ROBUST CONTROL OF A TWO-INPUT TWO-OUTPUT (TITO) MULTI-STATE CAVITY RF SYSTEM WITH MISMATCHED UNCERTAINTY*

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Abstract

A superconducting RF cavity is well modeled as a linear two input two output (TITO) system in the Inphase/Quadrature (IQ) coordinates and is both controllable and observable. Perturbation of cavity resonant frequency, due to beam loading or Lorentz force detuning, can be modelled as a matched uncertainty. The cavity field of a TITO cavity system with a matched uncertainty is controlled by output feedback or state feedback, whose performance function, given by the error bound, is made arbitrarily small. Because of the building cost of the RF system, the single RF source (single klystron)-multicavity structure is widely used. This structure is described as a parallel composite of multiple TITO cavity systems, resulting in a two input and two output multi-state system, where each output is a vector sum of multiple individual cavity I/Q components. The resulting control problem is not a simple extension of the single TITO system. Though the composite vector-sum system is controllable, the matched uncertainty of the TITO cavity system caused by cavity detuning becomes mismatched. Consequently, although the vector-sum error bound of the system output may be made arbitrary small, the resulting controller is blind to cancelling cavity field errors, which occur if two cavities are offset in opposite directions.

INTRODUCTION

A RF cavity is well modelled as a linear two input two output (TITO) system in the Inphase/Quadrature (IQ) coordinates and is both controllable and observable [1], [2]. When there is a time invariant or time varying frequency detuning caused by Lorentz force, microphonics, or beam loading, the nominal system state matrix is changed. The feedback control system design compensating for this perturbation plays a crucial role in accelerator field control. Because of the building cost of RF systems, the single RF source (single klystron)-multicavity structure is widely used [3], [4], [5], [6]. This structure is described as a parallel composite of multiple TITO cavity systems, resulting in a two-input two-output multi-state system, where each output is a vector sum of multiple individual cavity I/Q components. The resulting control problem is not a simple extension of the single TITO two-state system. Though the composite system is controllable, the matched uncertainty of the TITO cavity system caused by cavity detuning becomes mismatched. Consequently, although the vector-sum error bound of the system output may be made arbitrary small, the resulting controller is blind to cancelling cavity field errors, which occur if two cavities are offset in opposite directions. In the following, a parallel composite system of two TITO cavities is studied. A robust state feedback control law is proposed and the closed loop system performance is investigated. Simulation results demonstrate a small vector-sum error while individual cavity performance is poor.

CONTROL OF TITO MULTI-STATE CAVITY SYSTEM WITH MISMATCHED UNCERTAINTY

Consider a parallel composite system which is composed of two TITO cavities connected in parallel. The first thing that must be checked is the controllability of the system. In many cases, the state uncontrollability does not imply the input-output controllable in a practical sense, since the uncontrollable states are not of concern from the perspective of the system behavior or are not practically important. Specifically, when the system is stable, the control purpose is mostly focused on the output not the system state, and so the state controllability is not a real concern [7]. The vector-sum control which is the widely used output feedback control for the cavity field control can be understood in this context. In the parallel composite system of two cavities, if the eigenvalues of the two cavities are folded (overlapped), then, the composite system is state uncontrollable [8], [9]. In this case, though the composite system is stable, because there are state uncontrollable modes, the characteristics of the individual systems, such as rise and settling time, cannot be decided properly by the output feedback controller of the vector-sum control.

When there exists uncertainty in the composite system, whether the states is controllable or not, the tolerance of the system states to the uncertainty must be considered. Even if the output behavior of the system is within the scope of desired performance, state behavior must be contained to within the design scope.

