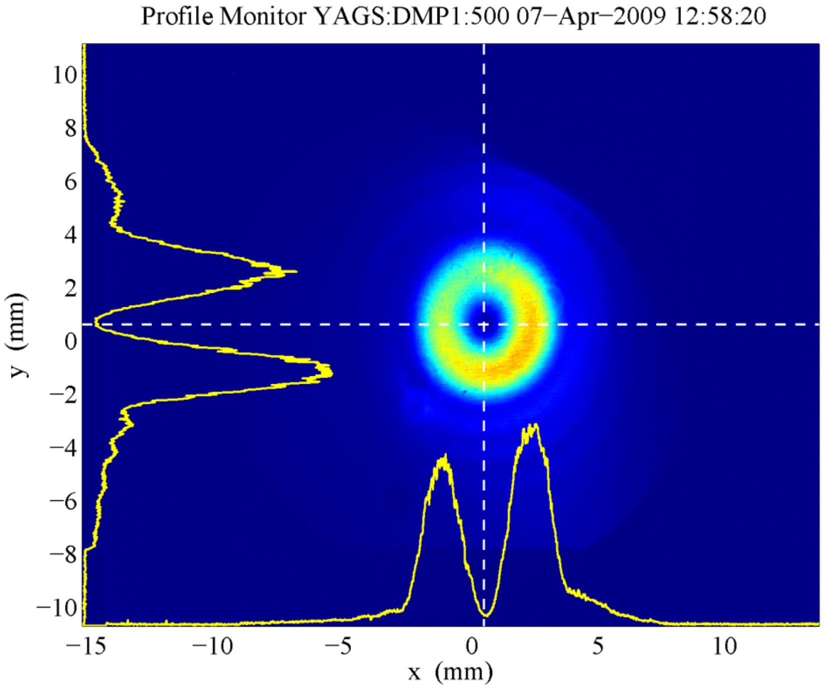
Modeling of Coherent Synchrotron Radiation using a Direct Numerical Solution of Maxwell's Equations.

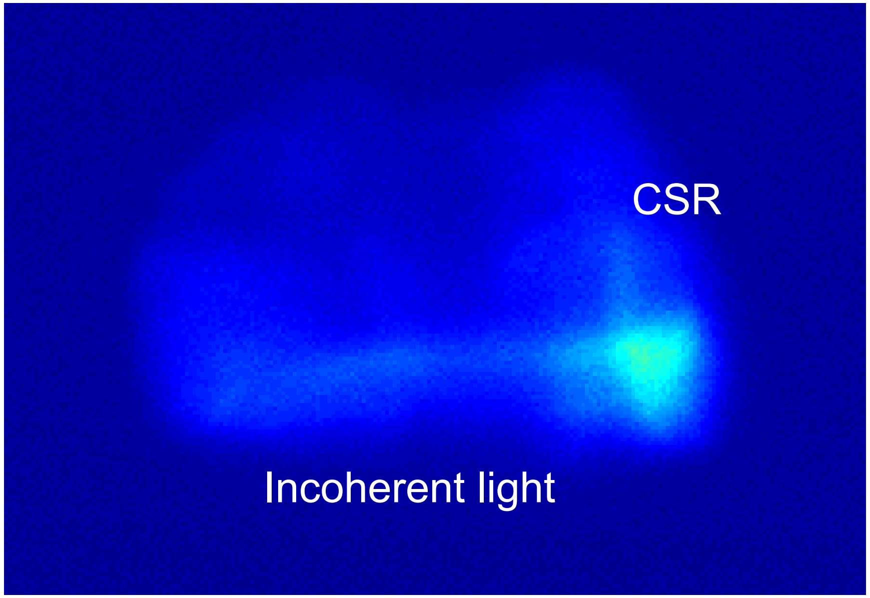
Sasha Novokhatski

11th International Computational Accelerator Physics Conference (ICAP)

August 19-24, 2012, Rostock-Warnemünde, Germany

The subject: Coherent Synchrotron Radiation





LCLS: CSR light on the YAG screen, when the electron bunch is bent down

Radiation from a horizontal bend

Classical Synchrotron Radiation

It is interesting to note that exactly 100 years ago, in 1912 G. A. Schott published his formula for intensity of radiation of a <u>relativistic</u> charged particle, which makes an instantaneously circular motion.

$$\frac{dI_n}{do} = \frac{e^2 n^2}{2\pi c}\omega_0^2\left[J_n^2\left(\frac{nv}{c}\cos\theta\right)tg^2\theta + \frac{v^2}{c^2}J'^2_n\left(\frac{nv}{c}\cos\theta\right)\right]$$

G. A. Schott, *Electromagnetic Radiation*
Cambridge University Press, Cambridge, 1912, p. 109.

Later in 1946, APS Meeting.

Here is an abstract of Julian Schwinger presentation at the American Physics Society Meeting, which was held at New York in September 1946

A8. Electron Radiation in High Energy Accelerator JULIAN SCHWINGER, *Harvard University*.*—The only fund mental limitation to the attainment of very high energ electrons in devices such as the betatron and synchrotro is the radiative energy loss accompanying the circul motion. For an electron of energy $E \gg mc^2$, moving in circular path of radius R, the energy radiated per revolu tion is

$$\delta E = \frac{4\pi}{3}\frac{e^2}{R}\left(\frac{E}{mc^2}\right)^4$$

which amounts to roughly 30 kev for an electron of 1 Bev in a magnetic field of 10^4 gauss. The radiation spectrum consists of harmonics of the rotation angular frequency $\omega = c/R$. The intensity in the nth harmonic is independent of E and varies as $n^{1/3}$ for $n < (E/mc^2)^3$, decreasing rapidly for higher frequencies. Thus the spectrum extends into the x-ray region for $E = 1$ Bev. The radiation is emitted in the direction of electron motion, within a narrow cone of angle $\theta \sim mc^2/E$ (0.03° for $E = 1$ Bev). In addition to the individual or incoherent radiation effects of the electrons, there exists, in the synchrotron, a coherent radiation arising from the electron bunching along the circular trajectory. This type of radiative energy loss is independent of E, for the coherent spectrum is limited by the length of the electron bunch. Since the latter is not a small fraction of the orbit circumference, the coherent radiation is emitted at long wave-lengths, and may be effectively suppressed by metallic shielding. Radiative energy loss in a conventional betatron results in an adiabatic decrease in the oribit radius, which is already appreciable for $E = 10^8$ ev. No difficulty is encountered in a synchrotron provided the radiofrequency voltage is adequate to supply the radiation losses. However, the required radiofrequency voltage reaches formidable proportions for energies in excess of 3 Bev. A further effect of importance at such high energies is a radiative damping of phase oscillations which is a stabilizing influence, however, only if the magnetic field index $n (H \sim r^{-n})$ is less than $\frac{3}{4}$.

* 15-minute invited paper.

According to J. Schwinger

spectrum

$$I(\omega)=\frac{\sqrt{3}}{2\pi}\frac{\omega_0 e^2}{c}\left(\frac{E}{mc^2}\right)F\left(\frac{\omega}{\omega_c}\right)$$

$$F(\xi)=\xi\int_{\xi}^{\infty}K_{5/3}(\eta)\,d\eta$$

$$\omega_c=\omega_0\frac{3}{2}\left(\frac{E}{mc^2}\right)^3$$

total power loss

$$\frac{dE}{dt}=\int_0^{\infty}I(\omega)\,d\omega=\frac{2}{3}\omega_0^2\frac{e^2}{c}\left(\frac{E}{mc^2}\right)^4$$

This result played a very important role in the progress of accelerator physics. It was the main argument for the development of Linear Colliders for electron-positron collisions at very high energies.

Is this strong energy dependence true?

