SVD-BASED METHOD FOR MEASUREMENT OF BEAM PARAMETERS AND FLAG RESOLUTION *

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Abstract

In NSLS II booster to storage ring transport line, the typical beam size in vertical plane is ~60 μm, which requires very high flag resolution to get good beam parameters measurement. This paper describes a new SVD-based method to measure transverse beam parameters and flag resolution simultaneously with double quads scan. Implementation simulations of the proposed method are performed for a dispersion free region in the NSLS-II booster to storage ring transport line. With this method, it breaks the limitation of beam parameters measurement accuracy due to the flag resolution.

INTRODUCTION

In the NSLS-II [1] booster to storage ring transport line [2], the typical beam size in the vertical plane is expected to be around 60 μm, which requires high resolution to get an accurate measurement of the beam emittance.

There are two error sources in the measurement of the beam size with flag. The systematic error is defined by the flag resolution, which is characterized by a point spread function. The random error source is noise which can happen because of beam parameters variation and/or during signal acquisition. The flag resolution is determined by a number of factors, such as the thickness and type of scintillator and the quality and design of the optics, and it constitutes systematic error. The second error source is random and can be suppressed with averaging.

For the YAG screen the resolution is in the range of 10 μm to 100 μm and it is hard to evaluate with sufficient accuracy. Measuring point spread function with a beam is also problematic because it is hard to focus beam into the small enough spot.

To overcome the deficiency of knowledge of the resolution, we propose a modification of the conventional quadrupole scan method [3] by introducing the second variable quadrupole.

MEASUREMENT SETUP

The simple layout is shown in Figure 1, including two quads and one flag. It is a drift space between the quads and the flag. Nevertheless, any known elements can be between the quadrupoles and flag, so that the beam transfer matrices are known.

In principle, both quadrupoles can be scanned together. In our method, one quadrupole is turned off while scanning the other quad.

To measure the flag resolution, two quads and one flag are necessary.

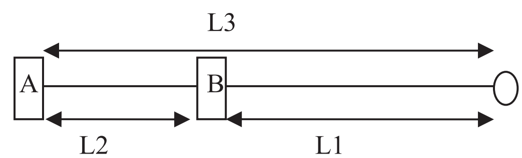

Figure 1: Layout of quads and flags for resolution measurement.

THEORY AND SIMULATIONS

The measured beam size S_x at flag is

$$S_x = \sqrt{S_{xActual}^2 + R^2} + \text{Noise} \qquad (1)$$

where $S_{xActual}^2 = \beta\varepsilon$ is the actual beam size defined by β-function and emittance ε. The measured beam size includes the flag resolution R and random noise Noise.

First, let's see the case without noise. The measured beam size can be written as

$$S_x^2 = S_{xActual}^2 + R^2 = \beta\varepsilon_x + R^2 \qquad (2)$$

We choose the point before quad A as the reference point. With the thin lens approximation, the transfer matrix from quad A to flag is

$$M = \begin{bmatrix} 1 & L_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_A} & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{L_3}{f_A} & L_3 \\ -\frac{1}{f_A} & 1 \end{bmatrix} \qquad (3)$$

where f_A is the focusing length of the quadrupole A, L_3 is the distance between quad A to the flag.

The scan with the quadrupole A gives the beam size at the flag as the function of quad A focusing length f_A is

$$S_x^2 = \frac{L_3\beta\varepsilon_x}{f_A^2} - \frac{2}{f_A}(L_3\beta\varepsilon_x - L_3^2\alpha\varepsilon_x) + (\beta\varepsilon_x - 2L_3\alpha\varepsilon_x + L_3^2\gamma\varepsilon_x + R^2) \qquad (4)$$

Where β, α, and γ are the Twiss parameter at the reference point, ε_x is the beam emittance and R is the flag resolution.

