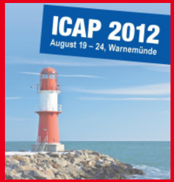
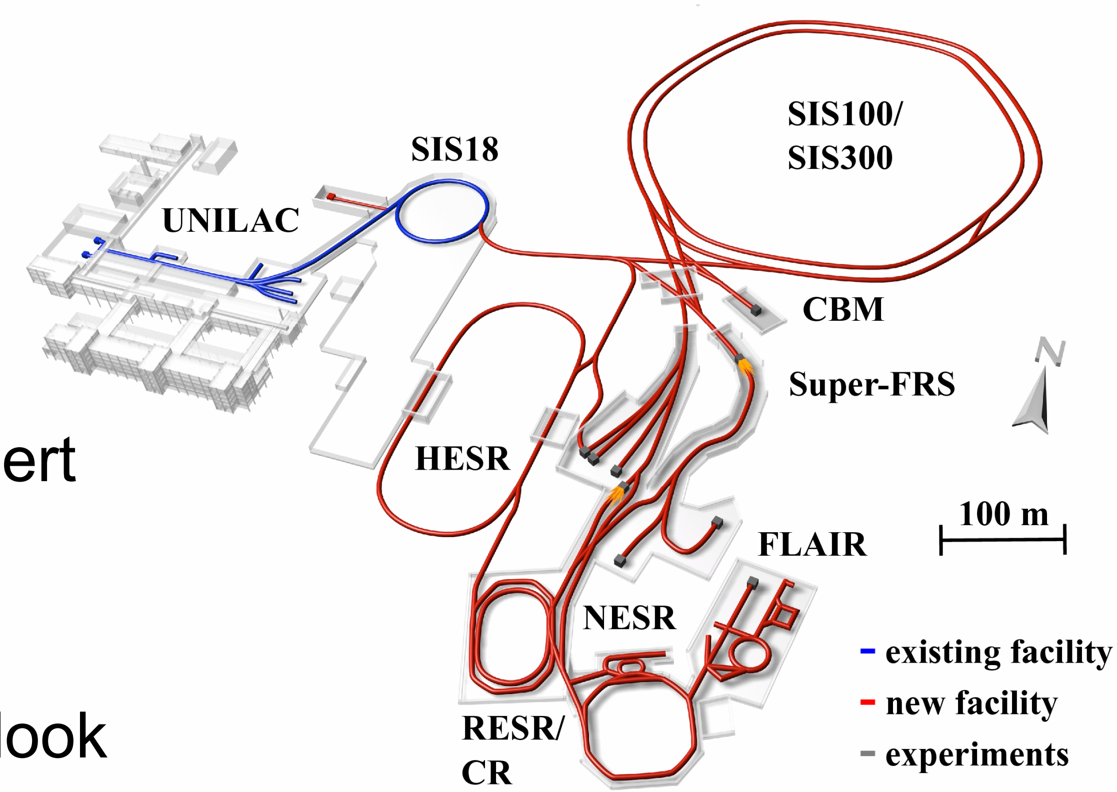
Numerical Calculation of Beam Coupling Impedances in the Frequency Domain using the Finite Integration Technique



Uwe Niedermayer and Oliver Boine-Frankenheim

Contents

- Motivation
- SIS100 impedance spectrum overview
- Definitions
- Full numerical simulation in FD
- Boundary Conditions
- Implementation
- Proposed inductive insert
- Kicker and its supply
- Bench measurements
- Current status and outlook



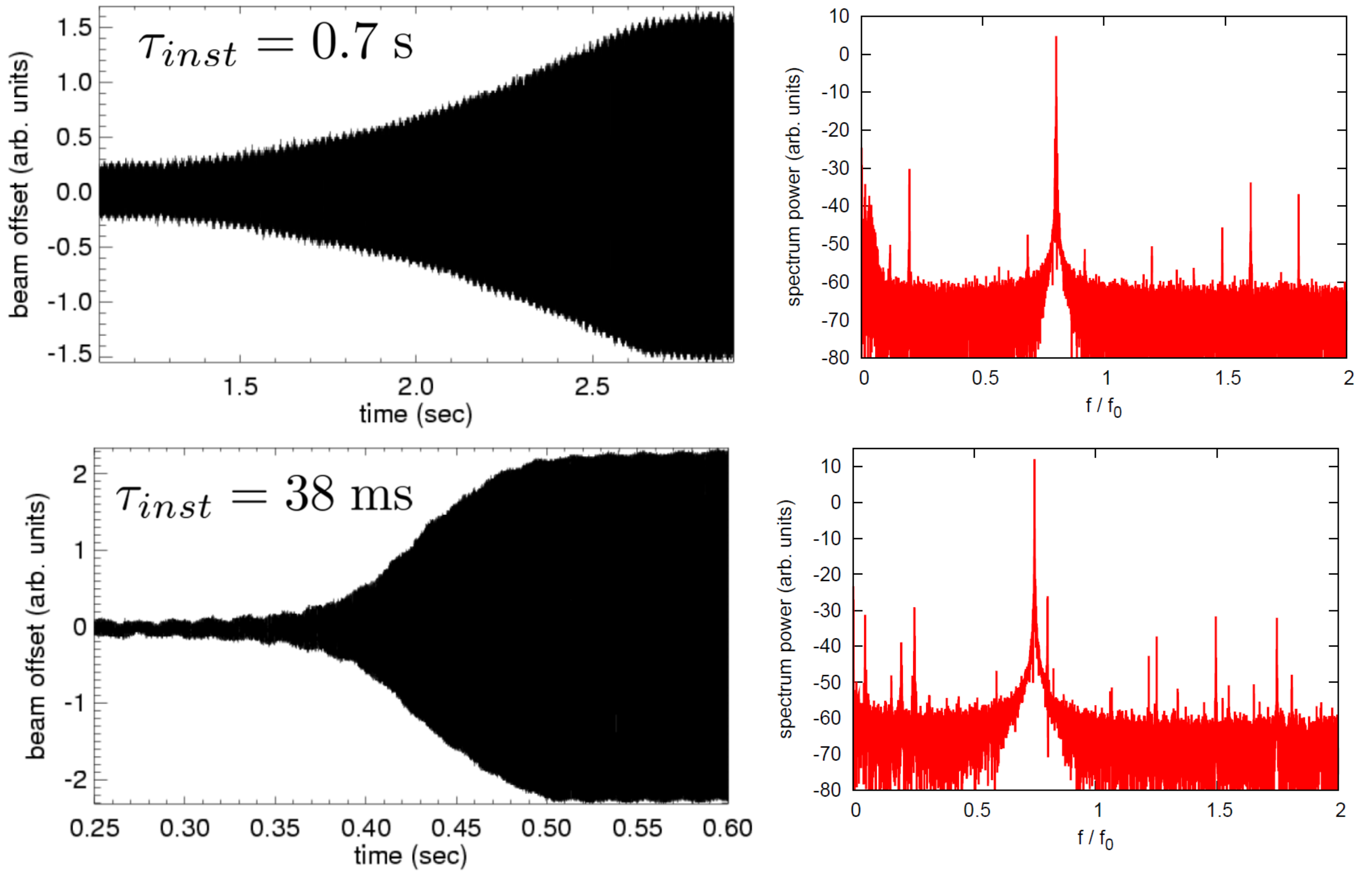
Motivation

- In SIS100, especially coasting beam and high intensity proton bunch are susceptible to impedance driven transverse instability
- The following components of SIS100 have been identified to cause large transverse impedance contribution:
  - Beampipe (especially thin, flat dipole sections)
  - *Ferrite-Kicker* and its supply network
  - Proposed *“Inductive Insert“* for long. Space-Charge compensation
  - Collimators
- Real part of longitudinal impedance causes heating (some kickers are in cold sections of SIS100)
- Heating in LHC kickers (Limitation of running time)