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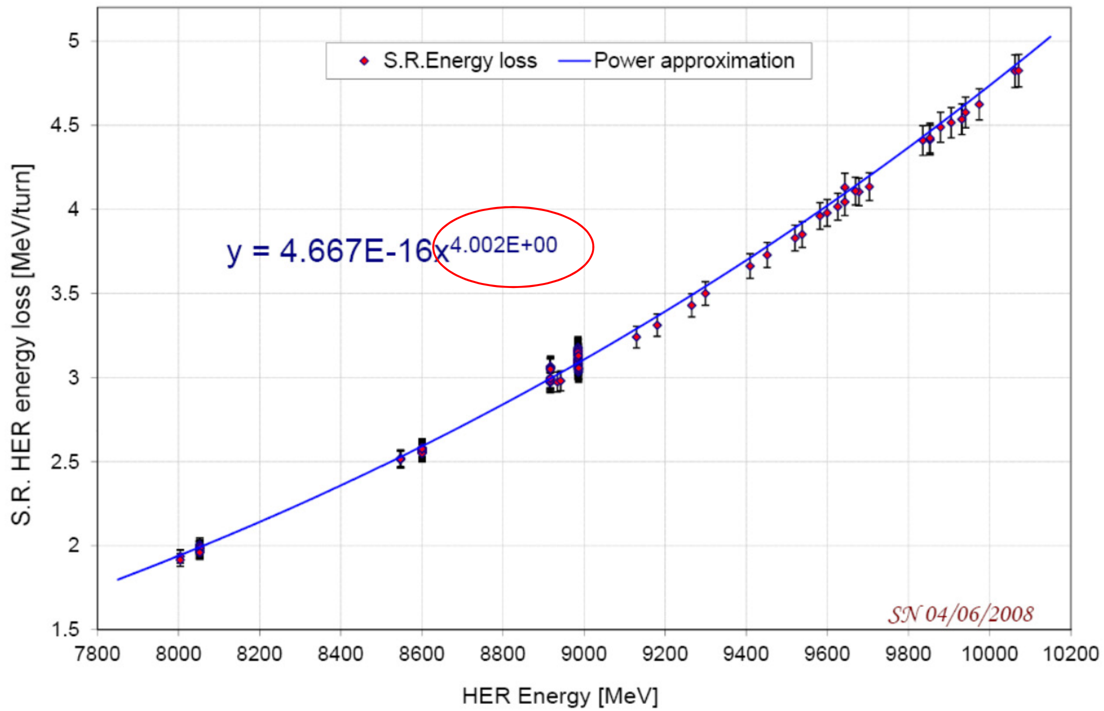
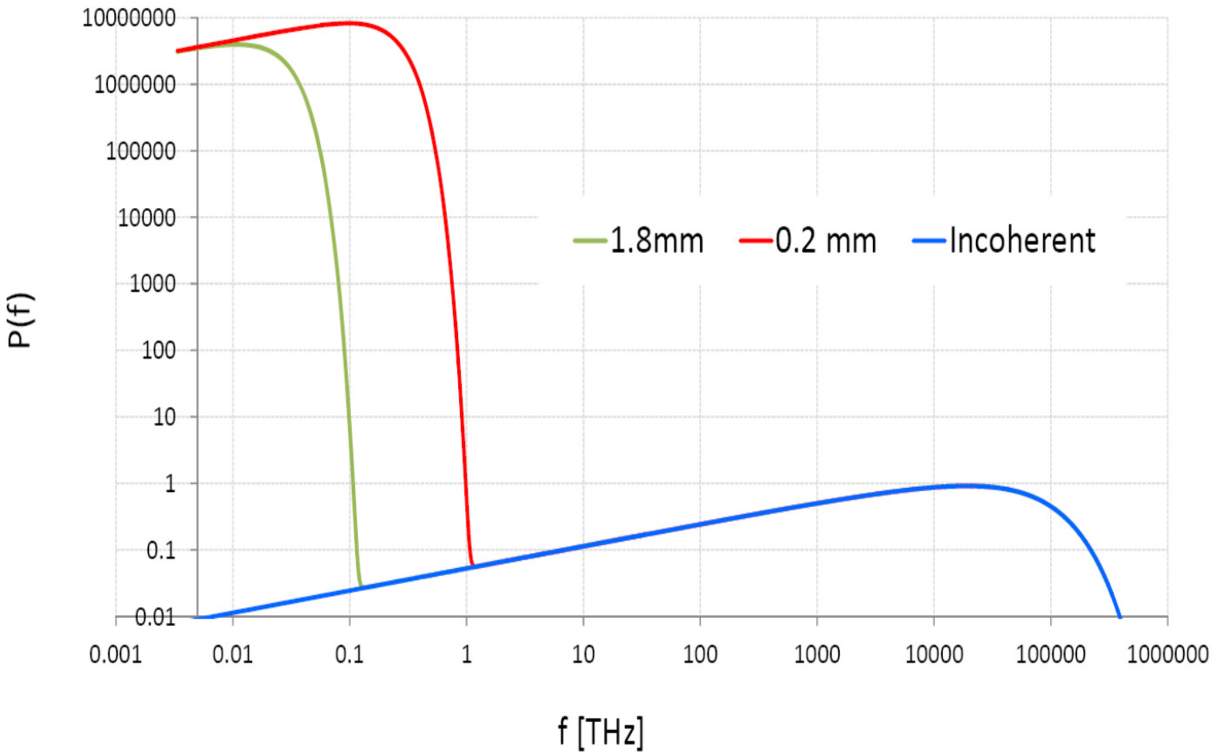


We had the opportunity to observe experimentally this strong dependence of the energy during the SLAC B-factory operations. During the 2008 energy scan to study 2S, 3S and above 4S resonance, we need to change the energy of HER (High Energy Ring) from 8 GeV to 10 GeV. Simultaneously we measured the energy loss due to synchrotron radiation (incoherent radiation, linear with the beam current) and coherent radiation (quadratic with the beam current) of wake fields. Here is a plot of an energy loss per turn for incoherent radiation. A fit of the experimental data points with a power function gives an exact fourth order dependence.

Why Coherent Radiation?



Because a number of particles even in a low charged bunch is usually more than hundred of millions

CSR simulations

The most popular method is to use integration of the Lienard-Wiechert potentials. Another approach is to numerically solve the approximate equations, which are a Schrodinger type equation

- R. Li. Nucl. Instrum. Meth. Phys. Res. A, 429, 310, 1998.
- M. Borland, Phys. Rev. ST Accel. Beams 4, 070701 (2001).
- T.Agoh and K.Yokoya, Phys. Rev. ST Accel. Beams, 7, 054403 (2004).
- G. Bassi et al., Nuc. Instrum. Methods Phys. Res. A, 557, pp. 189–204 (2006).
- …

We suggest to use Maxwell's and Newton's equations for CSR calculations

$$\nabla \times \mathbf{H} = J + \frac{\partial D}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot D = \rho.$$

$$\vec{J} = \sum_k e\vec{v}_k$$

$$\frac{d\vec{p}}{dt} = \vec{F} = e\vec{E} + \frac{e}{c}\left[\vec{v} \times \vec{H}\right]$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2}}$$

Maxwell's equations

Newton's equations

Main question:

How to solve Maxwell's equations?

Answer:

To use the Implicit algorithm, same algorithm that we have used for wake field calculations of very short bunches

Explicit and implicit finite difference schemes. 2-D example.

$$\frac{\partial^2\Phi}{c^2\partial t^2}=\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Phi}{\partial x^2}$$

Explicit

$$\Phi^{n+1}-2\Phi^n+\Phi^{n-1}=\left(\frac{c\Delta t}{\Delta z}\right)^2\Delta_z^2\Phi^n+\left(\frac{c\Delta t}{\Delta x}\right)^2\Delta_x^2\Phi^n$$

Implicit

$$\Phi^{n+1}-2\Phi^n+\Phi^{n-1}=\left(\frac{c\Delta t}{\Delta z}\right)^2\Delta_z^2\Phi^n+\left(\frac{c\Delta t}{\Delta x}\right)^2\Delta_x^2\frac{\Phi^{n+1}+\Phi^{n-1}}{2}$$

Dispersion

$$\sin^2\frac{\omega\Delta t}{2}=\left(\frac{c\,\Delta t}{\Delta z}\right)^2\times\sin^2\frac{\beta\,\Delta z}{2}+\left(\frac{c\,\Delta t}{\Delta x}\right)^2\times\sin^2\frac{\alpha\,\Delta x}{2}$$

$$\sin^2\frac{\omega\Delta t}{2}=\frac{\left(\frac{c\Delta t}{\Delta z}\right)^2\sin^2\beta\frac{\Delta z}{2}+\left(\frac{c\Delta t}{\Delta x}\right)^2\sin^2\alpha\frac{\Delta x}{2}}{1+2\left(\frac{c\Delta t}{\Delta x}\right)^2\sin^2\alpha\frac{\Delta x}{2}}$$

$$V_{wave}=\frac{\omega}{\beta}=c\frac{2}{\beta c\Delta t}\times\arcsin\left(\frac{c\Delta t}{\Delta z}\sin\frac{\beta\Delta z}{2}\right)$$