The relation between the beam size square and the quad's focusing $\frac{1}{f}$ is a parabolic curve. The fitted coefficients are

$$a_1 = L_3\beta\varepsilon_x$$
$$b_1 = -2\beta\varepsilon_x L_3 + 2\alpha\varepsilon_x L_3^2$$
$$c_1 = \beta\varepsilon_x - 2L_3\alpha\varepsilon_x + L_3^2\gamma\varepsilon_x + R^2$$

It is clear that scanning quad A is not enough to measure four parameters ($\beta\varepsilon_x$, $\alpha\varepsilon_x$, $\gamma\varepsilon_x$ and R). It is degenerate between $\gamma\varepsilon_x$ and R.

The scan with quadrupole B is performed in similar manner as quad A scan. The reference point is the same as quad A scan. With the thin lens approximation, the transfer matrix from the reference point to the flag is

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$$N = \begin{bmatrix} 1-\frac{L_1}{f_B} & L_1+L_2-\frac{L_1L_2}{f_B} \\ -\frac{1}{f_B} & 1-\frac{L_2}{f_B} \end{bmatrix} \quad (5)$$

where f_B is the quad B focusing length, L_1 is the distance between quad B to the flag and L_2 is the distance between quad A and quad B.

The beam size due to scan quads B is

$$S_x^2 = \frac{L_1^2(\beta\varepsilon_x - 2L_2\alpha\varepsilon_x + L_2^2\gamma\varepsilon_x)}{f_B^2} - \frac{2}{f_B}[L_1\beta\varepsilon_x - L_1(L_1 + L_2)\alpha\varepsilon_x + L_1L_2(L_1+L_2)\gamma\varepsilon_x] + [\beta\varepsilon_x - 2(L_1+L_2)\alpha\varepsilon_x + (L_1+L_2)^2\gamma\varepsilon_x + R^2] \quad (6)$$

The fitted coefficient is

$$a_2 = L_1^2(\beta\varepsilon_x - 2L_2\alpha\varepsilon_x + L_2^2\gamma\varepsilon_x)$$

Including the previous results for scan with quad A, we can be written

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} L_3 & 0 & 0 & 0 \\ -2L_3 & 2L_3^2 & 0 & 0 \\ 1 & -2L_3 & L_3^2 & 1 \\ L_1^2 & -2L_2L_1^2 & L_1^2L_2^2 & 0 \end{bmatrix} \begin{bmatrix} \beta_A\varepsilon_x \\ \alpha_A\varepsilon_x \\ \gamma_A\varepsilon_x \\ R^2 \end{bmatrix} \quad (7)$$

b_2 and c_2 can be also added in Eq. (7). But the above four equations are sufficient to find the four unknown parameters.

For more exact expression, the beam size at flag by scanning quads A and B can be written as

$$\begin{bmatrix} \sigma_x^{A1} \\ \sigma_x^{A2} \\ \vdots \\ \sigma_x^{An} \\ \sigma_x^{B1} \\ \sigma_x^{B2} \\ \vdots \\ \sigma_x^{Bm} \end{bmatrix} = \begin{bmatrix} M_{11}^{A1\,2} & -2M_{11}^{A1}M_{12}^{A1} & M_{12}^{A1\,2} & 1 \\ M_{11}^{A2\,2} & -2M_{11}^{A2}M_{12}^{A2} & M_{12}^{A2\,2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ M_{11}^{An\,2} & -2M_{11}^{An}M_{12}^{An} & M_{12}^{An\,2} & 1 \\ M_{11}^{B1\,2} & -2M_{11}^{B1}M_{12}^{B1} & M_{12}^{B1\,2} & 1 \\ M_{11}^{B2\,2} & -2M_{11}^{B2}M_{12}^{B2} & M_{12}^{B2\,2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ M_{11}^{Bn\,2} & -2M_{11}^{Bn}M_{12}^{Bn} & M_{12}^{An\,2} & 1 \end{bmatrix} \begin{bmatrix} \beta\varepsilon_x \\ \alpha\varepsilon_x \\ \gamma\varepsilon_x \\ R^2 \end{bmatrix} = M \begin{bmatrix} \beta\varepsilon_x \\ \alpha\varepsilon_x \\ \gamma\varepsilon_x \\ R^2 \end{bmatrix} \quad (8)$$

