PARAMETRIC STUDY OF THE CLIC DAMPING RINGS DELAY RING FOR REACHING ISOCHRONICITY CONDITIONS

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Abstract

A delay ring in the CLIC damping rings complex is necessary for recombining the two trains to one with the nominal bunch separation of 0.5ns. The preservation of the longitudinal bunch distribution demands an optics design, which eliminates momentum compaction factor up to high order, allowing the delay ring to function under isochronous conditions. Taking into account thin lens approximation, a qualitative estimation of parameters of the cell that will be used in the delay ring, is given, so as to obtain isochronicity conditions. Considerations on the possibility of tuning the cell under those requirements are finally presented.

INTRODUCTION

For the consecutive recombination of both particle species' trains, a single delay ring is considered downstream of the CLIC damping rings, in order to form the bunch pattern with 0.5 ns time structure, required by the collider. In order to preserve the longitudinal beam characteristics achieved in the damping rings, for the best collider performance, and especially respect the tight synchronous phase tolerance, the delay ring must function under isochronous conditions. The conditions for the required elimination of the momentum compaction factor up to second order is studied in the case of a Theoretical Minimum Emittance (TME) cell, for which a detailed optics analysis can be found in [1]. In addition, sextupoles have been added, to the analytical solution of the TME cell, to compensate second order momentum compaction factor. In this paper, the parametrisation of the components of the momentum compaction factor with respect to the TME cell's characteristics is being studied, using thin lens approximation. The optics functions at the centre of the bending magnet for achieving isochronicity conditions are estimated as a function of the different drift space lengths, the dipole characteristics and the sextupole strengths, bounded by the required optics stability conditions.

MOMENTUM COMPACTION FACTOR EQUATIONS FOR THE TME CELL

The main cell of the CLIC delay loop is chosen to be the Theoretical Minimum Emittance (TME) cell, mainly due to its compactness. A schematic layout of the cell is shown in Fig. 1. It consists of one dipole D of length l_{dip}, two quadrupoles Q1 and Q2 of focal lengths f_1 and f_2, two sextupoles, SX1 and SX2, with sextupole strengths λ_1 and λ_2 and three drift spaces s_1, s_2, s_3. An analytical

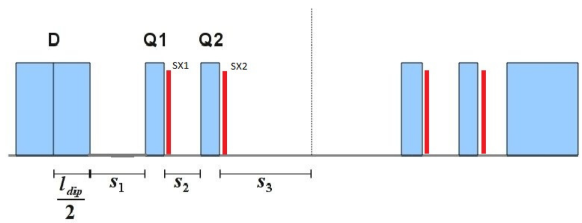

Figure 1: Schematic layout of the TME cell.

parametrisation of the TME cell is described in [1], where the quadrupole focal lengths, in thin lens approximation, are given by:

$$\begin{aligned} f_1 &= \frac{s_2(4s_1l_{dip}+l_{dip}^2+8\eta_{c,x}\rho)}{(4s_1l_{dip}+l_{dip}^2+8\eta_{c,x}\rho)+4s_2l_{dip}-8\eta_s\rho} \\ f_2 &= -\frac{8s_2\eta_s\rho}{(4s_1l_{dip}+l_{dip}^2+8\eta_{c,x}\rho)-8\eta_s\rho} \end{aligned} \quad , \qquad (1)$$

where, η_s is the dispersion at the center of the cell, which has a complicated dependence on s_1, s_2, s_3 and also on the dipole's characteristics, the horizontal beta function at the center of the dipole, $\beta_{x,c}$, and the dispersion at the center of the dipole, $\eta_{x,c}$. The dipole's field was chosen to be B=0.5 T, due to the fact that small magnetic field induces large bending radius and smaller absolute values of zero order momentum compaction factor, as showed in [3]. The momentum compaction factor α_c, with respect to the relative momentum deviation $\delta = \delta p/p$, can be written in the form:

$$\alpha_c = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 + O(\delta^3) \qquad (2)$$