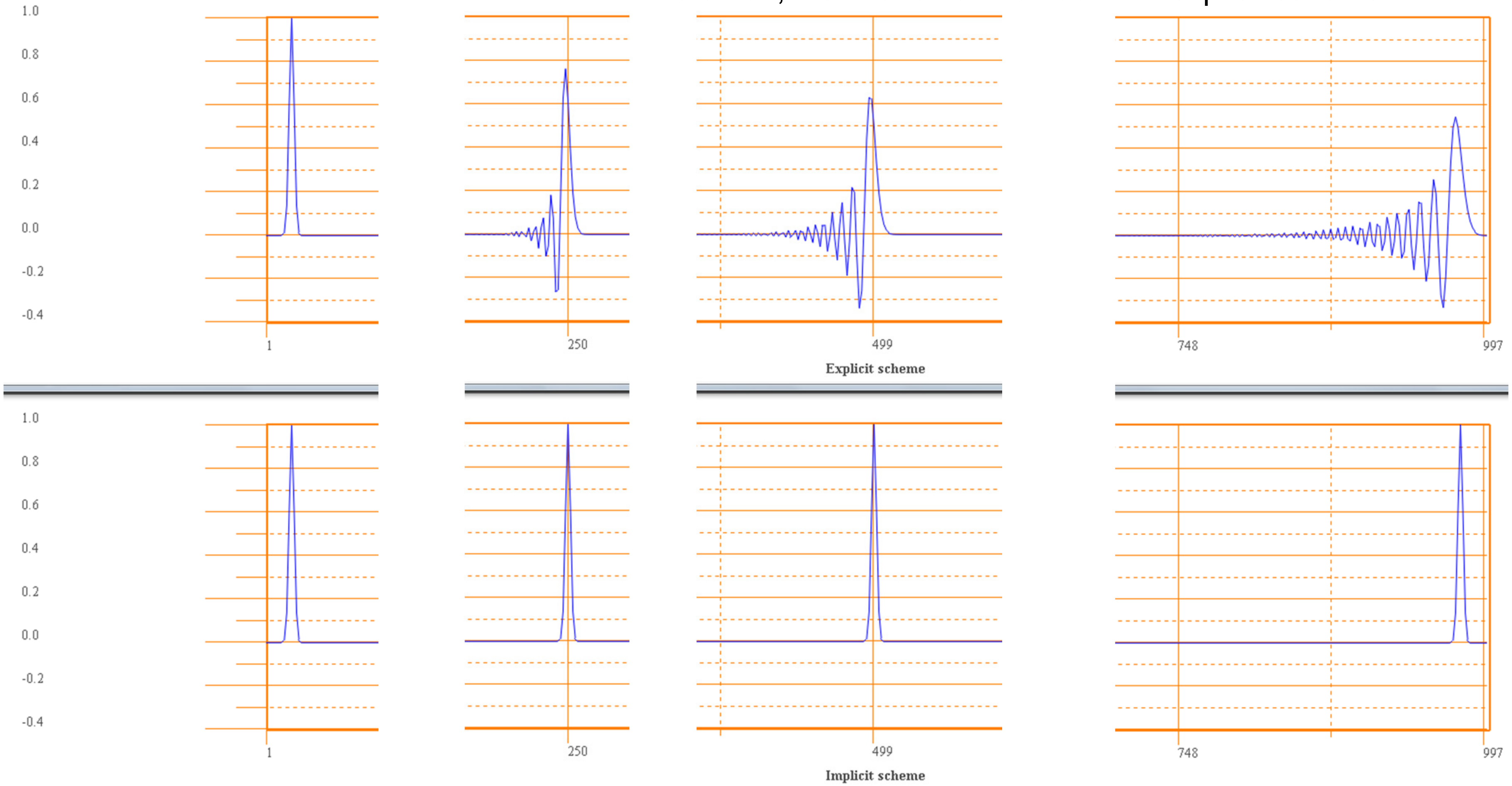
Stability

$$\left(\frac{c\Delta t}{\Delta z}\right)^2+\left(\frac{c\Delta t}{\Delta x}\right)^2\le 1 \quad \text{for } \Delta x=\Delta z \quad c\Delta t\le\frac{1}{\sqrt{2}}\Delta z$$

Any step size is OK

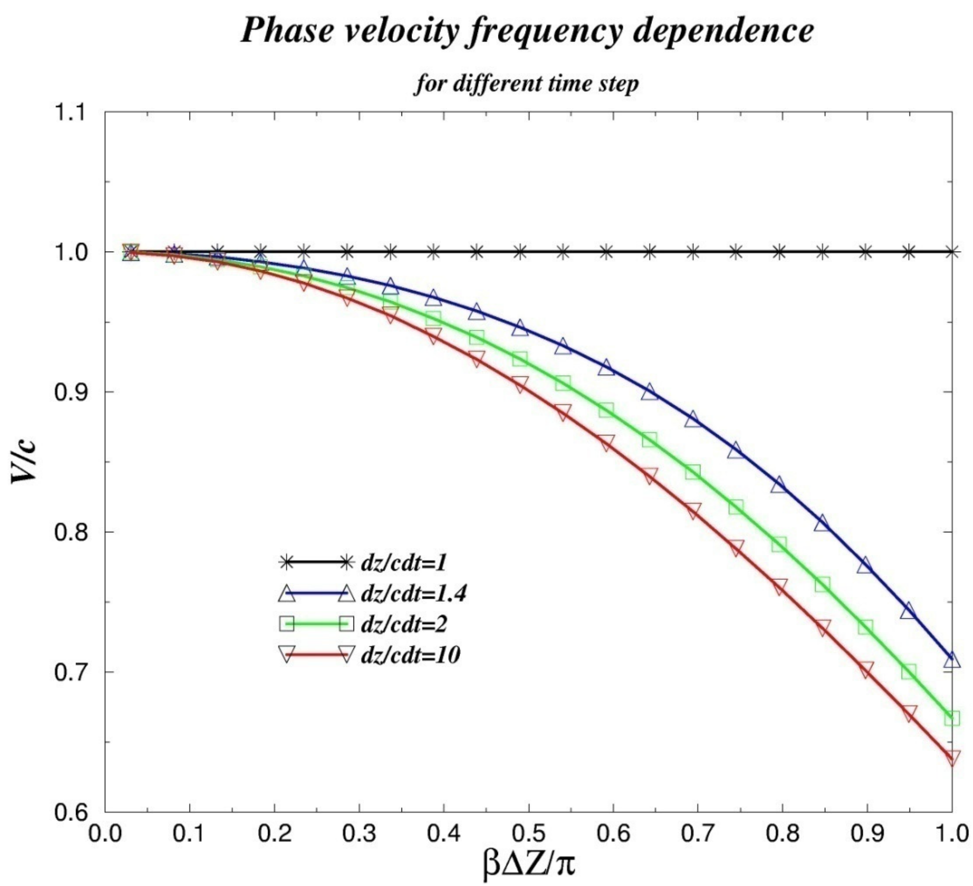
Implicit scheme as opposed to explicit for the case of a free propagating bucket

Distortion, modulation and diffusion in explicit scheme



Bucket length is equal to two mesh steps

Dispersion of a finite-difference scheme



Implicit scheme is always stable

Unstable condition for explicit scheme

Stable condition for explicit scheme creates dispersion

First wake field calculations using implicit scheme, 1976 (36 years ago)

An implicit algorithm has been used in the computer code designed in 1976 for wake field dynamics studies at the Novosibirsk Electron-Positron Linear Collider VLEPP (σ=1.8 mm).

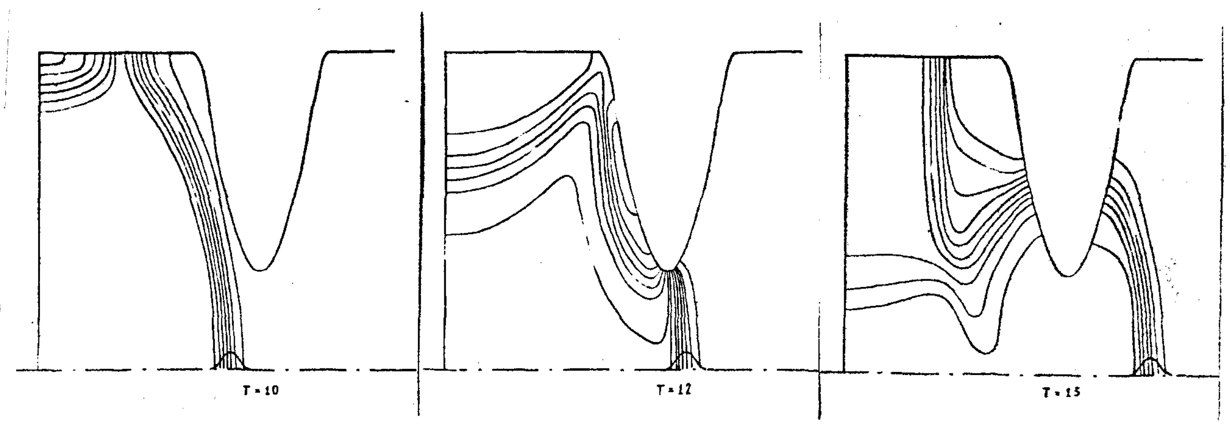


Fig. 1.

SLAC TRANS - /88

TRANSLATION

TITLE : – as translated into ENGLISH

BEAM DYNAMICS OF A COLLIDING LINEAR ELECTRON-POSITRON BEAM (VLEPP)

– as translated from RUSSIAN

ДИНАМИКА ПУЧКА ВЛЭППа

AUTHOR/S/ : V.E. BALAKIN, I.A. KOOP, A.V. NOVOKHATSKIĬ, A.N. SKRINSKIĬ, V.P. SMIRNOV

SOURCE : A 5-PAGE TYPESCRIPT.