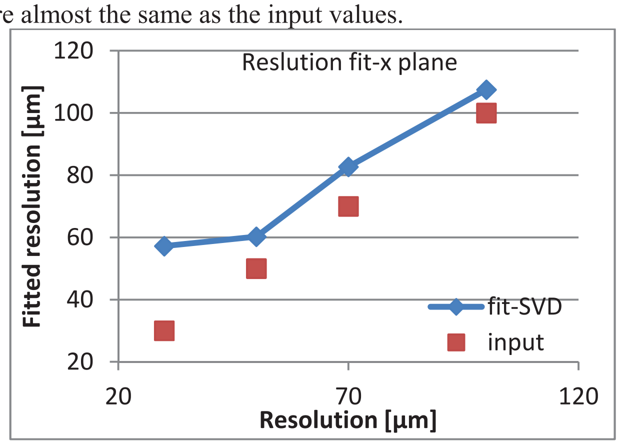
$\sigma_x = S_x^2$. The indices Ai, Bi represent different strength of quads A and B and n+m is larger than 4. With the SVD method, the beam Twiss parameters and emittance at the reference point and the flag resolution R can be found from

$$\begin{bmatrix} \beta\varepsilon_x \\ \alpha\varepsilon_x \\ \gamma\varepsilon_x \\ R^2 \end{bmatrix} = PseudoInverse[M] \cdot \begin{bmatrix} \sigma_x^{A1} \\ \sigma_x^{A2} \\ \vdots \\ \sigma_x^{An} \\ \sigma_x^{B1} \\ \sigma_x^{B2} \\ \vdots \\ \sigma_x^{Bm} \end{bmatrix} \quad (9)$$

Now we include the noise effect, which is a random number.

In the transport line, the expected flag resolution is ~70 µm. In simulation, the input flag resolution changes from 30 µm to 100 µm. The typical beam size is 300 µm in x plane and 60 µm in y plane. The noise in simulation is 30 µm in the horizontal plane and is 60 nm in the vertical plane, estimated with NSLS II beam parameters. To estimate the fitted parameters' accuracy, three parameters are used, the fitted resolution, the Twiss mismatch $(\beta_f\gamma_i + \beta_i\gamma_f - 2\alpha_i\alpha_f)/2$, and the ratio of fitted emittance and input emittance $\varepsilon_f/\varepsilon_i$. The index i stands for the values used in simulation and the index f corresponds to the fitted value. If the fitted value is exactly the same as input value, the Twiss mismatch and ratio of emittances should be 1.

We use Elegant [4] to simulate the quads scan process. The dependence of the fitted parameters versus the different input flag resolution with the above SVD method and traditional quad scan method are shown in Figure 2 to Figure 4. In Figure 2, the blue dot shows the fitted resolution with the above SVD method in the horizontal plane and in the vertical plane. The red dots show the input resolution used in simulation. In x plane, due to the big noise, with the input flag resolution increase, the fitted resolution is closer to the input value. This is because at larger fitted resolution, the noise is smaller, comparing with the resolution and size. In y plane, due to the very small noise error, the fitted values are almost the same as the input values.



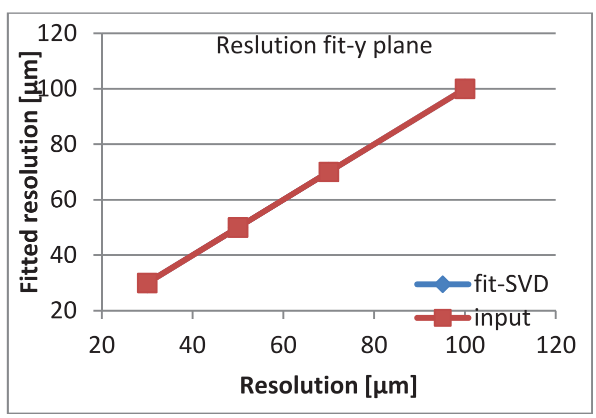


Figure 2: The fitted flag resolution vs. input flag resolution in x and y plane.