Transverse coasting beam instability in SIS18

H

V



Courtesy of
V. Kornilov, GSI

$$f/f_0 = 4/5$$

Revolution frequency

$$f_0 = 214\ kHz$$

$$f/f_0 = 3/4$$

$$n = \lceil Q_x \rceil$$

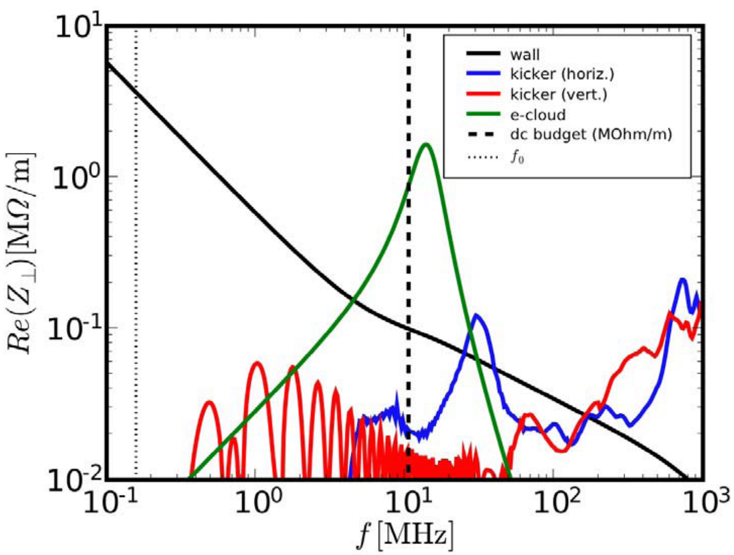
Betatron tune

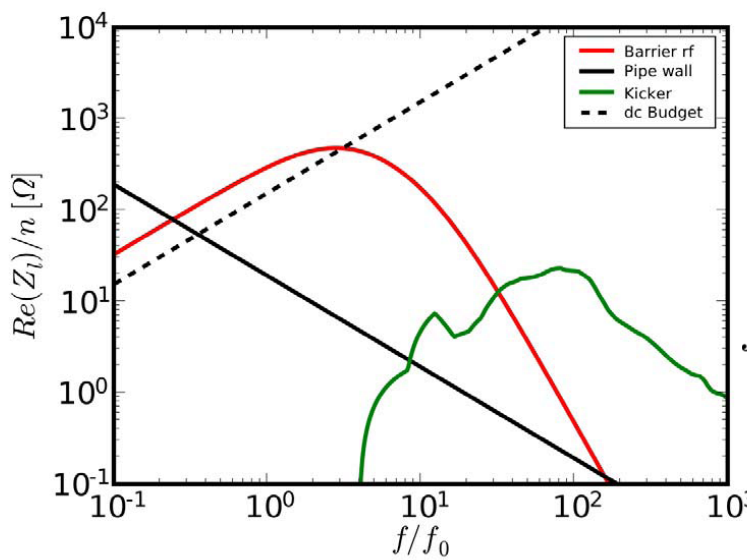
Transverse impedance provides coherent force
→ coherent instability

$$\frac{1}{\tau_{inst}} = \frac{N_{ions} q_{ion}^2}{4\pi l Q_x m_{ion} \gamma} \mathrm{Re}\left\{ \underline{Z}_{\perp,x}[(n - Q_x)\omega_0] \right\}$$

The coupling impedance spectrum in SIS18 and SIS100

Dominating devices	< 1 MHz	1 MHz – 50 MHz	50 MHz – 1 GHz	> 1 GHz (f)
$\mathrm{Re}\{\underline{Z}_\perp\}$	Beampipe (wall current)	Kicker, loading network (PFN)	Kicker, Ferrite yoke	Waveguide cutoff of beampipe (structural dependence)
$\mathrm{Re}\{\underline{Z}_\parallel\}$	MA Cavities (Broadband)	Ferrite Cavities (Narrowband)	Kicker, Ferrite yoke	





Rough estimate!

$$f_0 = 156\text{kHz}$$

Taken from SIS100 Technical Design Report, 2008

Time domain vs. Frequency domain

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- Time domain calculations e.g. by commercial software CST Particle Studio (Wake Potential)
- Impedances obtained by FFT
- Limitation by uncertainty relation

$$\Delta t \Delta f \geq 1$$

- Long wake length for low frequency $\Delta z \geq \frac{\beta c}{\Delta f} \approx 100 \text{ m} @ 1 \text{ MHz}$
- Large Gaussian bunchlength → impossibly long computation
- High computational effort for low velocity (large extension of source fields)

→FD approach pursued
for low and medium frequencies (below pipe cutoff)

Contents

Definition of coupling impedances in FD

Uniform cylindrical beam:

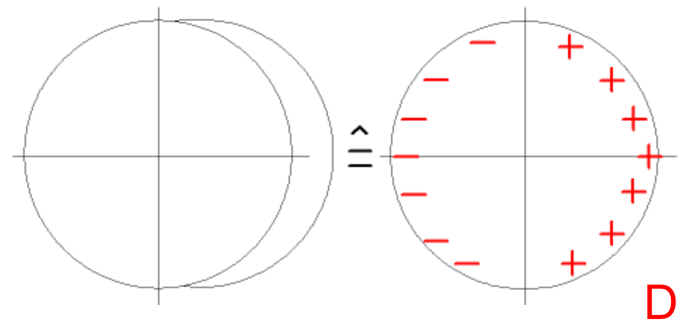
Radius of the beam

Displacement of the beam

$$\sigma(\varrho,\varphi) \approx \frac{q}{\pi a^2}\left(\Theta(a-\varrho) + \delta(a-\varrho) d_x \cos\varphi\right)$$

$$\underline{J}_{s,z}(\varrho,\varphi,z,\omega) = \sigma e^{-i\omega z/v}$$

$$\underline{\varrho}_s(\varrho,\varphi,z,\omega) = \frac{1}{v}\sigma e^{-i\omega z/v}$$



- Rigid beam
- Finite integration length due to decay of scattered fields

Dipolar beam current

$$\underline{Z}_{\parallel}(\omega) = -\frac{1}{q^2}\int_{beam} \underline{\vec{E}} \cdot \underline{\vec{J}}_{\parallel}^{*} \mathrm{d}V$$

$$\underline{Z}_{\perp,x}(\omega) = -\frac{v}{(qd_x)^2\omega}\int_{beam} \underline{\vec{E}} \cdot \underline{\vec{J}}_{\perp}^{*} \mathrm{d}V$$

Details: See e.g. R. Gluckstern, *CAS*, 2000 or T. Weiland and R. Wanzenberg, CAS, 1992

Imaginary part dominated by SPACE CHARGE!

Contents

Full numerical simulation in FD

- Before starting, the constitutive work of B. Doliwa between 2004 and 2007 has to be acknowledged
- New development is mainly due to computational needs (fast linear algebra (PETSc) on modern 64-bit machines)
- From Maxwell's equations we have

$$\nabla \times \frac{1}{\underline{\mu}} \nabla \times \underline{\vec{E}} + i\omega\kappa\underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_{ext}$$

- Charge implicitly included by continuity eq.

$$\underline{\mu} = \mu' - i\mu'' \qquad \underline{\varepsilon} = \varepsilon' - i\varepsilon''$$

Magnetization / Polarization Losses

→ Linear and Lossy
→ Hysteresis loop approximated by ellipse in H-B space
→ Excitation of Higher Order Harmonics neglected

Full numerical simulation in FD

$$\nabla \times \frac{1}{\underline{\mu}} \nabla \times \underline{\vec{E}} + i\omega\kappa \underline{\vec{E}} - \omega^2 \varepsilon \underline{\vec{E}} = -i\omega \underline{\vec{J}}_{ext}$$

$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H} \cdot d\vec{s} = \int_A \left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right) \cdot d\vec{A}$$

$$\oint_{\partial V} \vec{D} \cdot d\vec{A} = \int_V \rho \, dV$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

FIT

$$\mathbf{C}\widehat{\mathbf{e}} = -\frac{d}{dt} \widehat{\widehat{\mathbf{b}}}$$

$$\widetilde{\mathbf{C}}\widehat{\mathbf{h}} = \frac{d}{dt} \widehat{\widehat{\mathbf{d}}} + \widehat{\widehat{\mathbf{j}}}$$

$$\widetilde{\mathbf{S}}\widehat{\widehat{\mathbf{d}}} = \mathbf{q}$$

$$\mathbf{S}\widehat{\widehat{\mathbf{b}}} = \mathbf{0}$$

FIT is a mimetic discretization based on the INTEGRAL FORMULATION of Maxwell's equations (Weiland 1977)

$$\widetilde{\mathbf{C}} \mathbf{M}_{\mu^{-1}} \mathbf{C} \underline{\widehat{\mathbf{e}}} + i\omega \mathbf{M}_{\kappa} \underline{\widehat{\mathbf{e}}} - \omega^2 \mathbf{M}_{\epsilon} \underline{\widehat{\mathbf{e}}} = -i\omega \underline{\widehat{\widehat{\mathbf{j}}}}_{ext}$$