Translated November 1978

for Stanford Linear Accelerator Center by AD-EX®

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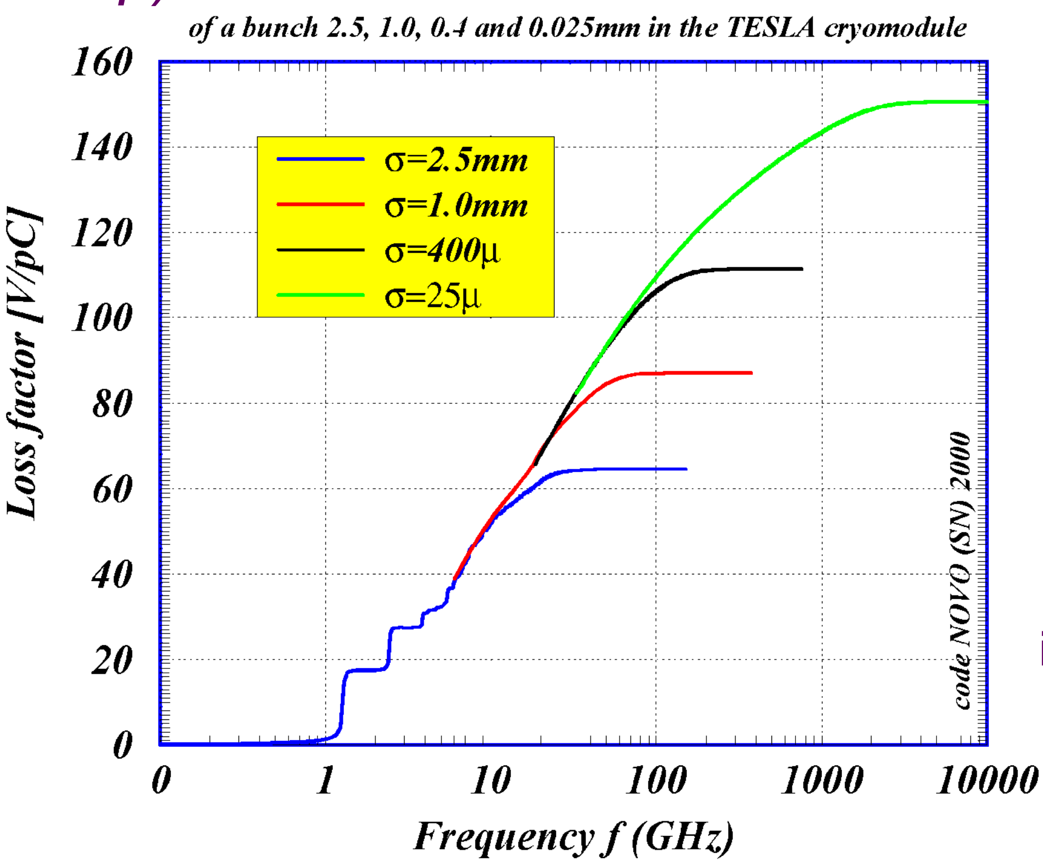
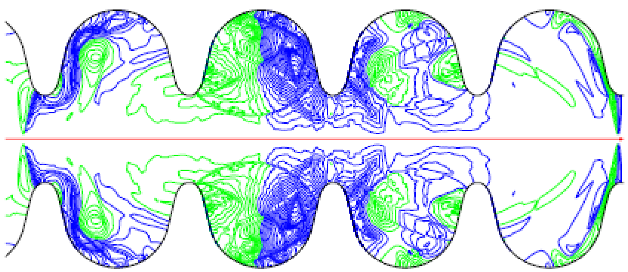
The code played a very important role in the discovery of BNS-damping.

Wake field calculations for a TESLA cryo-module, 1996

The code was used for calculating wake fields of very short bunches at the TESLA Linear Collider (σ~0.7mm) and TTF FEL (σ~25 μ).

Loss factor frequency integral





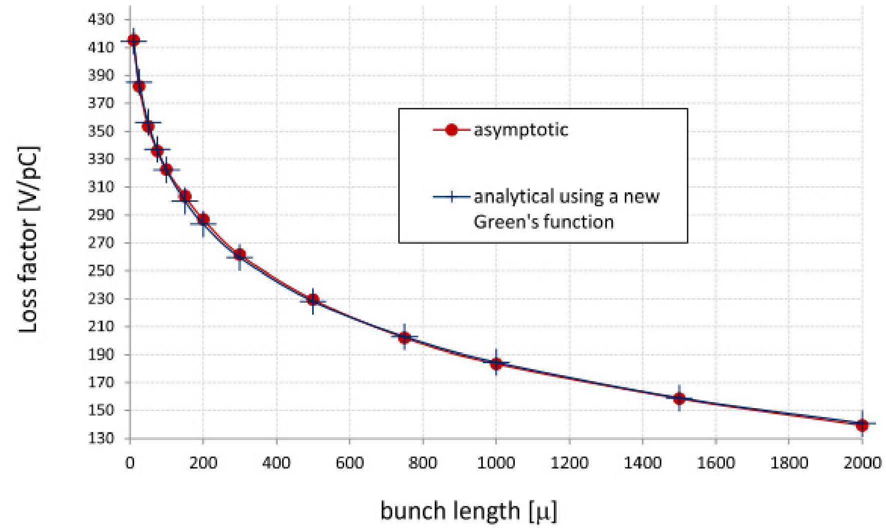
These calculations were important to prove the feasibility of the project.

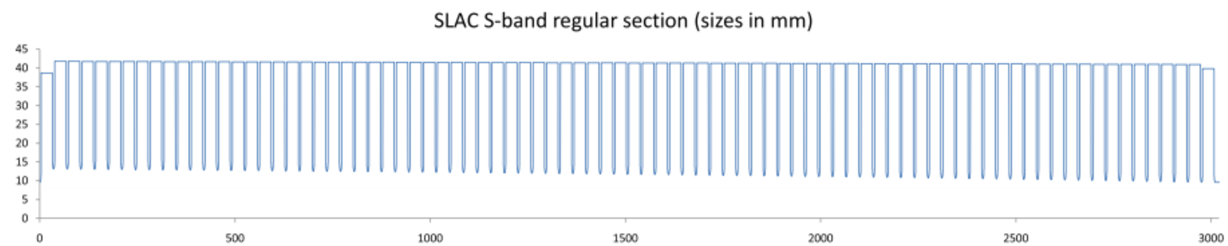
A new formula for the Green's function of the wake potential of the SLAC accelerating section

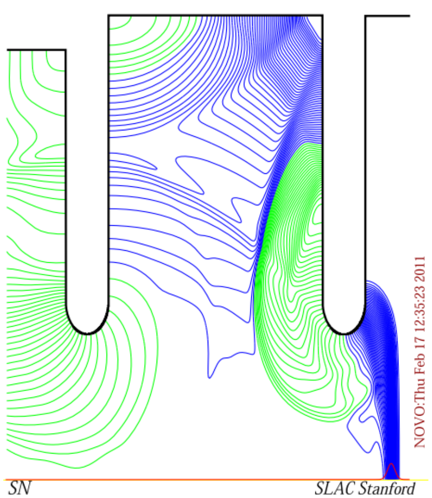
- Wake field calculations of the SLAC accelerating sections (6X3m=18m) for a bunch length of 10 micron.
- While analyzing the computational results we have found a new formula for the Green's function.

$$G_p(s) = \frac{Z_0 c}{\pi a^2}\left(1+\frac{s}{4s_0}\right)\exp\left(-\sqrt{\frac{s}{s_0}}\right)$$

a=0.01068 m, ***s0*** =0.435 mm





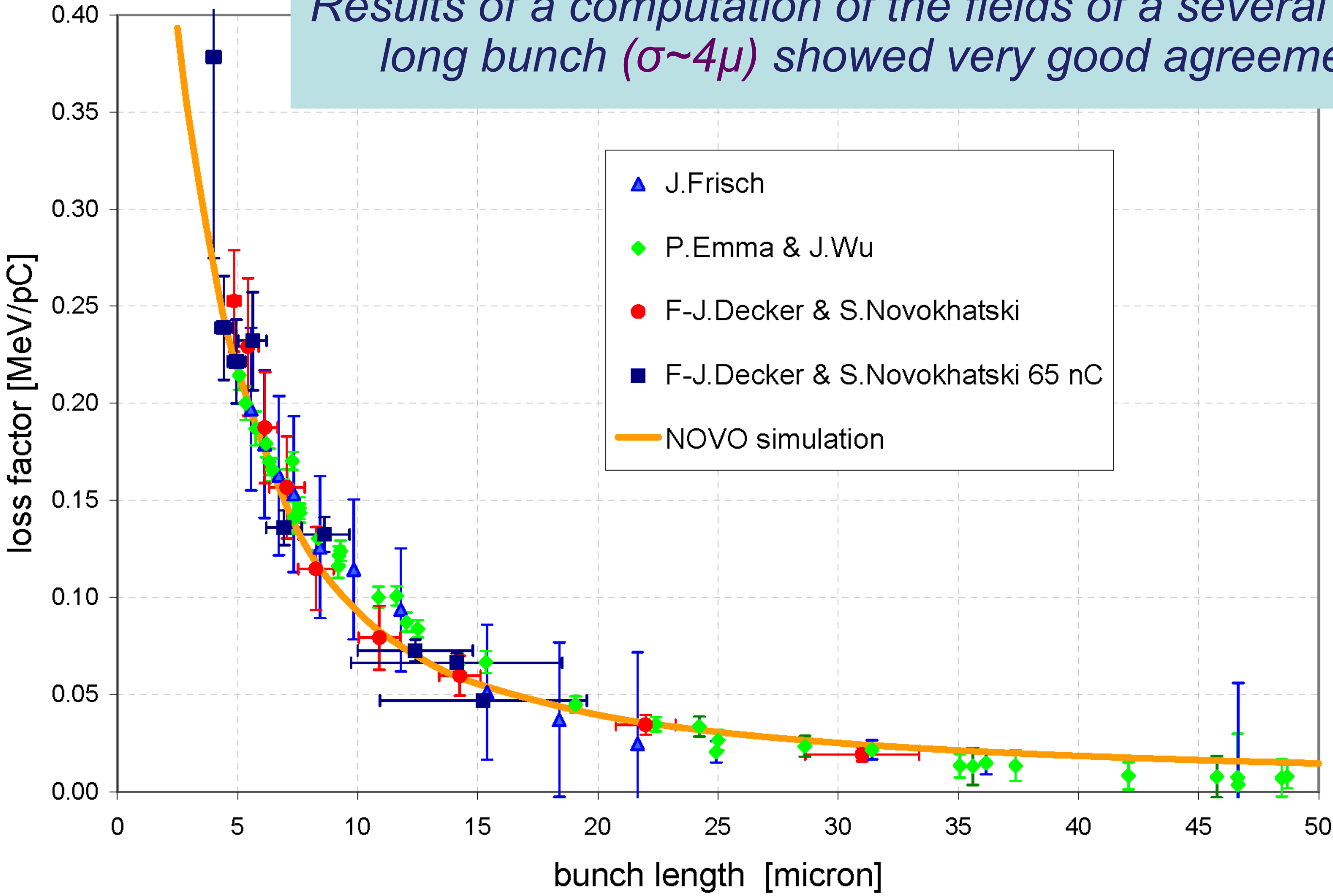


Electric field lines of the wake field excited by a short bunch in a SLAC accelerating section. The bunch has passed the two first disks of a section. The blue lines show field lines with the longitudinal projection opposite to the bunch velocity. These lines describe the deceleration forces and the green lines describe the acceleration forces.