In Figure 3 and Figure 4, the blue dots show the result with the SVD method and the red dots show the result with the traditional method. With the traditional method, it only fits the Twiss parameters and emittance. The flag resolution is considered as an unknown error. In x plane,

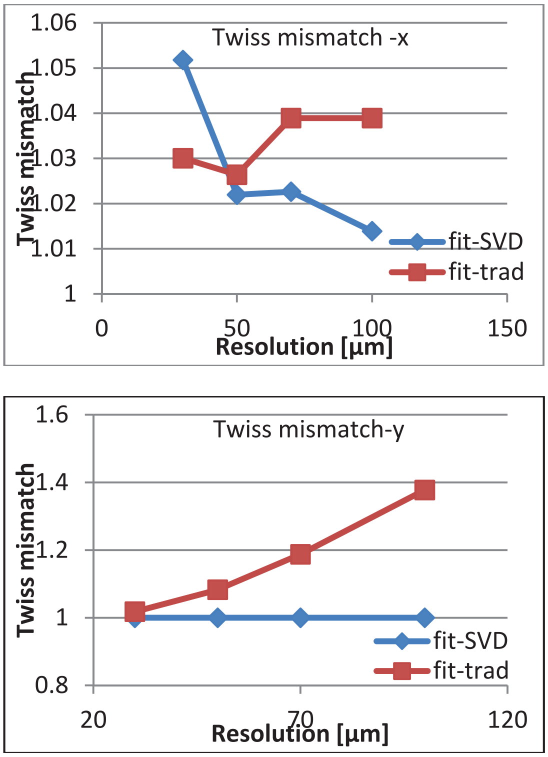


Figure 3: The fitted Twiss mismatch parameter vs. input flag resolution in x and y plane.

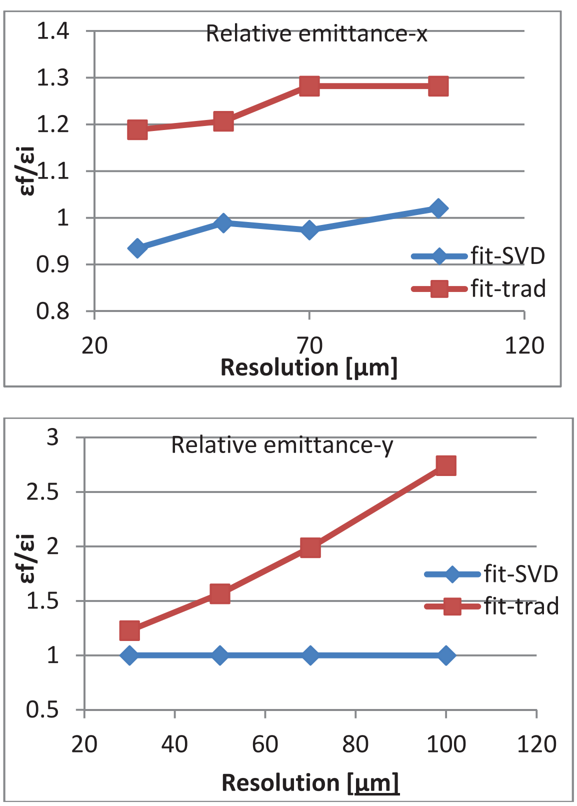


Figure 4: The fitted relative emittance vs. input flag resolution in x and y planes.

with the increase of the resolution, the fitted Twiss parameters and emittance are closer to the input values with the new method. It is similar as the conclusion of flag resolution fit. But with the traditional method, with the flag resolution increase, the fitted parameters are farther away from the input value. In y plane, with SVD method, the fitted emittance and Twiss parameter are very close to the input value with small noise error.

CONCLUSION

The proposed method allows measuring the beam emittance and flag resolution with two quads scans. The simulation results on NSLS II booster to storage ring transport line show that the fitted flag resolution and Twiss parameters have good agreement with the input values. The fitted emittance accuracy is within 10% of input value. In the vertical plane, due to the small noise, it is possible to measure the beam size which is substantially smaller than resolution.

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