Complex linear system of size $3n_p$, indefinite ill-conditioned matrix

Symmetrization

$$\widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C}\underline{\widehat{\mathbf{e}}} + i\omega\mathbf{M}_{\kappa}\underline{\widehat{\mathbf{e}}} - \omega^2\mathbf{M}_{\epsilon}\underline{\widehat{\mathbf{e}}} = -i\omega\underline{\widehat{\mathbf{j}}}_{ext}$$

$$\underline{\widehat{\mathbf{e}}} = \mathbf{M}_{\epsilon}^{-1/2}\underline{\widehat{\mathbf{e}}}' \qquad \widetilde{\mathbf{C}} = \mathbf{C}^T$$

$$(\mathbf{M}_{\epsilon}^{-1/2}\widetilde{\mathbf{C}}\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{M}_{\epsilon}^{-1/2} - i\omega\mathbf{M}_{\epsilon}^{-1/2}\mathbf{M}_{\kappa} - \omega^2 I)\underline{\widehat{\mathbf{e}}}' = -i\omega\mathbf{M}_{\epsilon}^{-1/2}\underline{\widehat{\mathbf{j}}}_{ext}$$

$$\left(A^T A - \omega^2 D\right)\underline{\widehat{\mathbf{e}}}' = b$$

Contents

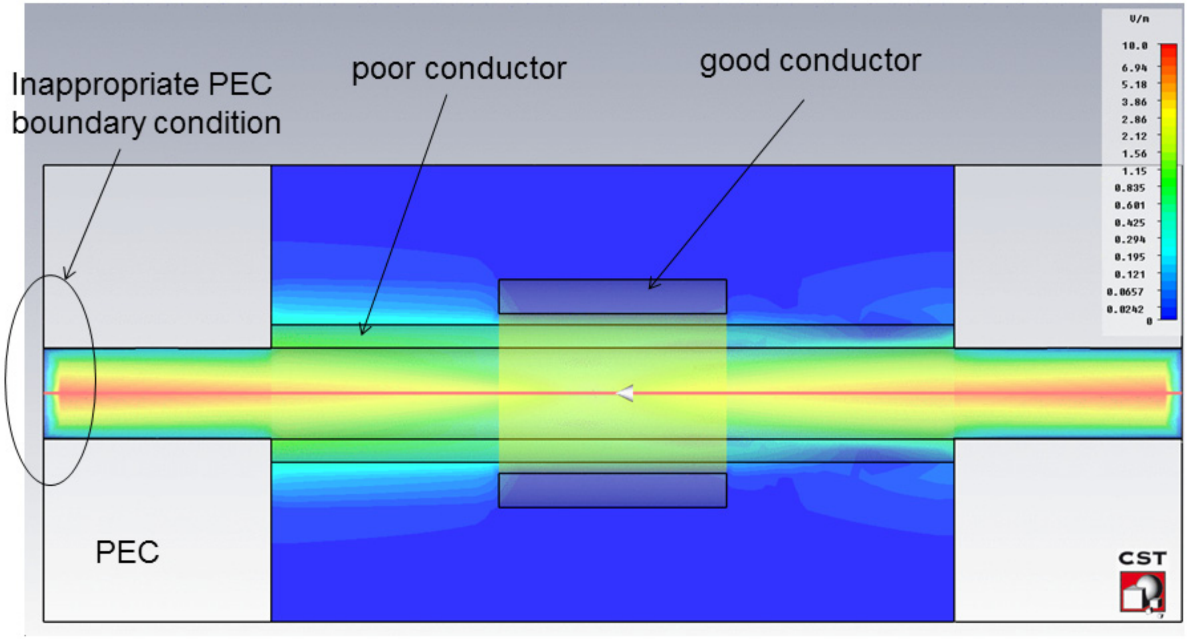
Boundary Conditions (BC) 1

Phase corrected periodic BC

- Field can be splitted in source and scattered part [(1)]
- Only source part at ports below cut-off frequency
- Calculation of the total phase advance in the structure

$$\partial_z \to -i\omega/v$$

$$P_{z,\mathrm{exp}} = e^{\frac{-i\omega(L+\Delta z_\mathrm{e})}{\beta c}}$$



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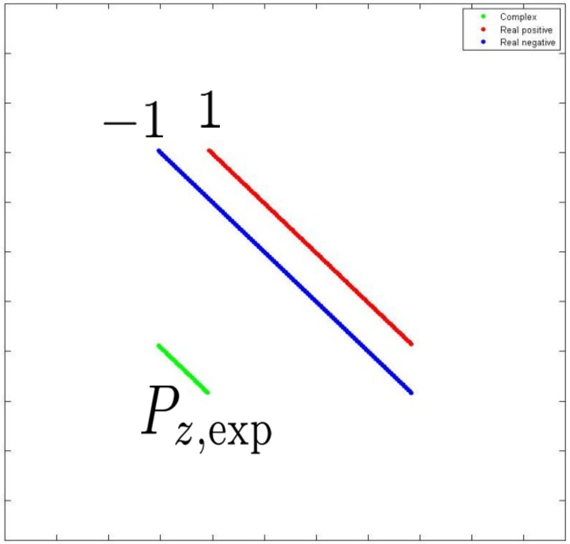
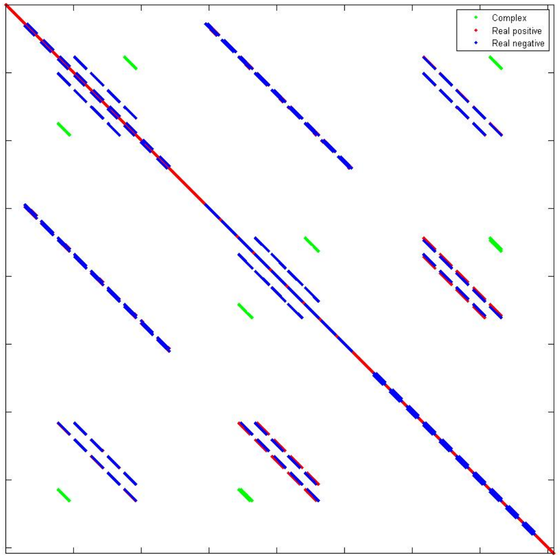
$\mathbf{P}_z$



$A^H A - \omega^2 D$



Boundary Conditions 2

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Infinite beam pipe BC [1]

- 2D solution with a priori known z-dependency
- Imprinted using the source equivalence theorem

$$\left(A^T A - \omega^2 D\right) \underline{\widehat{\mathbf{e}}}' = b + i\omega \mathbf{M}_\epsilon^{-1/2} \underline{\widehat{\mathbf{j}}}_e^{\mathrm{eq}} - \mathbf{M}_\epsilon^{-1/2} \widetilde{\mathbf{C}} \mathbf{M}_{\mu^{-1}} \underline{\widehat{\mathbf{j}}}_m^{\mathrm{eq}}$$

$$\underline{\widehat{\mathbf{j}}}_e^{\mathrm{eq}} = \widetilde{\mathbf{C}}_R \underline{\widehat{\mathbf{h}}}^{\mathrm{SG}} \qquad \underline{\widehat{\mathbf{j}}}_m^{\mathrm{eq}} = \mathbf{C}_R \underline{\widehat{\mathbf{e}}}^{\mathrm{SG}}$$

Residual curl operators

Solved on a supplementary 2D grid with

$$\mathbf{P}_z = diag\left(e^{-i\frac{\omega \Delta z}{\beta c}}\right)$$

- Sometimes called 2.5D simulation
  since longitudinal dependence known a priori but nonzero
- Can also be used for longitudinally homogeneous impedance calculations
- Limit $v \to \infty$ gives “radial model“ (purely 2D)

(1) M.C. Balk, Feldsimulation starrer Teilchenstrahlen beliebiger Geschwindigkeit und deren Anwendung in der Schwerionenbeschleunigerphysik, PhD at TU-Darmstadt, 2005.

