# 3-DIMENSIONAL MODELING OF ELECTRON CLOUDS IN NON-UNIFORM MAGNETIC FIELDS

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# Abstract

Electron clouds have the potential to pose serious limitations on accelerator performance in both hadron and lepton beams. Experiments using rf diagnostics are being performed to measure electron cloud densities at a number of accelerator facilities. However, it is difficult to calibrate plasma density with signal strength in these experiments, and modeling involves a number of technical and numerical challenges. Typically 2-Dimensional electrostatic methods have been used to model cloud buildup under beam crossing conditions. However, since traveling-wave rf experiments typically occur over many meters of beam pipe where magnetic fields are changing, one needs to develop 3-Dimensional electromagnetic models in order to accurately simulate rf diagnostics. We have developed accurate models of electron cloud-induced phase shifts in rf in a system with spatially varying magnetic field configurations using the plasma simulation code Vorpal. We present here results for measuring phase shifts in the CESR wiggler with realistic, spatially non-uniform magnetic field configurations.

# TRAVELING WAVE RF DIAGNOSTICS MEASURE ELECTRON CLOUDS IN ACCELERATORS

Modulation of rf due to electron cloud buildup has been measured in a number of experiments e.g. [1], and have been modeled by a number of different researchers over the last few years. Electron plasmas in an accelerator beam pipe have dielectric properties that change the dispersion relation of injected rf signals, and phase shifts are induced by the plasma that are nominally linear with plasma density. This method has the potential to provide estimates of cloud density, as well as measuring the efficacy of mitigation techniques in a non-destructive manner. However, it is difficult to determine the actual plasma density from phase shift measurements for a number of reasons. First, reflections and attenuation of rf power can cause frequency shifts, amplitude modulation, as well as making the true path length from the rf source to the measurement point unknown. Second, magnetic fields can cause resonances which can enhance the phase shifts. Finally, non-uniform density clouds can have an effect on the phase shifts that are not averaged out over the measurement cycle of many revolutions periods.

We have performed traveling wave rf diagnostic simulations using a plasma dielectric model [2] implemented in Vorpal [3], and have extracted plasma-induced phase shifts in different magenetic field configurations. In addition, we have implemented *port boundary conditions* that replace current sources previously used to generate rf signals in simulations, as well as unphysical layers used to absorb rf power at the simulation boundaries in the direction of the wave vector (PMLs or MALs). These improvements allow us to more efficiently produce results because this particular problem depends on the dielectric properties of the electron plasma. However, kinetic particle modeling of electron clouds are needed to model cloud build up, density evolution, and wall interactions.

# SIMULATION RESULTS

We have modeled plasma-induced phase shifts for travelling wave rf in order to understand the effects due to different magnetic field configurations and cloud density profiles. We have performed these simulations as well using the actual beam pipe geometry in the CESR wiggler, with similar results. Hence results shown here are for the simpler case of a rectangular cross section waveguide. The waveguide has transverse dimensions of 9 cm by 5 cm (xand y-direction respectively), and is 80 cm long in the zdirection. Figure 1 shows the simulation domain and the wiggler magnetic field lines, colored by field strength. Note from the figure that the strongest fields are almost completely aligned with the y-direction, which is the shortest transverse direction. There is almost no magnetic field in the other transverse direction (x-direction), and there is moderate magnetic field strengths in the longitudinal direction (z-direction) in the transition regions between the poles of the wiggler magnets.

For the nominal case, with a uniform density cloud that fills the entire beam pipe, linear theory predicts that the phase shift per unit length is

$$\Delta \phi/\ell = \frac{\pi}{c} \frac{f_p^2}{\sqrt{f^2 - f_c^2}} \tag{1}$$

where  $f_p$  is the plasma frequency, f is the rf frequency, and  $f_c$  is the cutoff frequency for the particular mode. The linear theory predicts that the phase shifts are linear in electron density, which we observe in the simulations here in all cases. In all of the simulations we drive the rf at 10%

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Figure 1: Wiggler field lines colored by field strength. The field is strongest in the y-direction

above the cutoff fequency. We consider two modes here: (1) rf polarized in the y-direction with a cutoff frequncy of 1.66 GHz, and (2) rf polarized in the x-direction with a cutoff frequency of 3 GHz. The first case is the lowest order mode for this waveguide, where the rf is polarized along the shortest transverse direction. The nominal case is referenced in the following figures as a solid blue line.

#### Upper Hybrid Resonances

Phase shifts can be suppressed or enhanced by a uniform dipole magnetic field depending on the strength and orientation of the field with the rf polarization [4, 5, 6]. If the magnetic field is oriented parallel to the rf polarization, then an ordinary wave is excited, which does not show a resonance. However, if the magnetic field is normal to the rf polarization, then an extraordinary wave is present, which has a (upper hybrid) resonant frequency given by

$$\omega_{uh}^2 = \omega_p^2 + \Omega_e^2 \tag{2}$$

where  $\omega_p$  is the plasma frequency and  $\Omega_e = eB/m_ec$  is the electron cyclotron frequency. Since the plasma frequency is much smaller than the electron cyclotron frequency, it can be ignored here. Thus in our simulations, the magnetic field strength that produces the cyclotron resonance is approximately 0.06 T in the first case referenced above (rf polarized in the y-direction) and 0.11 T in the second case (rf polarized in the x-direction).