The parallel composite system of two RF cavities is expressed as the two-input, 4-state, two-output system:

$$\dot{x} = \left(A + \Delta A(\Delta\omega)\right)x + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where

$$A = diag(A_1, A_2),$$

$$A_1 = \begin{bmatrix} -\frac{1}{\tau_{L1}} & 0 \\ 0 & -\frac{1}{\tau_{L1}} \end{bmatrix}, A_2 = \begin{bmatrix} -\frac{1}{\tau_{L2}} & 0 \\ 0 & -\frac{1}{\tau_{L2}} \end{bmatrix} \tag{3}$$

$$\Delta A(\Delta\omega) = diag(\Delta A_1(\Delta\omega_1), \Delta A_2(\Delta\omega_2)),$$

$$\Delta A_1(\Delta\omega_1)=\begin{bmatrix}0 & -\Delta\omega_1\\ \Delta\omega_1 & 0\end{bmatrix},\ \Delta A_2(\Delta\omega_2)=\begin{bmatrix}0 & -\Delta\omega_2\\ \Delta\omega_2 & 0\end{bmatrix},$$

$$B=\begin{bmatrix}B_1\\ B_2\end{bmatrix},\quad B_1=\begin{bmatrix}b_{11} & 0\\ 0 & b_{12}\end{bmatrix},\quad B_2=\begin{bmatrix}b_{21} & 0\\ 0 & b_{22}\end{bmatrix}$$

$$C=\frac{1}{2}\begin{bmatrix}C_1 & C_2\end{bmatrix},\ C_1=C_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$x=\begin{bmatrix}x_1 & x_2 & x_3 & x_4\end{bmatrix}^T,\qquad u=\begin{bmatrix}u_1 & u_2\end{bmatrix}^T,$$

where $\Delta\omega=\begin{bmatrix}\Delta\omega_1 & \Delta\omega_2\end{bmatrix}^T\in\Omega_{\Delta\omega}\subset R^2$

For the linear TITO uncertain system of a cavity, the uncertainty is matched. When an uncertain TITO cavity system is connected in parallel with another uncertain TITO cavity system, the uncertainty becomes mismatched [10], [11]. That is, the uncertainty is not in the range of the input matrix B. We decomposed the uncertainty into the sum of matched and unmatched components by projecting the uncertainty to the range of the matrix B, for $\Delta\omega\in\Omega_{\Delta\omega}$

$$\Delta A(\Delta\omega)=BB^+\Delta A(\Delta\omega)+\left(I-BB^+\right)\Delta A(\Delta\omega),\quad (4)$$

Here, B^+ is the pseudo inverse of the matrix B. Note that $BB^+\cdot\left(I-BB^+\right)=0$. For the decomposition of the uncertainty as Eq. (4), it is assumed that there exist nonnegative symmetric matrices, $F_{\Delta\omega}$ and $H_{\Delta\omega}$

$$\left(B^+\Delta A(\Delta\omega)\right)^T\left(B^+\Delta A(\Delta\omega)\right)\le F_{\Delta\omega}\quad (5)$$

$$\left(\left(I-BB^+\right)\Delta A(\Delta\omega)\right)^T\left(I-BB^+\right)\Delta A(\Delta\omega)\le H_{\Delta\omega}\quad (6)$$

Consider the nominal state equation

$$\dot{x}=Ax+Bu\quad (7)$$

$$y=Cx\quad (8)$$

Let the desired output be $y_r\in\Omega_r$. In steady state, the corresponding state and input, x_r, u_r, can be obtained by solving

$$\begin{bmatrix}A & B\\ C & 0\end{bmatrix}\begin{bmatrix}x_r\\ u_r\end{bmatrix}=\begin{bmatrix}0\\ y_r\end{bmatrix},\quad (9)$$

Since $y_r\in\Omega_r$ is bounded, there exist bounded sets Ω_{xr} and Ω_{ur}, such that we can define

$$\rho_1=\max\left\{\left\|B^+\Delta A(\Delta\omega)x_r\right\|,\Delta\omega\in\Omega_{\Delta\omega},x_r\in\Omega_{xr}\right\}\quad (10)$$

$$\rho_2=\max\left\{\left\|\left(I-BB^+\right)\Delta A(\Delta\omega)x_r\right\|,\Delta\omega\in\Omega_{\Delta\omega},x_r\in\Omega_{xr}\right\}$$

Theorem For positive constants ξ_1, ξ_2, and a positive definite matrix Q, if the Riccati equation

$$A^TP+PA-\xi_1PBB^TP+\xi_2P^2+\frac{1}{\varepsilon_1}F_{\Delta\omega}+\frac{1}{\varepsilon_2}H_{\Delta\omega}+Q=0$$

has positive definite solution P, then the control law

$$u=-\gamma B^TPe-B^+Ax_r\quad (11)$$

$$e=x-x_r\quad (12)$$

where γ is a positive constant, makes the state of system Eq. (1), (2) uniformly bounded.