Space charge impedance as test-case for boundary conditions

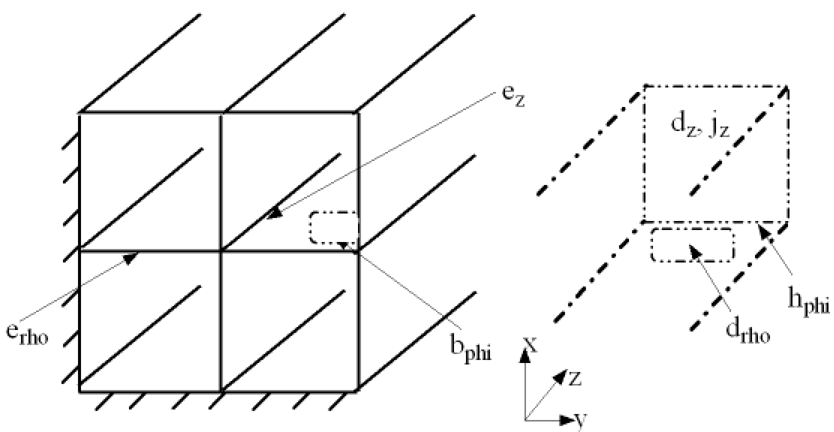
$$1-\frac{1}{\beta^2}=-\frac{1}{\beta^2\gamma^2}$$

$$\left(I-\frac{\Delta z\Delta\tilde{z}}{v^2}\mathbf{M}_{\mu^{-1}}\mathbf{M}_{\epsilon^{-1}}\right)=-\frac{1}{\beta^2\gamma^2}I$$

$$\left(\triangle_\perp-\frac{\omega^2}{\beta^2\gamma^2c^2}\right)\underline{E}_z=-\frac{i\omega\mu\sigma}{\beta^2\gamma^2}e^{-i\omega z/v}$$

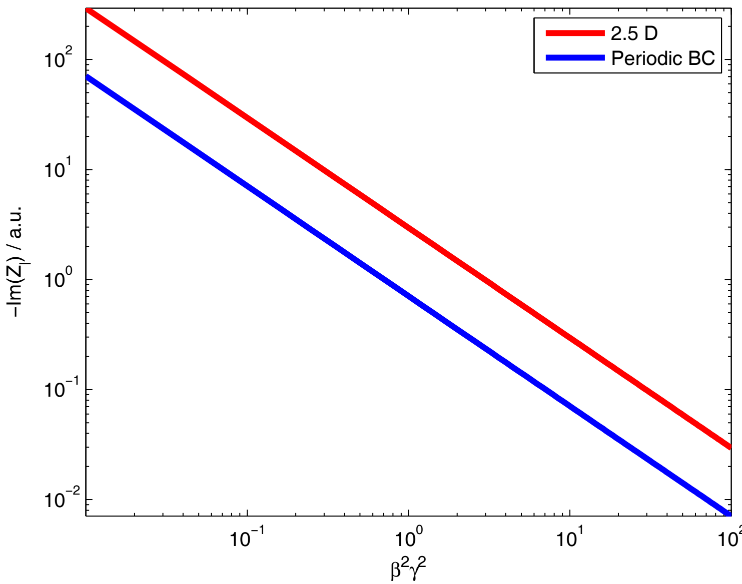
$$\underline{Z}_\parallel^{SC}=-i\omega\frac{\mu_0 gl}{4\pi\beta^2\gamma^2}$$

$$\underline{\widehat{\mathbf{e}}}_z=\frac{-i\omega}{4\beta^2\gamma^2-\omega^2\delta_\perp^2/c^2}\mathbf{M}_\mu\underline{\widehat{\widehat{\mathbf{j}}}}_{ext}$$



Primary mesh with PEC boundary

Dual mesh



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Implementation

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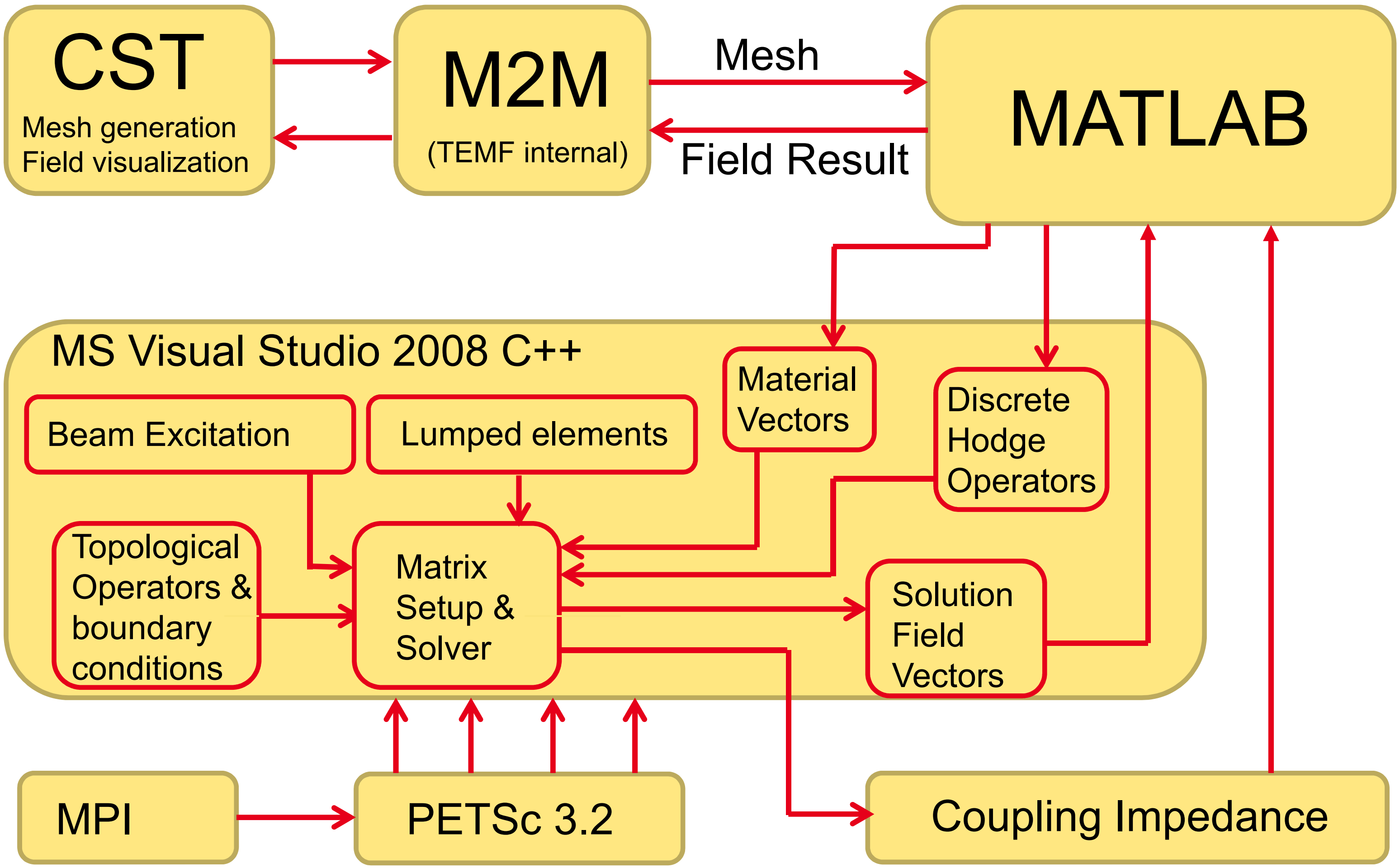

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Proposed SC compensation insert