Comparison with wake field measurements

Recently we got an opportunity to make a comparison with wake field measurements at LCLS.

Results of a computation of the fields of a several micron long bunch (σ~4μ) showed very good agreement.



Creating a prime algorithm to study CSR

I. Fourier expansion in vertical direction

Assuming that there is no beam motion in the vertical direction and the beam chamber has a constant vertical size)

II. Second order field equation

III. Implicit algorithm

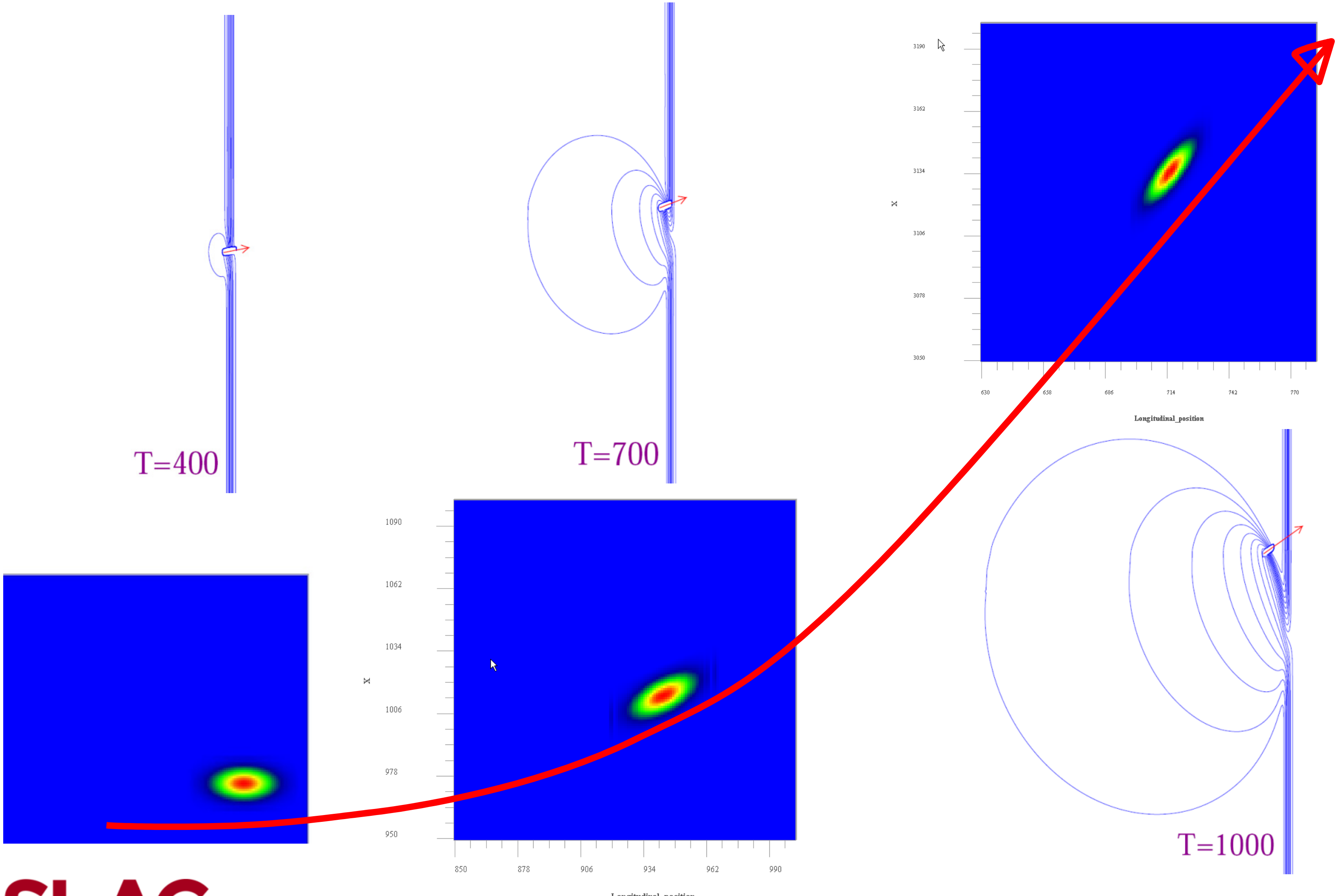
IV. Moving mesh

V. Particle equation

We need many particles to have a smooth charge distribution of a bunch

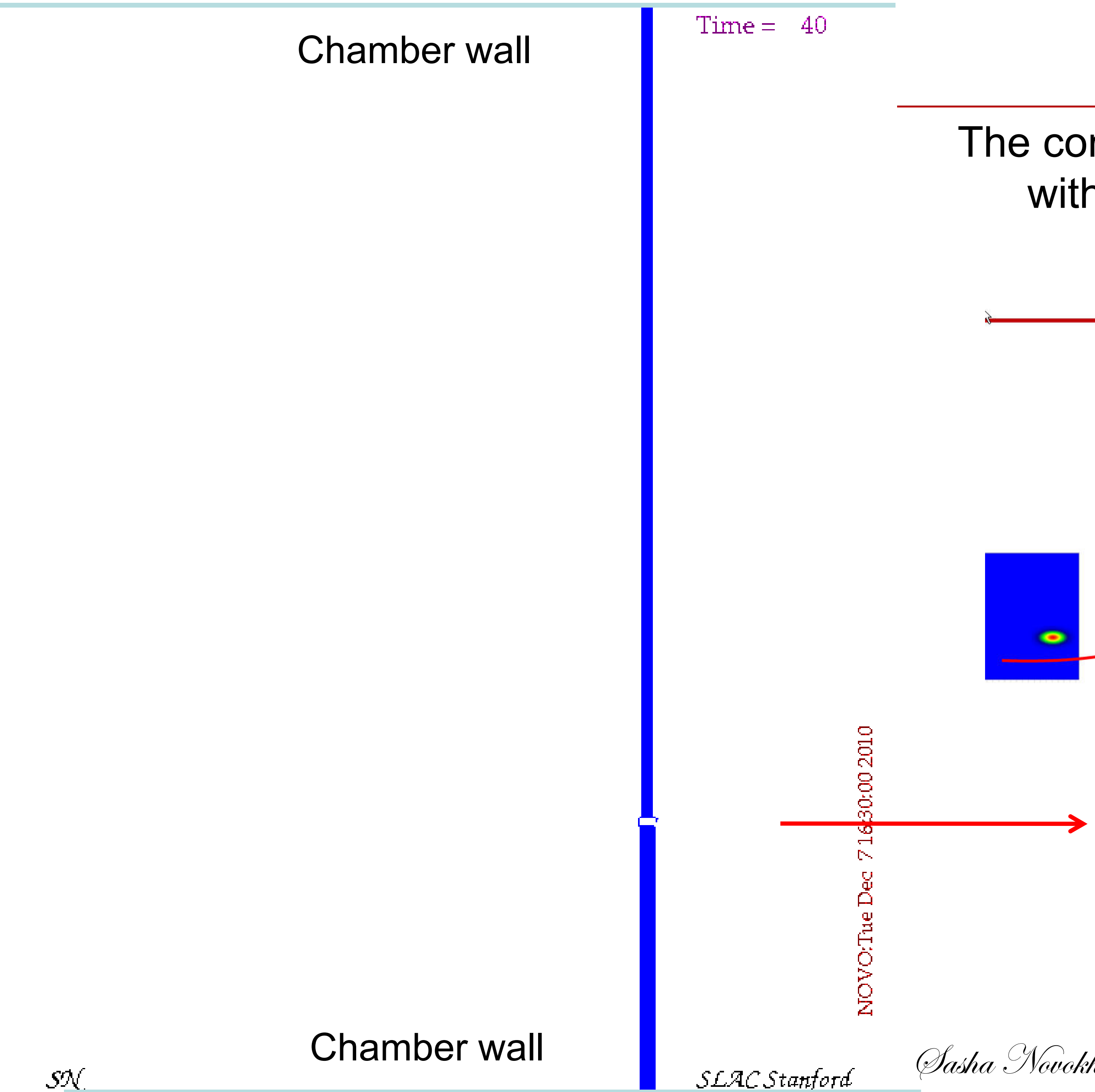
VI. Parallel code*

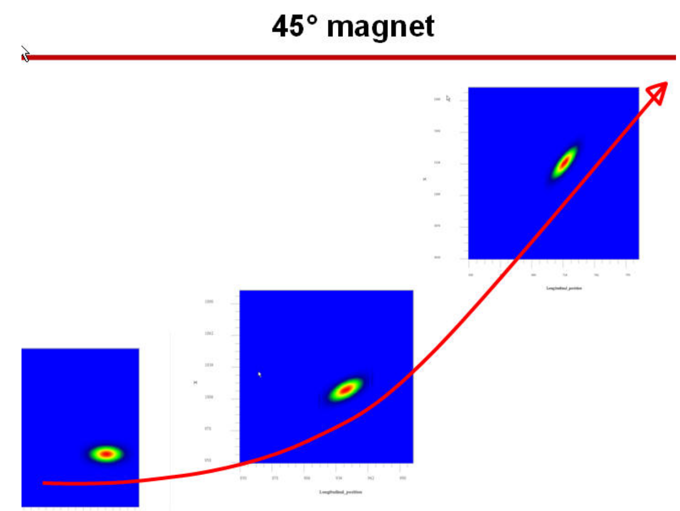
Field dynamics in a 45   magnet



Sasha Novokhatski August 21, 2012

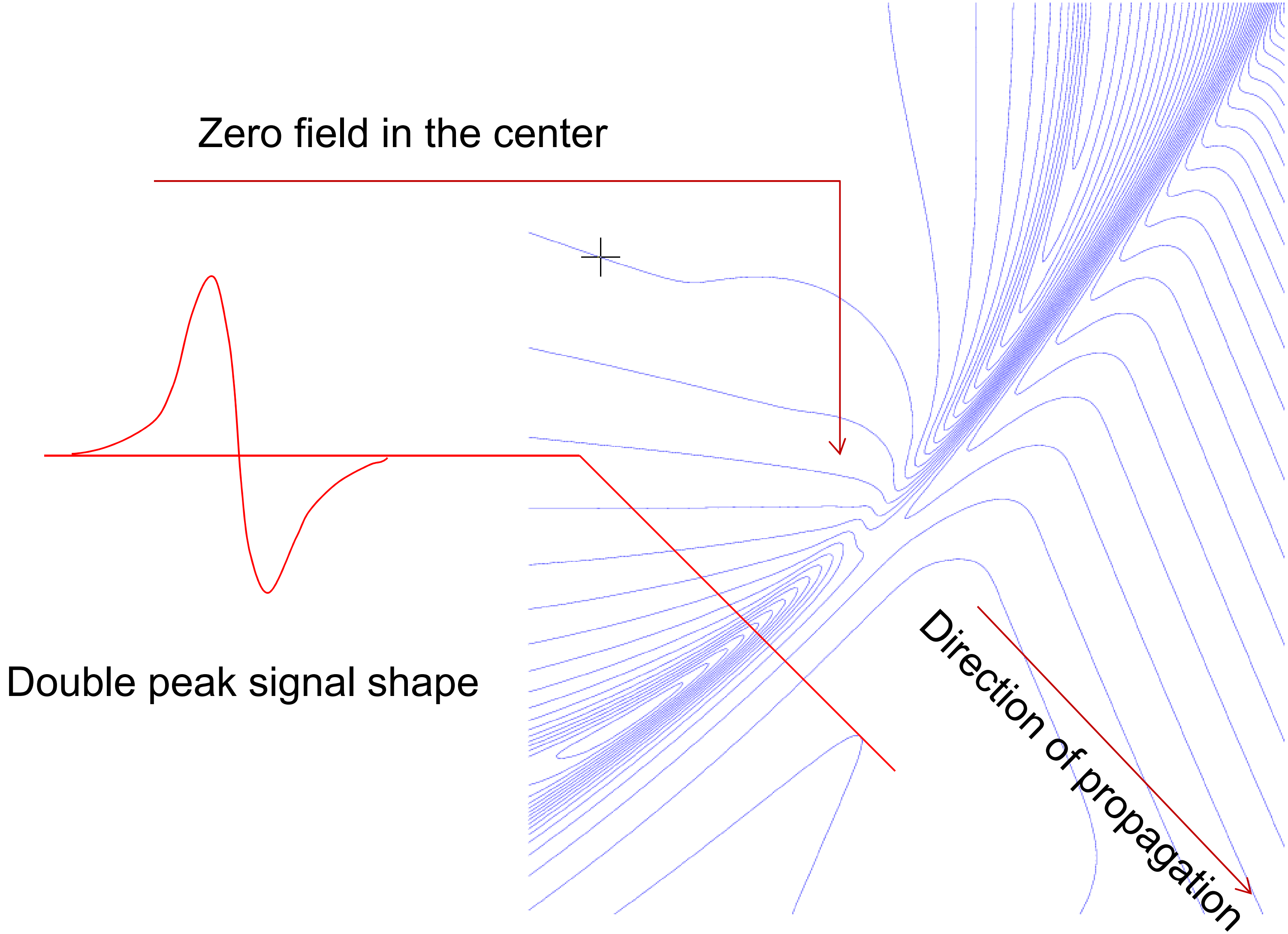
The computer mesh moves with a speed of light



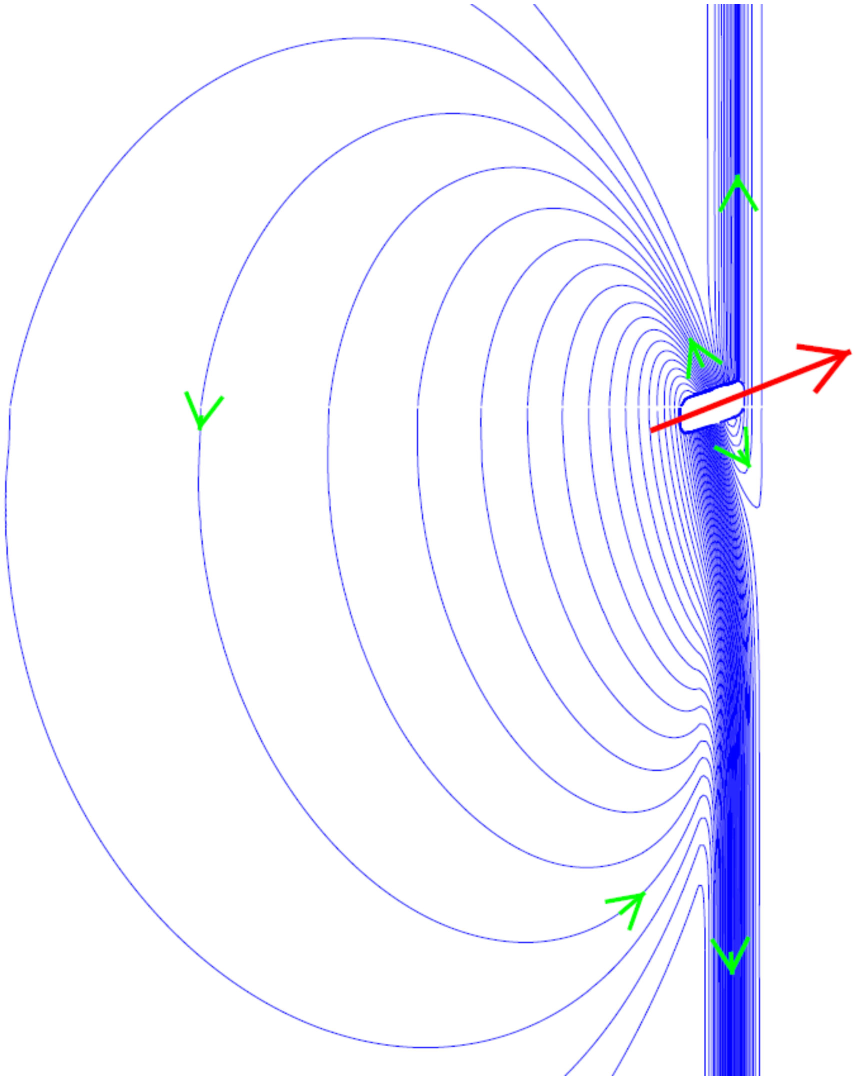


Initial direction

Transition radiation. For free.



