NUMERICAL SIMULATION STUDY OF THE MONTAGUE RESONANCE AT THE CERN PROTON SYNCHROTRON*

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Abstract

The Montague resonance provides a coupling between the vertical and the horizontal dynamics of beam and can cause particle losses due to unequal aperture sizes of the accelerator. In this paper, we present a new numerical simulation study of a previous Montague resonance crossing experiment at the CERN PS including detailed three-dimensional space-charge effects and machine nonlinearity. The simulation reproduces the experimental data well and suggests that the longitudinal synchrotron motion played an important role in enhancing transverse resonance coupling.

INTRODUCTION

The Proton-Synchrotron (PS) as one of the oldest accelerators at CERN has served more than 50 years and will continue to support the LHC for another 25 years [1-2]. Space-charge effects have been identified as the most serious intensity limitation in this machine [3], since nonlinear space-charge effects in high intensity hadron beams can cause significant emittance growth and particle losses. Due to the unequal aperture size of the machine, the coupling between the horizontal and the vertical dynamics driven by the space-charge effects (so-called Montague resonance [4]) can lead to losses of particles inside the ring. During the period between 2002 and 2004, a number of experiments were done at PS to study the Montague resonance. These data were also used to benchmark a number of space-charge simulation codes [5-7]. In those previous studies, constant focusing approximation, a linearized version of the PS lattice, a fully nonlinear lattice of the PS with coasting beam were used to study those Montague resonance measurements. None of those simulations included the fully nonlinear lattice and synchrotron oscillation with three-dimensional space charge effects and tried to account for the dynamic resonance crossing measurements in the experiment. In this paper, we used a new updated version of the IMPACT code to include the fully PS nonlinear lattice with both RF focusing and 3D space-charge effects to study both the static resonance crossing and the dynamic resonance crossing from the experimental measurements at PS.

COMUPTATIONAL MODEL

The IMPACT code was used in this study. The IMPACT is a parallel particle-in-cell code that was originally developed to model the dynamics of multiple charged particle beams in linear accelerators [8]. The code includes the effects of externally applied fields from

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ISBN 978-3-95450-115-1
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magnets and accelerating cavities as well as the effect of self-consistent space charge fields. It has been applied to a number of studies such as beam dynamics studies in the SNS linac, JPARC linac, RIA driver linac, CERN superconducting linac, and LEDA halo experiment. For the purpose of studying space-charge effects in a synchrotron ring, the IMPACT code was extended to include thin lens kicks for nonlinear elements and RF cavities, multi-turn simulation/injection, RF ramping, etc. The 3D space-charge solver used in this study is based on a combination of the spectral method and the finite difference method proposed in reference [9].

SIMULATION RESULTS

The physical parameters of the PS used in this study are summarized as follows: the RF frequency is 3.5 MHz, the RF voltage is 27.0 kV; the proton beam normalized rms emittance is 7.5 mm-mrad (30 mm-mrad for 2 sigma emitance) in the horizontal plane and 2.5 mm-mrad in the vertical plane (10 mm-mrad for 2 sigma emitance); the longitudinal rms bunch length is 45 ns with rms dp/p of 0.0017; the vertical tune is kept at 6.21 while the horizontal tune is scanned between 6.15 and 6.245 using two groups of quadrupoles; the synchrotron oscillation period is about 1.5 ms; the half aperture sizes are 7cm by 3.5 cm; the number of protons per bunch is 1.0×10^{12} with 1.4GeV kinetic energy. Using those parameters and the nonlinear PS lattice at a working point (6.245,6.21), we generated an initial matched distribution with zero current using a normal form of the one turn transfer map from the ML/I code [10]. Due to the non-zero dispersion, the initial matched distribution shows strong correlation between the horizontal position and the relative energy deviation (pt) as shown in Figure 1. Such a correlation might have significant impact for the final emittance exchange that will be discussed in the following section.



Figure 1: Initial matched phase space distribution.

05 Beam Dynamics and Electromagnetic Fields D03 HIgh Intensity in Circular Machines In this study, we adopted a set of numerical parameters with 100,000 macroparticles and 65x65x129 mesh grid points for space-charge calculation. The number of space-charge calculations per turn is 60 distributed roughly uniformly along the ring. As a test of our choice of numerical parameters, we did numerical convergence test. The transverse emittance evolution from two different numerical parameters are shown in Figure 2.