Outline of Proof) Definite a Lyapunov function

$$V(e,t)=e^T(t)Pe(t)\quad (13)$$

Then the time derivative of $V(e,t)$ is

$$\dot{V}(e,t)\le e^T\Big(A^TP+PA-(2\gamma-\varepsilon_1-\varepsilon_3)PBB^TP+(\varepsilon_2+\varepsilon_4)P^2$$
$$+\frac{1}{\varepsilon_1}F_{\Delta\omega}+\frac{1}{\varepsilon_2}H_{\Delta\omega}\Big)e+\frac{1}{\varepsilon_3}x_r^T\left(B^+\Delta A\right)^T\left(B^+\Delta A\right)x_r$$
$$+\frac{1}{\varepsilon_3}x_r^T\left(B^+\Delta A\right)^T\left(B^+\Delta A\right)x_r+\frac{1}{\varepsilon_4}x_r^T\left(\left(I-BB^+\right)\Delta A\right)^T\left(I-BB^+\right)\Delta Ax_r$$

If $\xi_1=2\gamma-\varepsilon_1-\varepsilon_3>0$, $\xi_2=\varepsilon_2+\varepsilon_4$, then, by the Riccati equation, the above inequality is reduced to

$$\dot{V}(e,t)\le-\lambda_{\min}(Q)\|e\|^2+\frac{\rho_1^2}{\varepsilon_3}+\frac{\rho_2^2}{\varepsilon_4}\quad (14)$$

$$\dot{V}(e,t)\le-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}V(e,t)+\frac{\rho_1^2}{\varepsilon_3}+\frac{\rho_2^2}{\varepsilon_4}\quad (15)$$

Eq. (15) shows that the bound is made small by choosing the design parameters ε_3 and ε_4 properly. By the definition of the Lyapunov Function, the error bound is given by

$$\|e\|\le\left(\frac{\rho_1^2}{\varepsilon_3}+\frac{\rho_2^2}{\varepsilon_4}\right)^{1/2}\left[\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)\lambda_{\min}(Q)}\right]^{1/2}\quad (16)$$

This proves the theorem.

Now consider two SNS superconducting cavities connected in parallel. The nominal system matrices are

$$A_1=\begin{bmatrix}-1726.3 & 0\\ 0 & -1726.3\end{bmatrix},\ A_2=\begin{bmatrix}-1723.7 & 0\\ 0 & -1723.7\end{bmatrix},$$

$$B_1=\begin{bmatrix}1726.1 & 0\\ 0 & 1726.1\end{bmatrix}\ B_2=\begin{bmatrix}1723.4 & 0\\ 0 & 1723.4\end{bmatrix},$$

$$C_1=C_2=0.5I,\quad C=\begin{bmatrix}C1 & C2\end{bmatrix}$$

For a given output set point $y_r=\begin{bmatrix}8.9493 & -4.4620\end{bmatrix}^T$, the state set point and input set point are obtained from Eq. (9).

$$x_r=\begin{bmatrix}8.9493 & -4.462 & 8.9493 & -4.462\end{bmatrix}^T$$

$$u_r=\begin{bmatrix}-8.9507 & 4.4626\end{bmatrix}^T$$

The Lorentz force detuning is the main frequency detuning and other detuning are ignored. Lorentz force detuning change their values within the ranges $\Delta\omega_1=\begin{bmatrix}-700 & 0\end{bmatrix}rad/\sec$ $\Delta\omega_2=\begin{bmatrix}-838 & 0\end{bmatrix}rad/\sec$ for a RF pulse period. The bounds Eq. (10) are obtained as $\rho_1=4.4583$, $\rho_2=984.21$. In order to find out the dependence of the error bound $\|e\|$ upon γ, an optimization based on gridding of the domain ε_2 and the domain ε_4 was run for several values of γ. Table 1 shows the result for values of $Q=27.137I$, $\varepsilon_1=2$, $\varepsilon_3=2\gamma-\varepsilon_1$. With $(\gamma,\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)=(1000,2,34986,64724)$, the Riccati equation solution is and Figure 1 shows the simulation result.