where α_0, α_1, α_2 are the zero, first and second order momentum compaction factors respectively and can be expressed as [2]:

$$\alpha_0 = \frac{1}{L}\int_0^L \left(\frac{\eta_0}{\rho}\right) \mathrm{d}s \qquad (3)$$

$$\alpha_1 = \frac{1}{L}\int_0^L \left(\frac{\eta_1}{2\rho} + \frac{\eta_0'^2}{2}\right) \mathrm{d}s \qquad (4)$$

$$\alpha_2 = \frac{1}{L}\int_0^L \left(\frac{\eta_2}{\rho} - \frac{\eta_0\,\eta_0'^2}{2\rho} + \eta_0'\,\eta_1'\right) \mathrm{d}s \qquad (5)$$

L is the length of the element at which each term is calculated, η_0, η_1, η_2 are zero, first and second order dispersion functions respectively and ρ the bending radius of the dipole.

The dispersion functions of Eqs. (3), (4), (5), were obtained for each element of the cell and for every order, by solving the differential equations describing their evolution [2]. The propagation starts from the centre of the dipole, where the derivative of the dispersion is zero [1] until the point of symmetry, at the middle of the cell, i.e. at the end of the drift space s_3, where the derivative is also zero.

ISOCHRONOUS CONDITIONS FOR THE TME CELL

Zero Order Momentum Compaction Factor

The zero order momentum compaction factor for the TME cell is [3]:

$$\alpha_0 = \frac{24\eta_{c,x} - (\eta_{c,x} + \rho)\theta^2}{24\rho} \tag{6}$$

Setting $\alpha_0 = 0$ and solving Eq. (6) with respect to $\eta_{c,x}$, the zero order isochronicity condition is:

$$\eta_{c,x} = \frac{\theta^2 \rho}{-24 + \theta^2} \tag{7}$$

The dispersion at the centre of the dipole, satisfying the isochronicity condition, depends only on the dipole characteristics and it is indeed negative for almost all reasonable bending angles θ. Substituting Eq.(7) expression in Eq.(1) the analytical expressions for f_1, f_2 that eliminate α_0 are obtained. Also by using Eq.(7) in the analytical expression of horizontal and vertical phase advances will lead to expressions that induce zero order isochronicity. The horizontal phase advance of the cell, eliminating α_0, is expressed by:

$$cos(\mu_x) = \frac{-16\beta_{c,x}^2(-24+\theta^2)^2 + \theta^2(-32+\theta^2)^2\rho^2}{16\beta_{c,x}^2(-24+\theta^2)^2 + \theta^2(-32+\theta^2)^2\rho^2} \tag{8}$$

Fig. 2, presents the horizontal phase advance, μ_x, with respect to the beta function at the center of the dipole, $\beta_{c,x}$, and for different bending angles, that satisfy the zero order isochronicity condition.

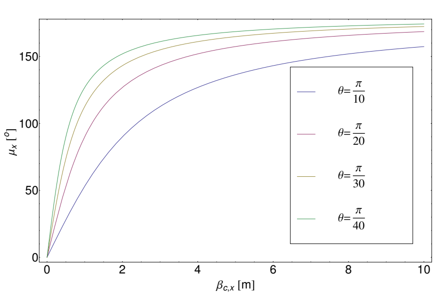

Figure 2: Horizontal phase advance μ_x versus horizontal beta function at the dipole centre, for which the zero order momentum compaction factor is eliminated.