Our simulations show that phase shifts are very close to the nominal theoretical values in the case of no magnetic field as well as for ordinary waves for both x- and ypolarized rf, as is expected, which is shown in figure 2. We also simulated ordinary waves with a magnetic field strenth of 2.0 T with the same result (not shown in the figure below), which is close to the maximum wiggler field strength considered below.

Extraordinary waves exhibit different behavior depending on the magnetic field strengths. For strong fields of 2.0 T, which are much larger than the resonant field strength in both cases, the phase shifts are reduced by a factor of about  $10^4$ , as is shown in figure 2. Phase shifts are significantly reduced even in the case where the magnetic field strength is only three tims the resonant field strength, which can be see in the top plot of figure 2. We interpret this result as a *freezing* effect, where the rf is unable to push electrons off of strong magnetic field lines, and so the electrons suppress the phase shifts. However, if the magnetic field is close to the resonant field strength, blue diamonds in the bottom plot of figure 2, an increase in the phase shifts is observed. Simulations exactly on the upper hybrid resonance produce phase shifts approximately 6.5 times greater than the case with no magnetic field at a density of  $9e^{12} e^{-}/m^{3}$  (not shown).



Figure 2: Phase shifts as a function of effective plasma density for uniform dipole magnetic fields. Top plot shows phase shifts for electric field polarized in the y-direction (lowest TE mode), and the bottom plot is for polarization in the x-direction (TE mode).

# Wiggler Magnetic Field Configuration

Spatially non-uniform magnetic fields can affect plasmainduced phase shifts in two primary ways. First, there may be locations in the field where the magnetic field strength and orientation relative to the rf polarization is in resonance with the cyclotron frequency. This would cause a local increase in the phase shift over some distance, which in turn would increase the total phase shift measured over then entire cloud. Second, non-uniform magnetic fields will affect how the clouds evolve over long time scales, creating non-uniform density clouds which could then change the phase shifts. We model wiggler fields here using a bmad model fit for a wiggler at the CesrTA accelerator. The strongest fields are in the y-direction in the wiggler, with a maximum strength of almost 2 Tesla, and a mean value

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of  $\langle B_y \rangle = 0.17$  T. There is nearly no magnetic field in the other transverse direction (x-direction), but there is a significant field parallel to the wave vector (z-direction) in the transition regions between magnet poles with a maximum strength of 0.5 Tesla.

Our simulations show a significant reduction in the phase shifts below the nominal theoretical value when measured in realistic wiggler fields, as is shown in figure 3. At a density of  $9e^{12} e^{-}/m^{3}$  the phase shift is about 20% lower than the nominal case with no magnetic field. Note that if the wiggler field were replaced by a pure dipole field with a strength equal to the average  $B_y$ , then there would be an ordinary wave, which would not change the phase shifts.



Figure 3: Phase shifts as a function of effective plasma density for modeled wiggler magnetic fields. Simulated phase shifts are about 20% lower than the nominal no magnetic field case at a density of  $9e^{12} e^{-}/m^{3}$ .

The interpretation of this result is that since the strongest magnetic fields in the wiggler are aligned with the rf polarization that the electron motion is mostly parallel to the rf polarization. Fairly strong fields in the z-direction suppress the phase shifts somewhat from the nominal value.

#### Non-Uniform Density

As mentioned above, another expected effect from nonuniform fields is to evolve the cloud into a spatially nonuniform configuration which could then affect the phase shifts. To test this, we performed simulations with the same wiggler field, but with a different dielectric distribution. We decreased the width (in the x-direction) of the dielectric by a factor of 4, while simultaneously increasing the equivalent density by the same factor. So the average density, measured over the whole waveguide, remains the same.

As is shown in figure 4 this increased the simulated phase shifts above the nominal value for no magnetic field. Note however that the larger phase shifts are much lower than one would expect if only considering the peak density. In that case, the nominal values would be four times the nominal values denoted by the blue line in figure 4 which is for overall density averaged over the entire waveguide. This result indicates that the effect of non-uniform

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cloud densities on measured phase shifts is potentially very large, further complicating the determination of densities via phase shift measurements. It seems probable that the effects of non-uniform densities is larger than effects due to cyclotron resonances in non-uniform magnetic fields.



Figure 4: Phase shifts as a function of effective plasma density for modeled wiggler magnetic fields with two different plasma dielectric configurations. Simulated phase shifts are higher when the peak density is larger, although the average density is the same in both cases.

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