- Longitudinal Space Charge is like a negative inductance

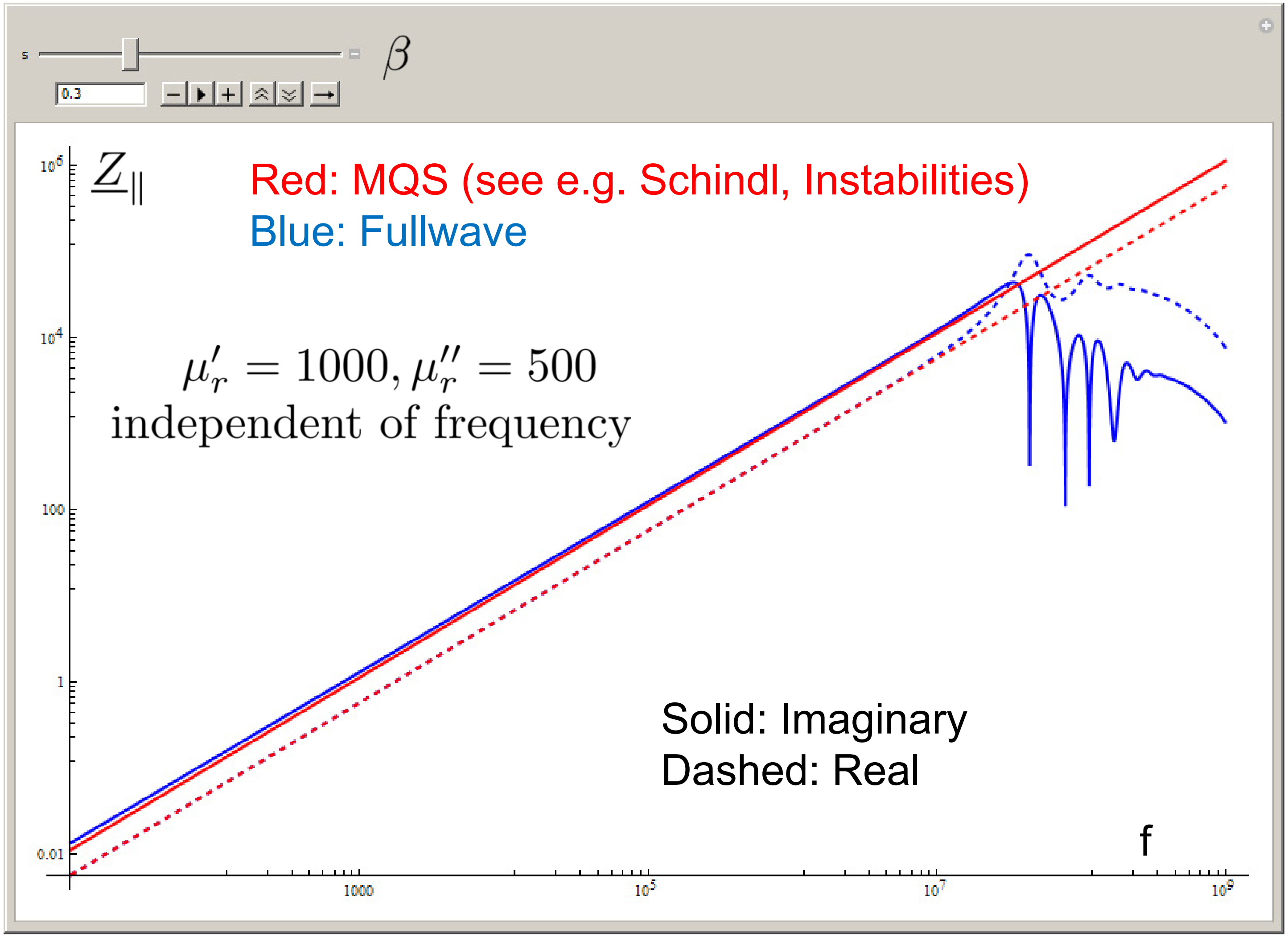
$$\underline{Z}_{\|}^{SC} = -i\omega \frac{\mu_0 g_0 l}{4\pi\beta^2\gamma^2}$$

- Causes potential well distortion / decrease of bucket-height
- Can be compensated by positive inductance

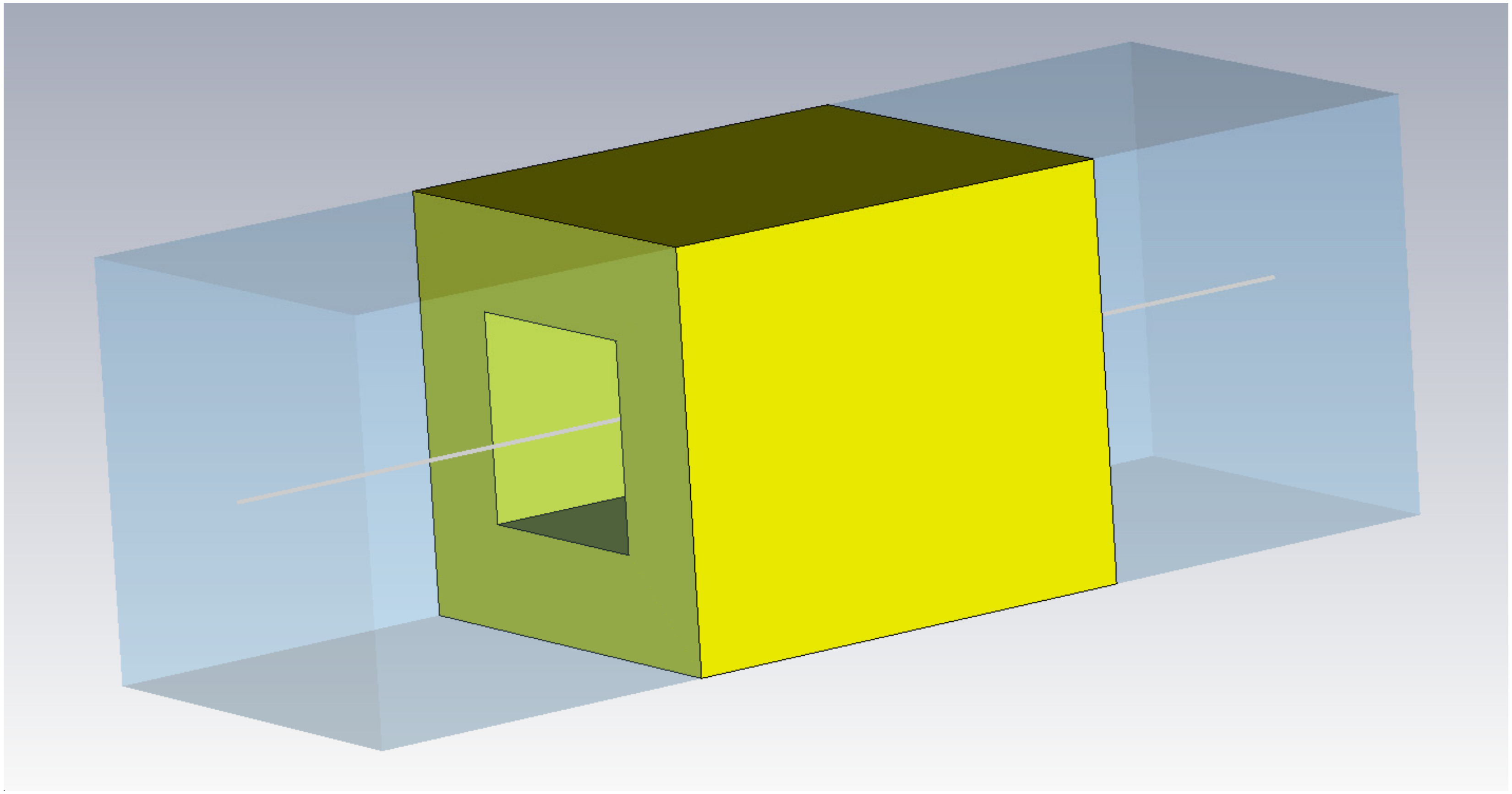
$$\underline{Z}_{\|}^{INSERT} \approx i\omega \frac{\mu}{2\pi} l \ln\frac{h}{b} \ , \quad f < 10\ \mathrm{MHz} \ \ , \quad \beta > 0.3$$

- Implemented in PSR / Los Alamos using highly permeable material (Ferrite)
- Magnetization losses cause real part of impedance
  →Negative mass instability @ PSR (PhD Thesis C. Beltran, 2003)
- Impact on transverse impedance?

Analytical calculations for cylindrical SC compensation insert (2D)