Bunch self-field remakes itself



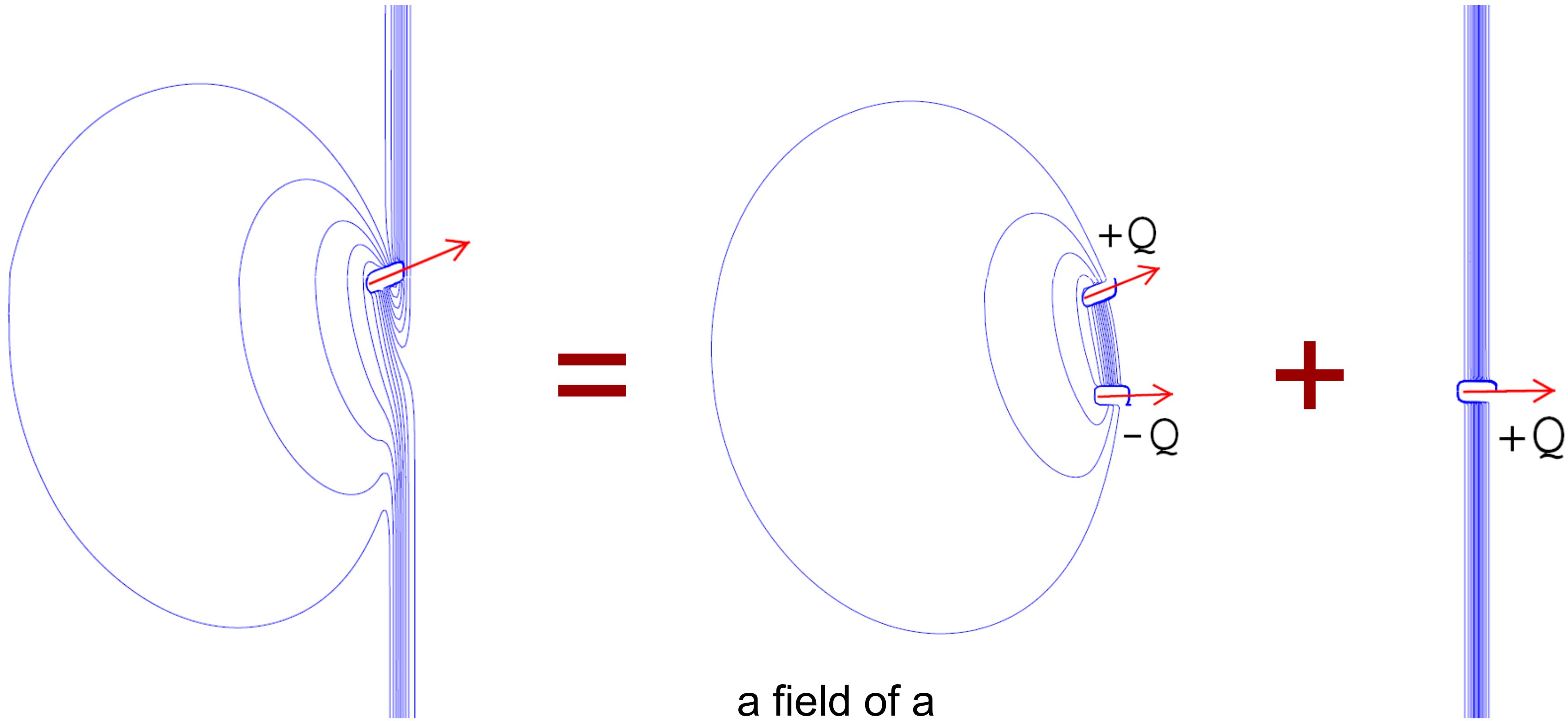
The upper field lines take the position of the lower lines

A red arrow shows a bunch velocity vector

Green arrows show field line directions

The lower field lines take the position of the upper lines

The picture becomes clear if we decompose the field in two parts



a field of a moving dipole

Decomposition of the field of a bunch moving in a magnetic field into two fields:

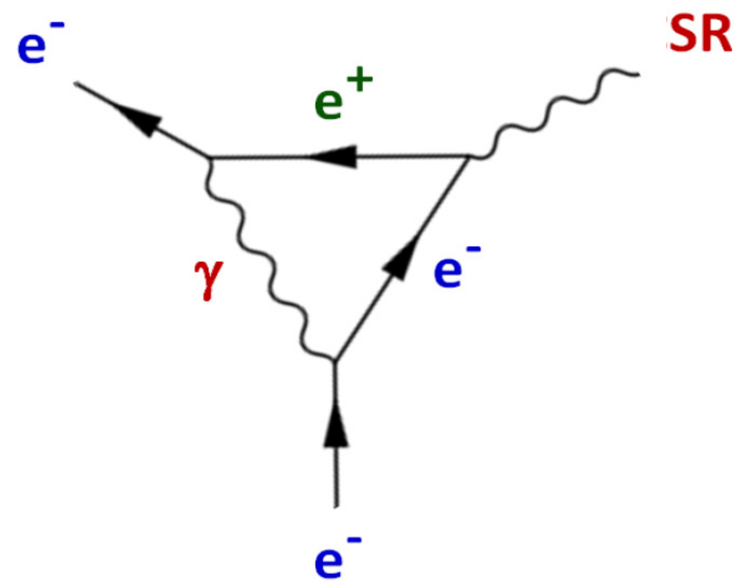
a field of a bunch moving straight in initial direction

Decomposition as an analog the Feynman diagram

There could be a close analogy between the field decomposition and the Feynman diagram.

A real electron produces a virtual photon, which decays into electron-positron pair, corresponding to a dipole.

The positron can annihilate with the ongoing scatted electron to emit a photon. This photon corresponds to synchrotron radiation.



An absolute dipole electric field in time

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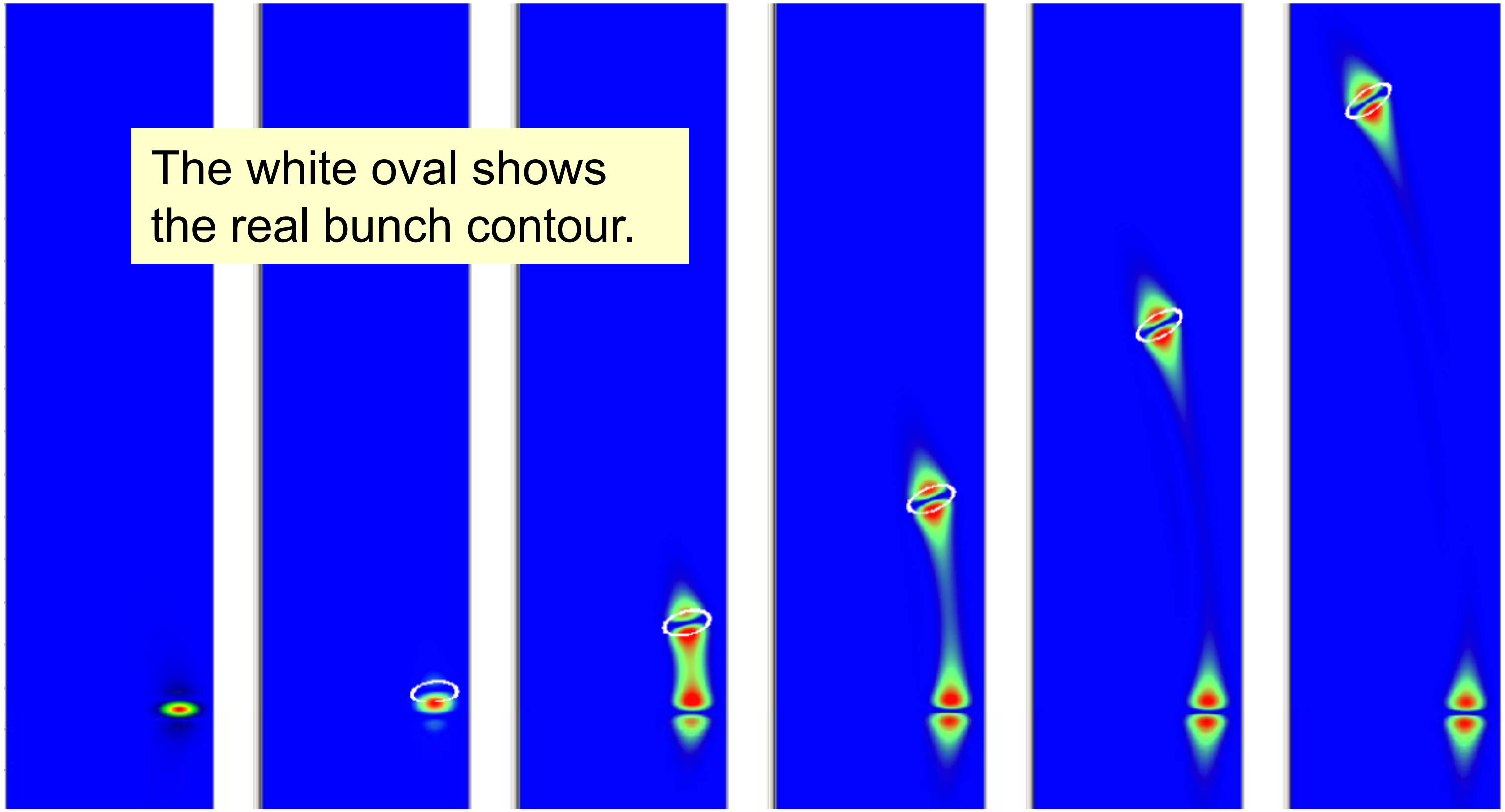
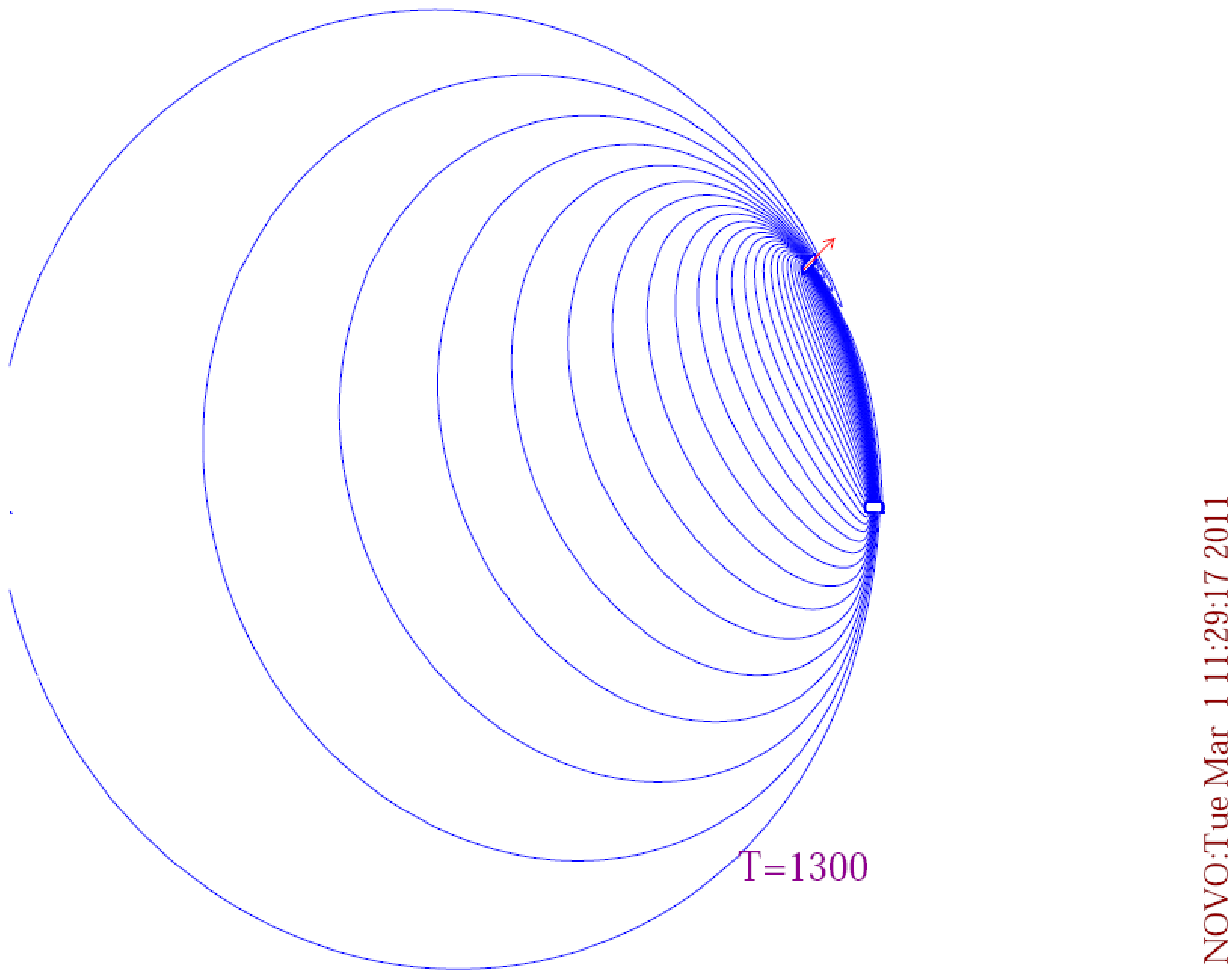
The white oval shows
the real bunch contour.