Even though, by definition, the solutions in the horizontal plane provide always stability, this is not the case for the vertical plane. An analytical expression for the vertical phase advance μ_y exists [1], parametrised with respect to θ, ρ, f_1, f_2 and drift spaces s_1, s_2 and s_3. Fig. 3, shows the

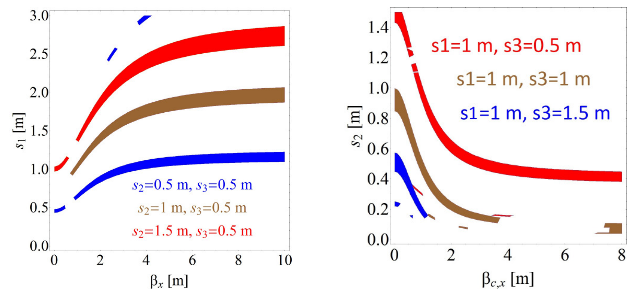

Figure 3: Drift lengths s_1 (left) and s_2 (right) with respect to $\beta_{c,x}$, achieving zero order isochronicity and satisfying the stability condition in the vertical plane, for $\theta = \frac{\pi}{20}$. Each color corresponds to different fixed values of s_2, s_3 (left) and s_1, s_3 (right).

regions of solutions that satisfy the stability criterion in the vertical plane, $|\cos\mu_y| \leq 1$, for the drift lengths s_1 (left) and s_2 (right), with respect to the horizontal beta function at the dipole centre, for which $\alpha_0 = 0$. The dependence of s_3 to $\beta_{c,x}$ is similar with the one of s_2. Each color corresponds to different fixed values of s_2, s_3 (left) and s_1, s_3 (right). The trend shows, that for a fixed s_3, higher values of s_2 impose higher s_1 values for stability. The dependence on $\beta_{c,x}$ is in general weak. For a fixed value of s_1, smaller values of s_2 and larger of s_3 satisfy the stability criterion.

First Order Momentum Compaction Factor

The first order momentum compaction factor, α_1, can be eliminated, only if the first term of Eq.(4), the dipole's contribution, becomes negative and equal to the second term, which is called the wiggling part. Using thin lens approximation, the dipole's contribution to α_1 can be expressed as:

$$\alpha_{1,dip} = \frac{9(-640+\theta^4)}{10(-24+\theta^2)^2} \tag{9}$$

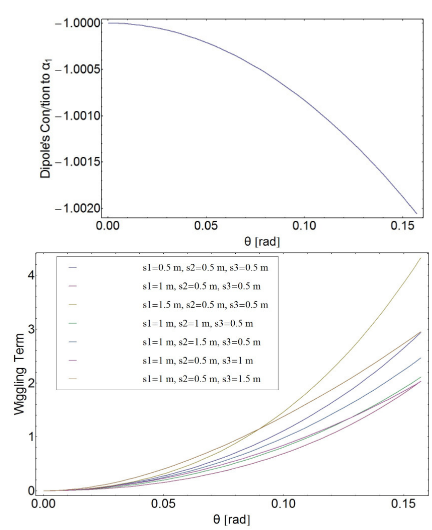

Figure 4: Top:Dipole's contribution to α_1 with respect to θ. Bottom: The wiggling term contribution to α_1, with respect to the dipole's bending angle.

Fig. 4 (top), shows the dipole contribution with respect

ISBN 978-3-95450-115-1

to θ while, Fig. 4 (bottom), the contribution of the wiggling part with respect to θ, for different combinations of s_1, s_2 and s_3. The dipole contribution is always negative and weakly depends on θ. Elimination of α_1, lies on finding appropriate combinations of the drift lengths and the bending angle, for which the two terms have equal and opposite in sign contributions.

Calculating the integral of Eq. (4) along the TME cell, using thin lens approximation, leads to an expression which has a complicated dependence on θ, ρ, f_1, f_2. Combining the latest with Eq. (1), the first order momentum compaction factor with respect to the drift lengths, the dipole characteristics and $\beta_{c,x}$ is obtained.

Fig. 5, shows α_1 with respect to θ, for different combinations of s_1, s_2 and s_3. For specific triplets of s_1, s_2 and s_3, there are always values of θ that eliminate α_1.