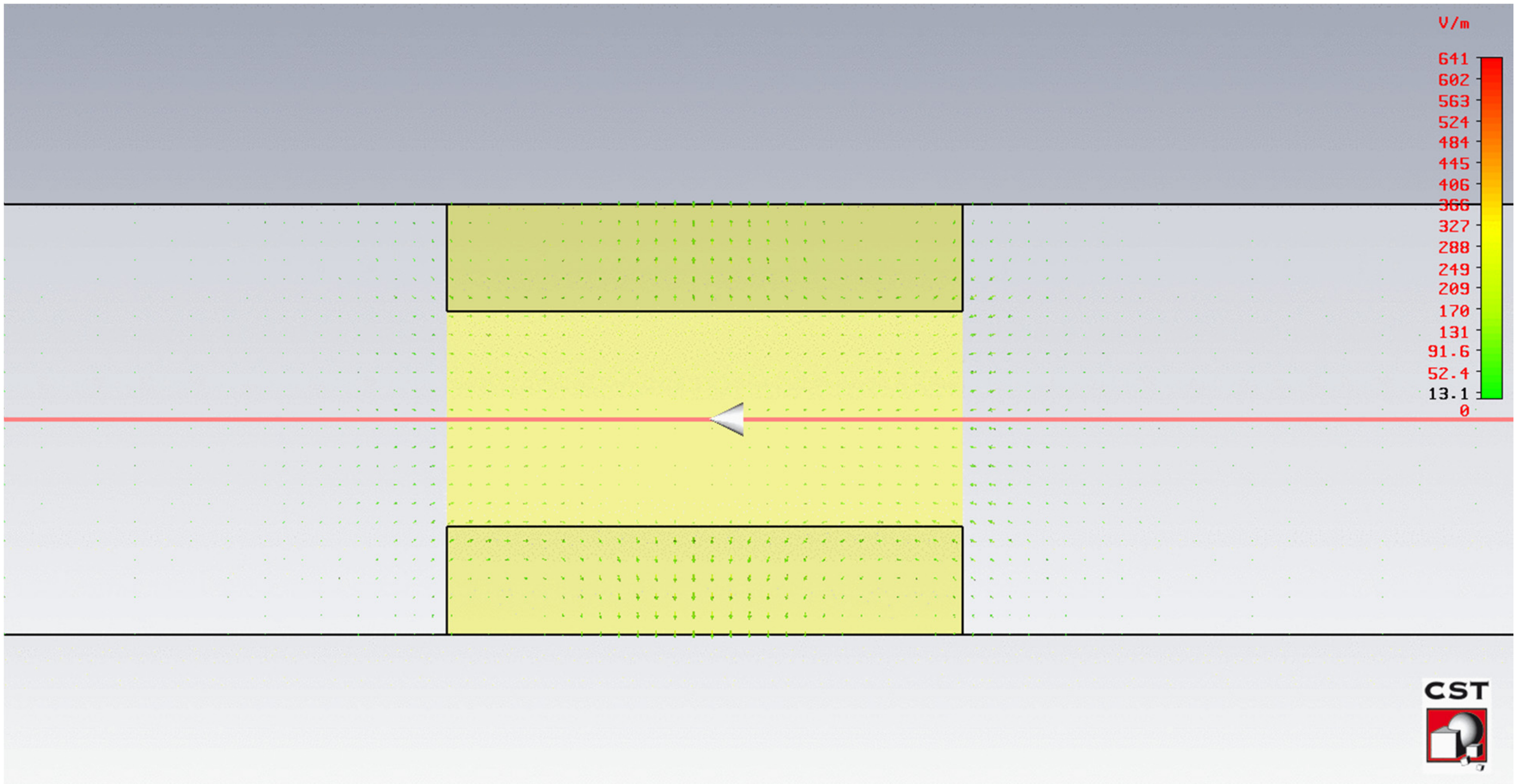
First Results for Inductive Test Structure



$$\mu_r' = 1000, \quad \mu_r'' = 0$$

Impedance purely inductive

Electric field



f=100MHz

Phase corrected periodic boundary

Done and To Do

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- Preprocessing in MATLAB, treating of PEC cells ✓
- Construction of CURL-Matrix and Electrical Boundary conditions ✓
- Construction of System Matrix ✓
- Preconditioner and solver for up to 10^6 DOFs ✓
- 2D solver for boundary conditions ✓
- Integration of the beam adapted boundary in the main grid
- Curl matrix with periodic boundary conditions ✓
- Reassembling of Hodge operators with frequency dependent material parameters and lumped elements ✗
- Solver optimization for larger systems ✗

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- **Bench measurements**
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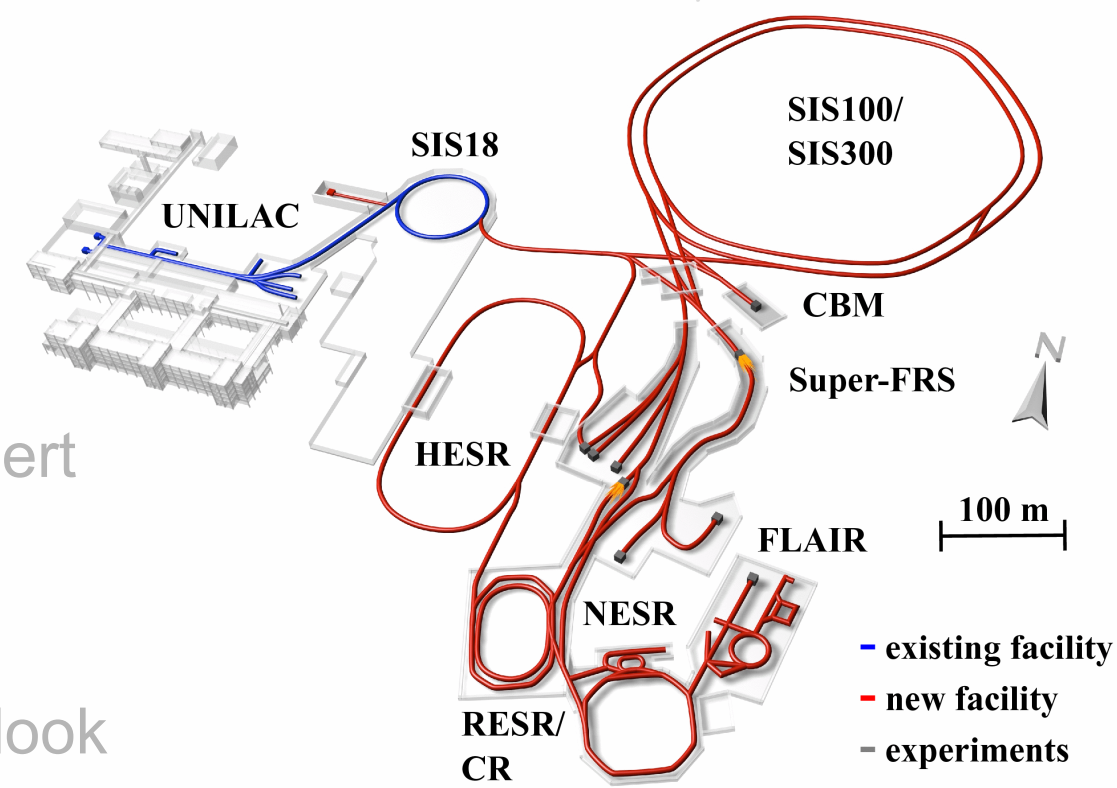

High Frequency Measurement setup

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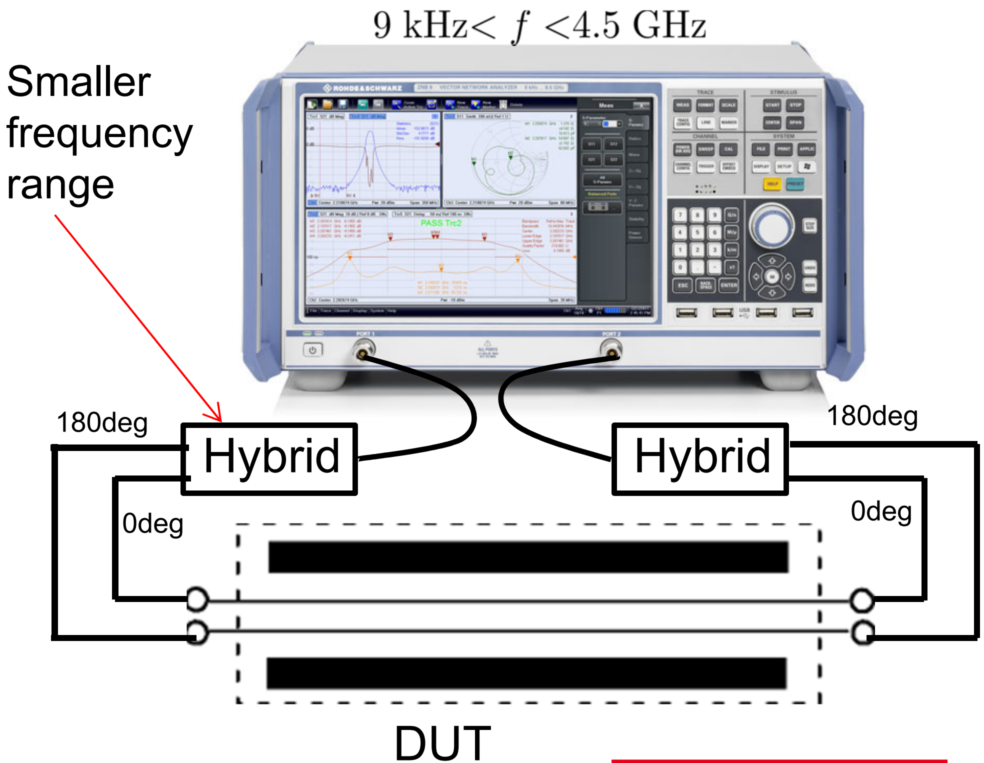


