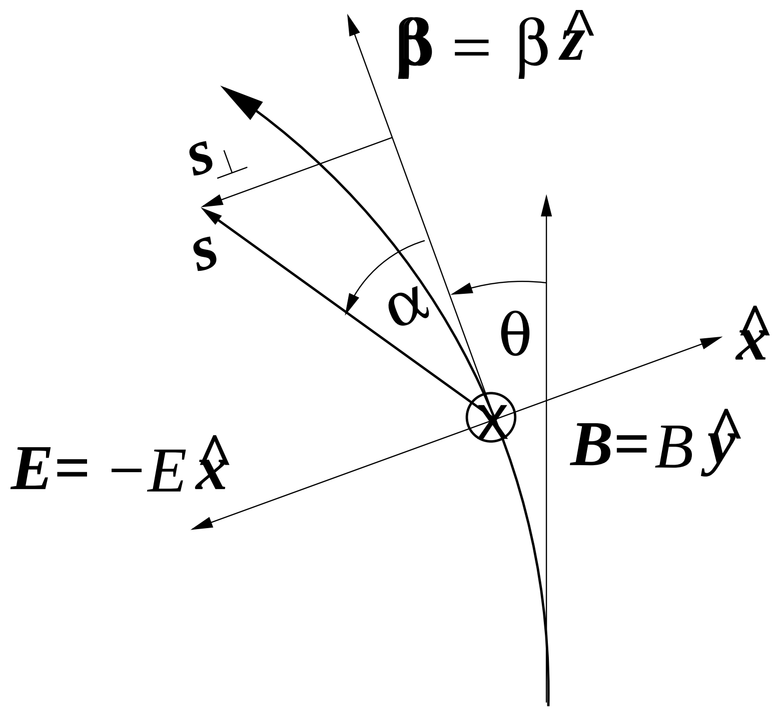

Figure 6: Spin vector **s** has precessed through angle α away from its nominal direction along the proton's velocity.

- The BMT equation gives the evolution of the spin precession angle α, relative to the proton direction;

$$\frac{d\alpha}{dt} = \frac{eE(x)}{m_p c}\left(\frac{g\beta(x)}{2} - \frac{1}{\beta(x)}\right), \tag{12}$$

  where $g = 5.5857$ is the proton g-factor. **This precession vanishes at the magic velocity.**

- "**Windshield wiper effect**": the spin oscillates back and forth as the particle executes synchrotron oscillations.

- The angular velocity depends on angular momentum L and radial coordinate r;

$$\frac{d\theta}{dt} = \frac{L}{\gamma m_p r^2}. \tag{13}$$

- Combining equations,

$$\frac{d\alpha}{d\theta} = \frac{eE(x)(r_0+x)^2}{Lc\beta(x)}\left(\left(\frac{g}{2}-1\right)\gamma(x) - \frac{g/2}{\gamma(x)}\right), \tag{14}$$

- The first factor is almost constant. The second factor vanishes on the design orbit.
- To find the evolution of α over long times we need to average this equation;

$$\left\langle\frac{d\alpha}{d\theta}\right\rangle = \left\langle\frac{eE(x)(r_0+x)^2}{Lc\beta(x)}\right\rangle\left(\left(\frac{g}{2}-1\right)\langle\gamma\rangle - \frac{g}{2}\left\langle\frac{1}{\gamma}\right\rangle\right). \tag{15}$$

- For bunched beam operation γ deviates sinusoidally during **synchrotron oscillations**, and only odd harmonics appear even at large amplitudes. So the average $\langle\gamma\rangle$ is equal to the magic value γ_0.
- But the $1/\gamma$ factor in Eq. (15) **does not average to $1/\gamma_0$.**

- The (relativistic) **virial theorem** can be used to perform the averaging.
- "**Virial**" G is defined to be the dot product of radius vector $\mathbf{r}$ and momentum $\mathbf{p}$;

$$G = \mathbf{r} \cdot \mathbf{p}. \tag{16}$$

- The time rate of change of $\mathbf{G}$ is given by

$$\left.\frac{dG}{dt}\right|_{\text{bend}} = m_p c^2 \gamma - m_p c^2 \frac{1}{\gamma} - eE_0 r_0 \frac{r_0^m}{r^m}. \tag{17}$$

- Averaging over time, presuming bounded motion, which requires $\langle dG/dt \rangle$ to vanish, one obtains

$$\left\langle \frac{1}{\gamma} \right\rangle = \langle \gamma \rangle - \frac{E_0 r_0}{m_p c^2/e} \left\langle \frac{r_0^m}{r^m} \right\rangle. \tag{18}$$

- Plug this back into Eq. (15);

- Applying this result and $r = r_0 + x$;

$$\left\langle \frac{d\alpha}{d\theta} \right\rangle \approx -\frac{E_0 r_0 \gamma_0}{(p_0 c/e)\beta_0} \left(\left\langle \frac{\gamma}{\gamma_0} - 1 \right\rangle + m \left\langle \frac{x}{r_0} \right\rangle - \frac{m^2 - m}{2} \left\langle \frac{x^2}{r_0^2} \right\rangle \right).$$

- Higher order terms in the expansion parameter $x/r_0 \approx 2 \times 10^{-4}$ have been dropped.

- If the parenthesized factor has a value of order 1, the spin coherence time would be measured in milliseconds, far too short for the EDM measurement to be feasible. Fortunately, for bunched beams, after averaging, the parenthesized factor is very small.

- Polarimeter/RF feedback forces the first term (in parenthesis) to cancel exactly.

- The factor $\langle x \rangle$ also tends to cancel over many betatron cycles. But changes of electric potential cause this cancellation to be imperfect.

- For **cylindrical electrodes** $m = 0$ and there is no decoherence. This suggests that the optimal electrode shape will be at least approximately cylindrical.

- To avoid a resonance m **cannot be exactly zero**.
- Taking $m = 1$ as a possible field index value, it can be seen that the parenthesized factor reduces to $\langle x/r_0 \rangle$. Already of order 10^{-4}, this factor further averages to zero for linear betatron and synchrotron oscillations.
- Linearizing chromatic dependencies, along with small beam emittances, is expected to produce acceptable SCT.
- Spin decoherence occurring on entrance to and exit from bend elements has been neglected. In fact **this is the dominant source of decoherence.**
- Chromatic linearization is expected to also reduce this decoherence mechanism to an acceptable level.