Figure 2: Transverse rms emittance evolution with two sets of numerical parameters. One set is 100,000 macroparticles, 65x65x129 grid points, and the other set is 200,00 macroparticles with 65x65x257 grid points.

It is seen that by doubling both the number of macroparticles and the numerical grid points, the transverse emittances do not show noticeable differences. This suggests that the numerical parameters used in this study might be sufficient. We also doubled the number of space-charge kicks per turn along the ring and did not see noticeable differences.

Using the above parameters and the nonlinear PS lattice, we carried out a number of simulation runs. Each run corresponds to a tune working point in the static resonance crossing measurements. Each simulation was run to 13,000 turns to account for the 30 ms real measurement time after injection. The simulation results together with the original measurements are shown in Figure 3.



Figure 3: Emittance exchange during the static crossing of the horizontal tune. The top one is from the measurement and the bottom one is from the simulation.

Here the measurements were done using a flying wire 30 ms after each injection for each working point. Several subsequent measurements under identical conditions were averaged in order to improve statistics of the data. It is seen that the simulation gives a reasonably good reproduction of the experimental measurements with a good match of the resonance stop band width. To check the effects of the longitudinal synchrotron motion, we also did simulation with a longitudinal frozen distribution. The transverse emittance evolutions for a working point (6.197,6.21) with/without the longitudinal oscillation are shown in Figure 4. It is seen that there is significantly



Figure 4: Transverse rms emittance evolution with/ without longitudinal synchrotron motion at working point (6.197,6.21).

more emittance exchange between the horizontal and the vertical plane with the longitudinal synchrotron oscillation. One feature to note is that there is a quick emittance exchange within the first 50 turns in both cases. After this period, the longitudinally frozen beam does not show further emittance exchange while the beam with synchrotron oscillation continues to show emittance exchange. This is because the synchrotron oscillation helps bring particles initially outside of the resonance stop band into the resonance stop band through the longitudinal motion. In contrast to previous studies, the $\overline{\Box}$ simulation results in the Figure 3 and 4 for the static crossing show no sign of "overshoot" of emittance exchange inside the stop band. This absence of full equipartitioning in the final emittance inside the resonance stop band could be related to the initial position-energy deviation correlation of the beam initially matched to the PS lattice as shown in Figure 1.



Figure 5: Transverse emittance evolution with different tunes and an initial upright distribution (without x-pt correlation).

By using an initial upright beam dispersion mismatched to the PS lattice (i.e. no position-energy deviation).

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correlation), we ran the simulation for a few working points inside the stop band. The simulation results of transverse emittance evolution are given in Figure 5. It is seen that the final transverse emittances do approach to full equipartitioning for a working point inside the resonance stop band. This is also consistent with the observation that the dispersion mismatch at PS injection cannot be disentangled from the Montague resonance effect due to the fast nature of the resonance and the assumption of fixed tunes.

Dynamic Montague resonance crossing was also measured at PS by slowly crossing the stop band in a time span of 100 ms. Numerical simulation was carried out including the three-dimensional space-charge effects and the full nonlinear lattice for a real time span of 100 ms. The transverse emittances as a function of the horizontal tune from both the experimental measurements and the simulation are shown in Figure 6. Here, the crossing started with an initial horizontal tune of 6.15 and ramped up linearly to 6.245. It is seen that the simulation results agree well with the experimental measurements. The dynamic resonance crossing shows quite different features compared with the static crossing. There is no clear stop band across the horizontal tune. The transverse emittances approach equipartitioning as the horizontal tune moves through the static resonance stop band.



Figure 6: Emittance exchange during the dynamic crossing of the horizontal tune. The top one is from the measurement and the bottom one is from the simulation.

We also carried out a dynamic crossing simulation starting with 6.245 and ramping down to 6.15. The results are shown in Figure 7 together with some experimental measurements. Again, the simulation agrees with the measurement reasonably well. The main discrepancy in the amplitude of the emittance is due to fact that the measurements were done in 2004 with a smaller starting emittance.



Figure 7: Emittance exchange during the dynamic crossing of the horizontal tune in opposite direction. The top one is from the measurement and the bottom one is from the simulation.

In summary, this study shows that numerical simulations including fully nonlinear lattice, synchrotron oscillation and 3D space-charge effects give pretty good reproduction of the measurements of the Montague resonance at PS. Longitudinal synchrotron oscillation helps the emittance exchange inside the stop band.

ACKNOWLEDGMENTS

This work used the computing resources from the National Energy Scientific Computing Center.

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