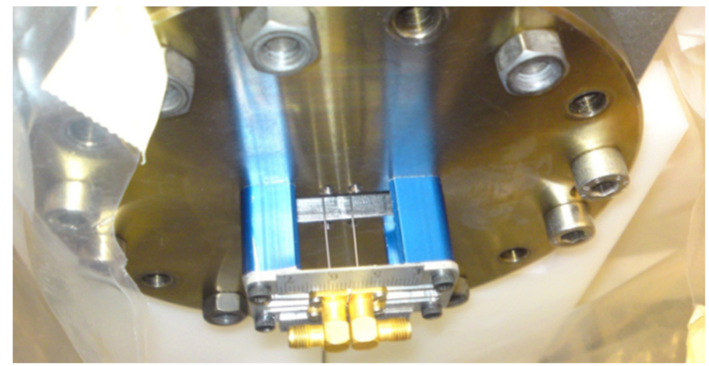
$$\underline{Z}^{dist} = 2Z_c \ln S_{21}$$

$$\underline{Z}^{lumped} = 2Z_c \frac{1-S_{21}}{S_{21}}$$

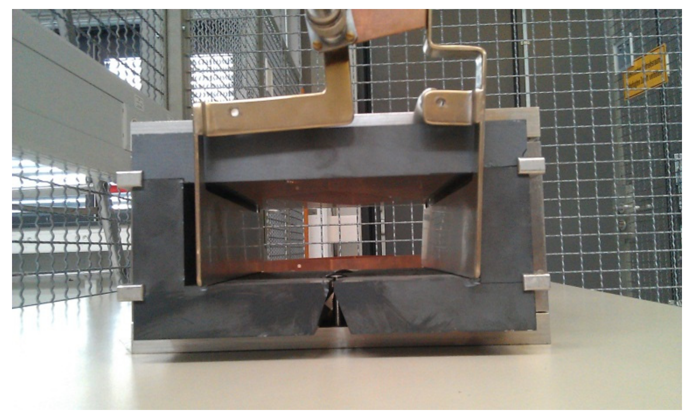
Normalized S-parameters (to REF)

$$\underline{Z}_{\perp} = \frac{c\underline{Z}}{\omega \Delta^2}$$

Walling 1989, Caspers 1992, Hahn 1978



Setup @ CERN (MKE-Kicker)



SIS18/SIS100 kicker with supply bar (High voltage connection)

Conclusions
(current status and outlook)

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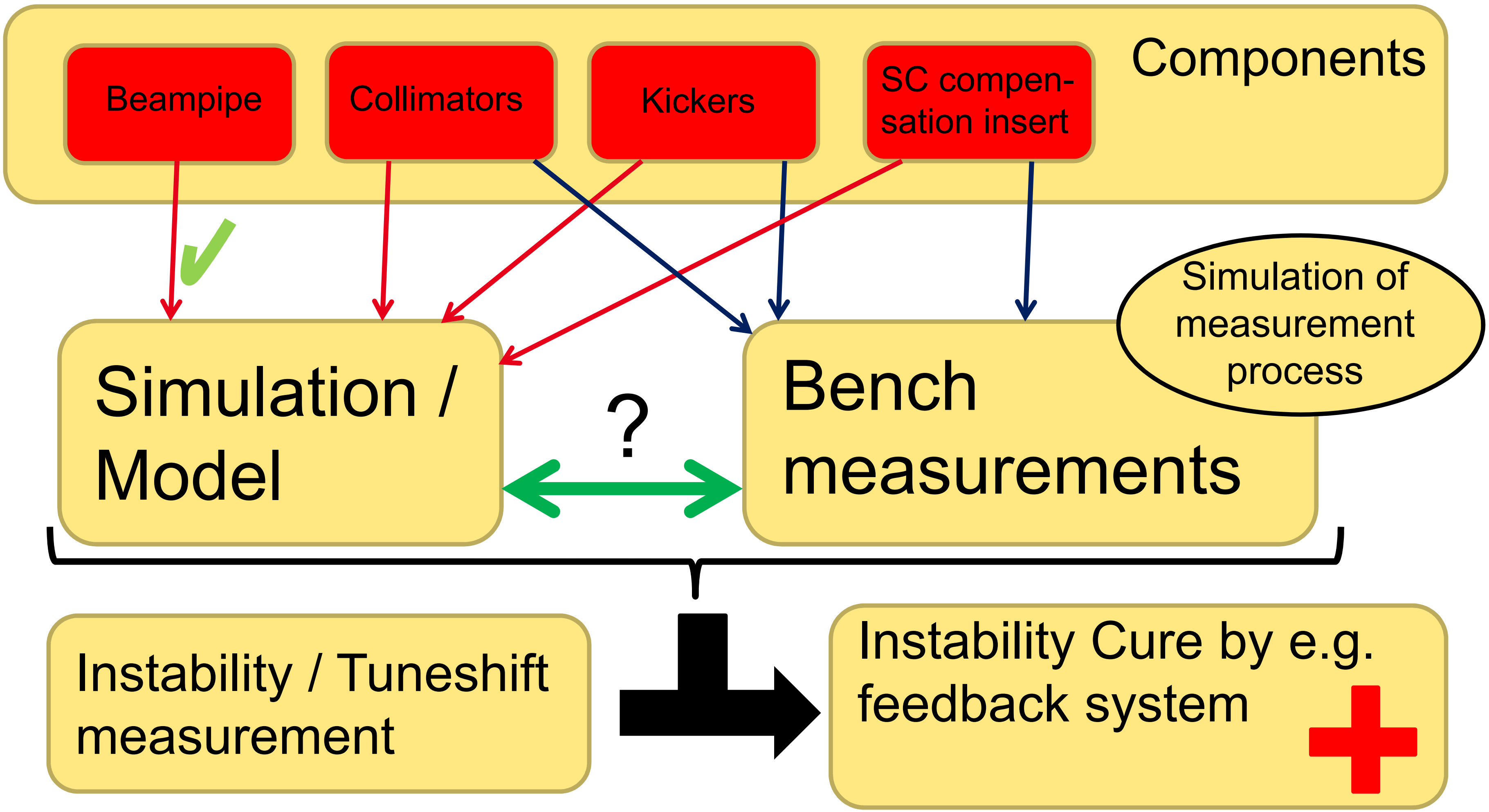
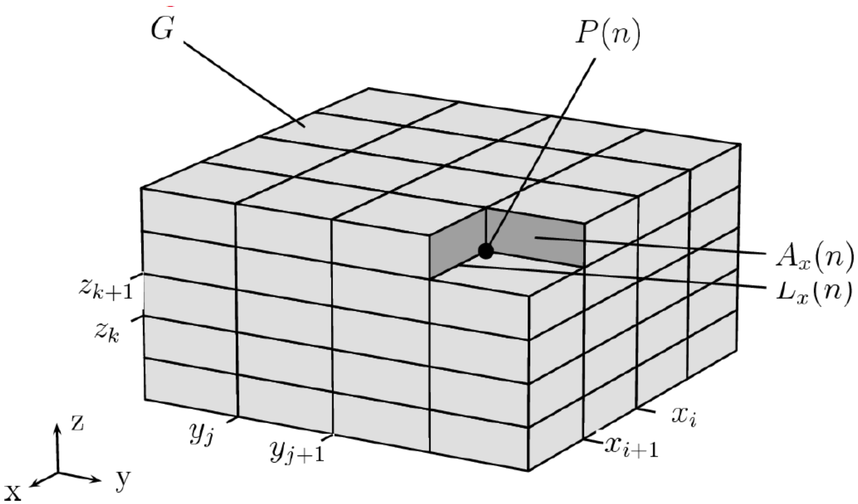


THE END

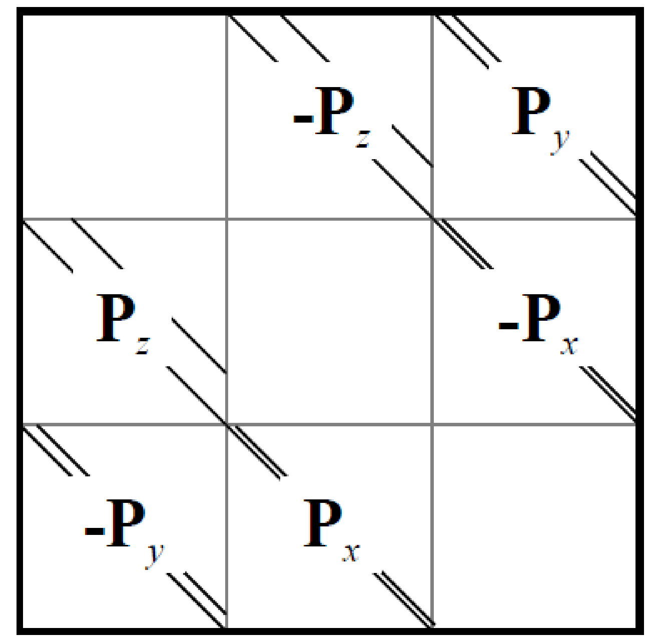
- Thank you for your kind attention
- Any questions?