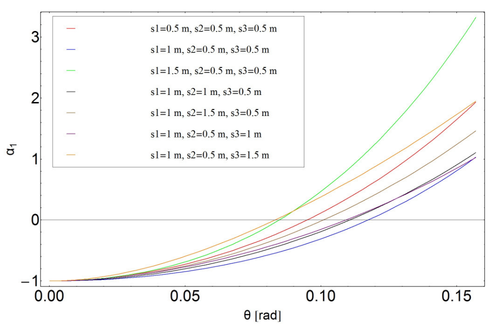

Figure 5: α_1 with respect to θ for different s_1, s_2, s_3 values.

Second Order Momentum Compaction Factor

The second order momentum compaction factor, α_2, can be expressed through Eq. (5). The first and second term of the equation depends only on the dipole characteristics, while the third term, $\eta_0'\eta_1'$, depends on the sextupole strengths. Calculation of the integral of Eq. (5) along the TME cell, leads to an expression for α_2, which depends on the cell characteristics ρ, θ, s_1, s_2, s_3 and the sextupole strengths λ_1, λ_2.

Fig. 6, shows values of λ_1 and λ_2 for a choice of $\theta=\frac{\pi}{12}$ and of drift spaces s_1=0.8 m, s_2=0.6 m and s_3=0.8 m. It is important to highlight than not every pair of λ_1 and λ_2 give solution to the elimination of α_2. For reasons of optics stability, only sextupole strengths with opposite sign are of interest. Fig. 7, shows the second order momentum compaction factor, α_2, with respect to θ for choices of λ_1= 100 m^{-3} and λ_2= -75 m^{-3}, according to the results of Fig. 6. Each color curve corresponds to a different triplet of s_1, s_2, s_3. Not all triplet combinations can eliminate α_2, while the dipole's bending angle which eliminates α_2, depends strongly on the choice of the drift spaces lengths. It is important to notice, that, only certain combinations of λ_1 and λ_2 eliminate α_2. After tuning the cell parameters in order to eliminate α_0 and α_1, the sextupoles' strengths can be uniquely defined in order to eliminate α_2.

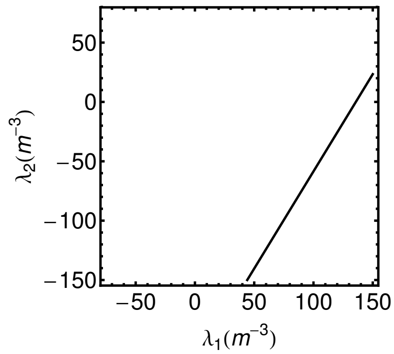

Figure 6: Values of λ_1 and λ_2 that eliminate α_2.

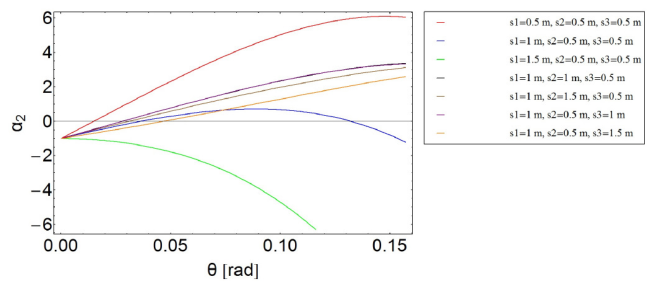

Figure 7: Plot of α_2 with respect to θ, for different s_1, s_2, s_3.

CONCLUSIONS

In order to preserve the longitudinal beam characteristics, for the best collider performance, the CLIC DR delay ring must function under isochronous conditions. The elimination of the momentum compaction factor up to high order is possible with the appropriate adjustment of the TME cell parameters. Zero order isochronicity condition, implies negative dispersion at the center of the dipole. For specific s_1, s_2, s_3, there are always solutions of the dipole's bending angle, satisfying the stability criterion in the horizontal and vertical planes, that eliminate the first and second order momentum compaction factors. However, the existence of those solutions does not guarantee the simultaneous elimination of the latest. A further numerical study is in progress for comparing the analytical results with simulations for high order isochronous cells.

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