$$P=\begin{bmatrix}0.0095433 & 0 & -0.00254 & 0\\ 0 & 0.009543 & 0 & -0.00254\\ -0.00254 & 0 & 0.0095703 & 0\\ 0 & -0.00254 & 0 & 0.0095703\end{bmatrix}$$

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CONCLUSION

For output tracking performance, state controllability and input-output controllability are not enough. The matching condition should be satisfied for practical tracking performance. In the case where the single RF power source feeds the single cavity, the matching condition is satisfied. Therefore, the state and output errors can be made arbitrarily small under the uncertainty caused by the frequency detuning of the RF cavity. When the single RF power source feeds two cavities, the parallel composite system, the system becomes mismatched. In this case, as shown in the simulation, the output tracking performance may be achieved. However, the states of the individual cavities are not constrained to a satisfactory boundary and therefore the single RF source-multi cavity topology is not desirable. This is true for other controller structures such as PID, H infinity, etc., unless local feedback controllers are applied to each cavity.

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Table 1: Optimization Parameters

γ	$\|e\|_{optimal}$	$\varepsilon_{2optimal}$	$\varepsilon_{4optimal}$
10	1.00750	32953	60963
100	0.97813	34817	64410
1000	0.97521	34986	64724
1500	0.97510	34992	64735
2000	0.97505	34993	64739
2500	0.97502	34996	64743
3000	0.97500	34998	64746

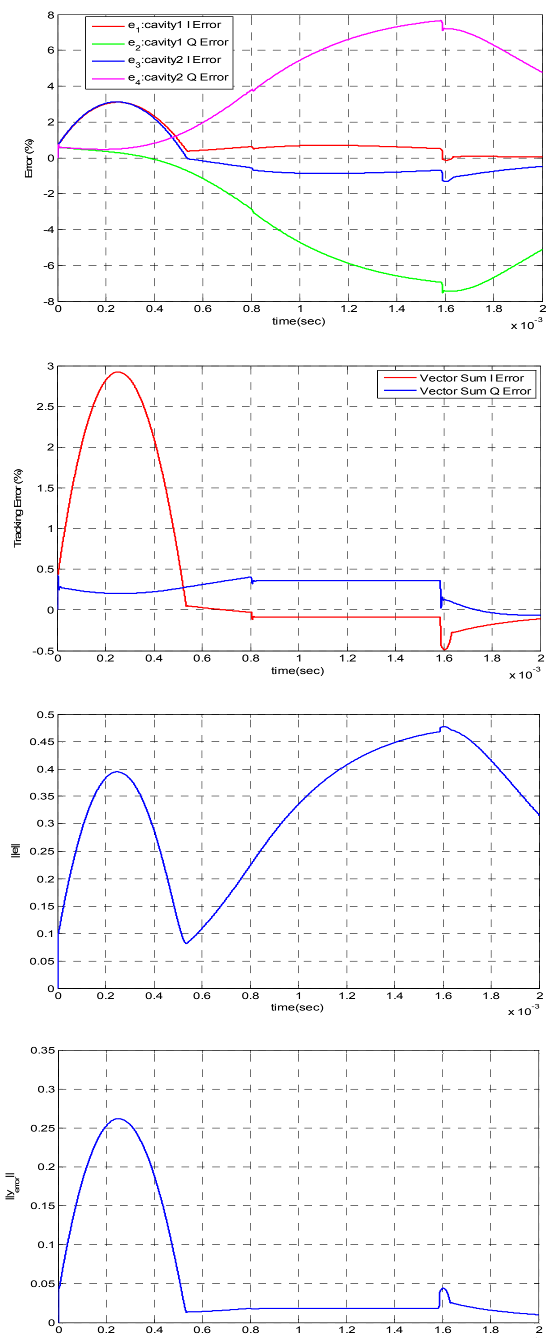

Figure 1: Cavity field I and Q errors of each cavity (top), vector sum output error (2nd), error norm of cavity field I and Q errors(3rd), and output error norm(bottom).