The FIT Grid



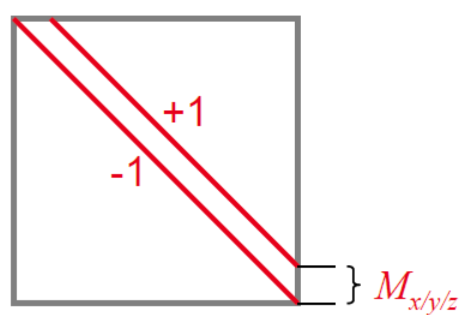
Discrete CURL operator (Matrix)

$\mathbf{C} =$



The same way:
Discrete DIV operator (Matrix)

$$\mathbf{S} = \begin{pmatrix} \mathbf{P}_x & \mathbf{P}_y & \mathbf{P}_z \end{pmatrix}$$

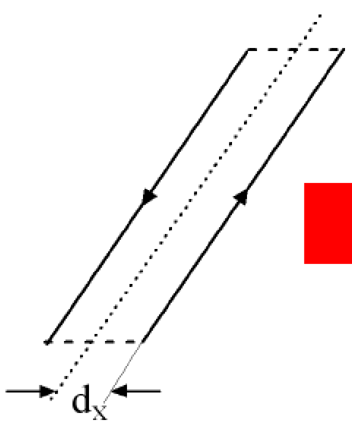


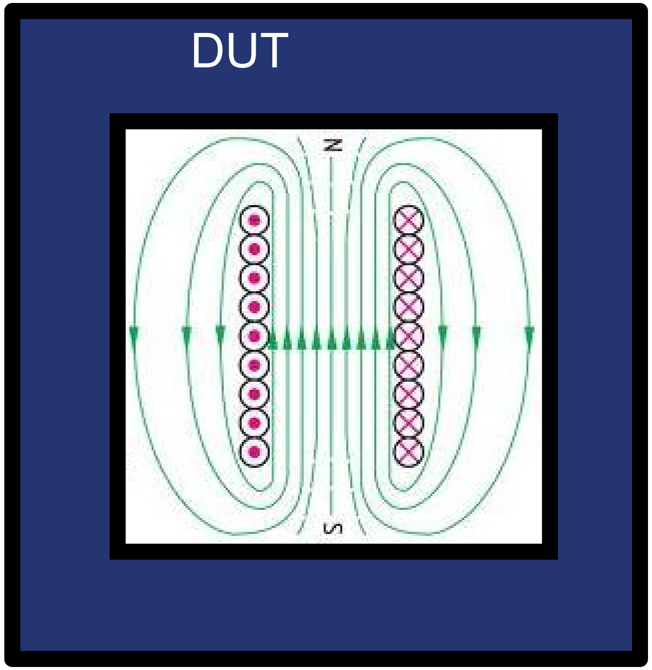
Discrete partial derivative operators

Pictures from T.Weiland, VAdF1

LF Transverse impedance

Coil Measurement:
Use coil instead of 2 wires





LCR Meter

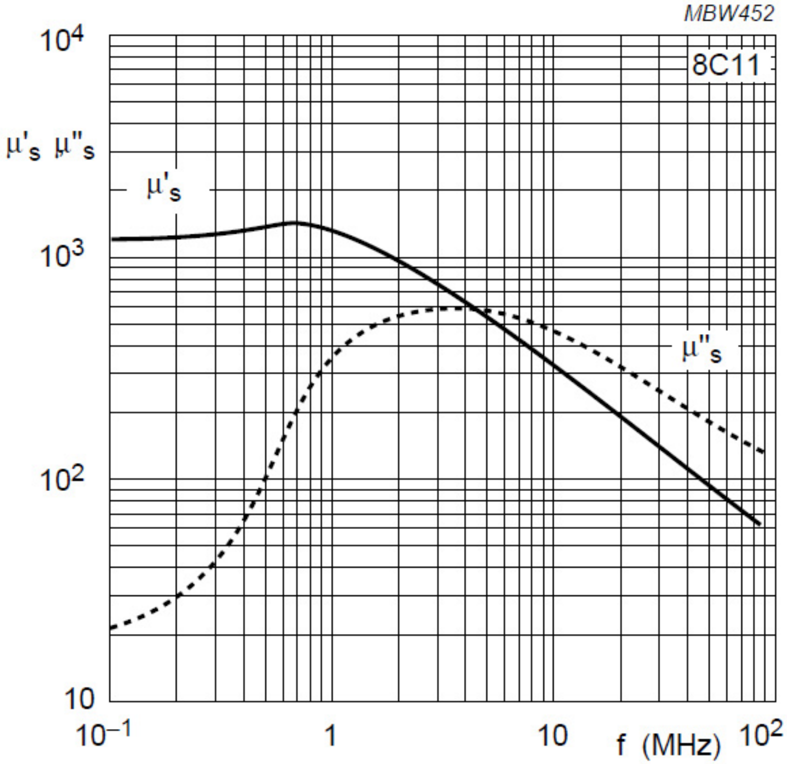
$$20\ \text{Hz} < f < 2\ \text{MHz}$$

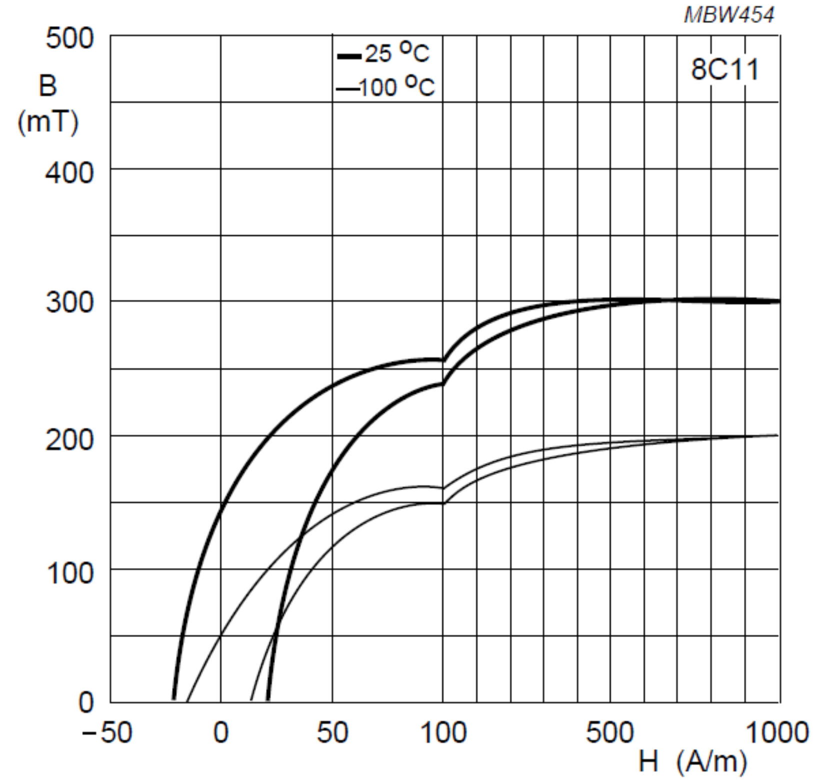
$$\underline{Z}_\perp = \frac{c(\underline{Z}^{DUT} - \underline{Z}^{REF})}{\omega N^2 \Delta^2}$$

Drawback: Upper frequency limit due to coil resonance @ approx. 1MHz

Sensitivity on material parameters

Ferroxcube 8C11





- Calculation of extremal cases
- Estimation of higher harmonics (nonlinear response)