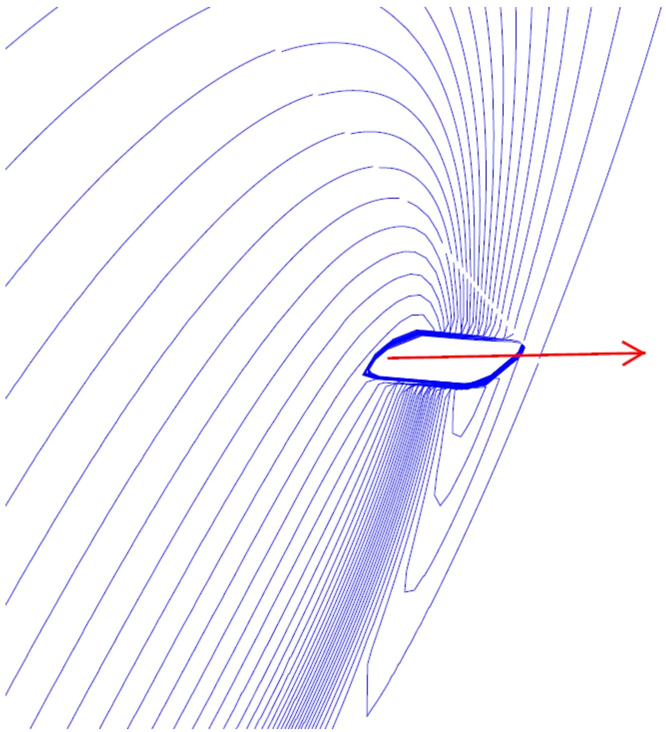
When a dipole is created an electric field appears between a real bunch and a virtual bunch. This field increases in value and reaches a maximum value when the bunches are completely separated and then it goes down as the bunches more apart leaving fields only around the bunches.

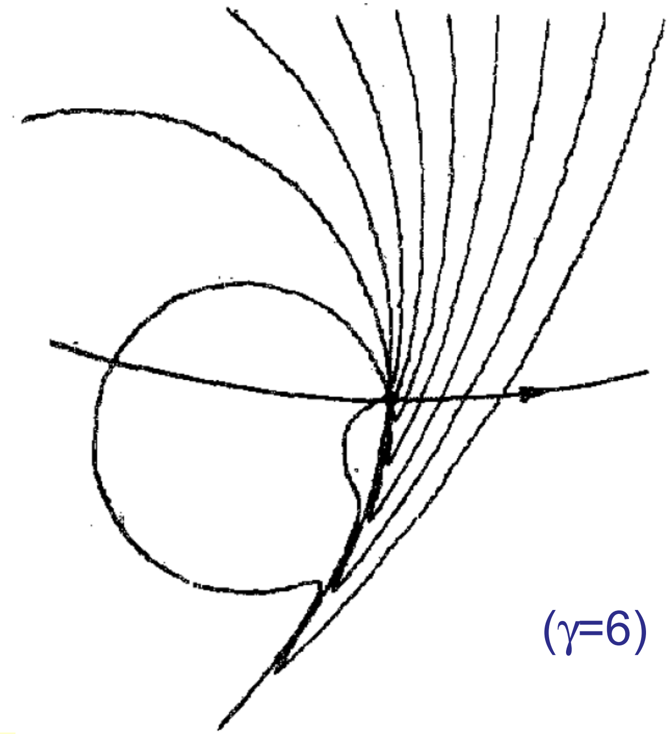
Detailed plot of a dipole field



Comparison with a classical synchrotron radiation

γ - region in front of a particle





(γ=6)

equivalent bunch length

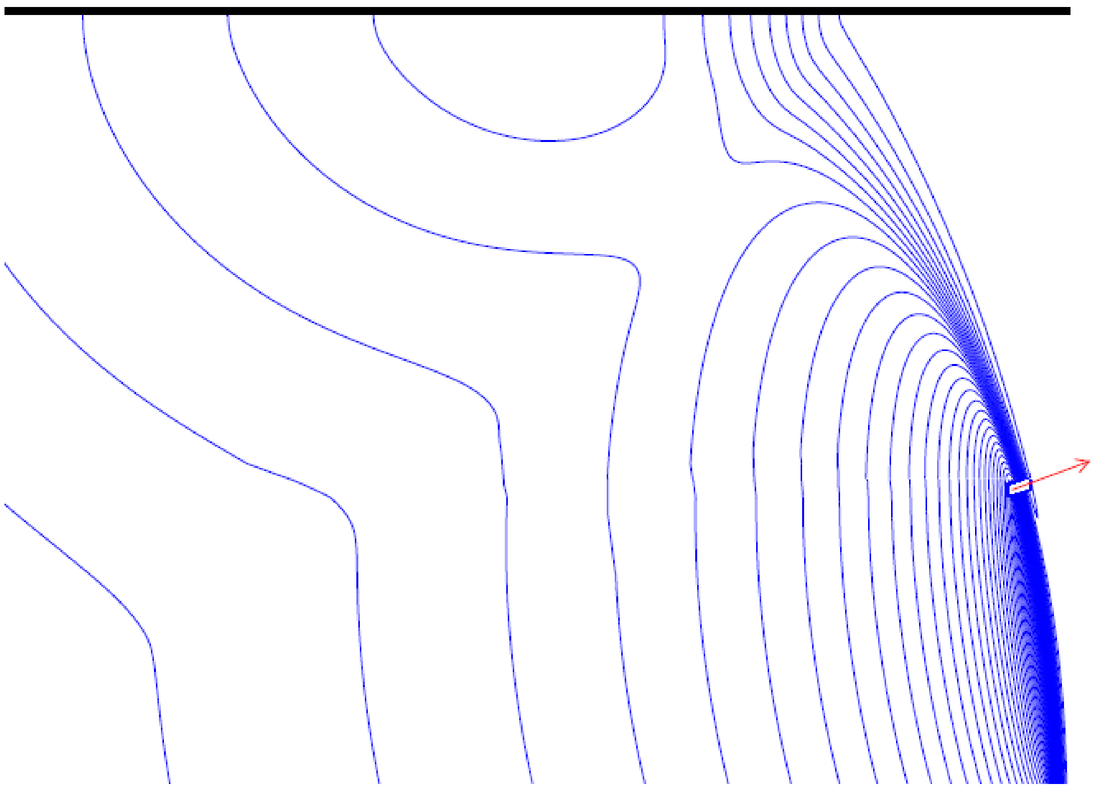
for a bending radius ρ

$$\sigma = \frac{2}{3}\frac{\rho}{\gamma^3}$$

FORCE LINES OF ELECTRIC AND MAGNETIC FIELDS
OF AN ARBITRARILY MOVING CHARGE

S. G. Arutyunyan

A new bunch field



A long way to a steady-state regime

Electrical forces inside a bunch

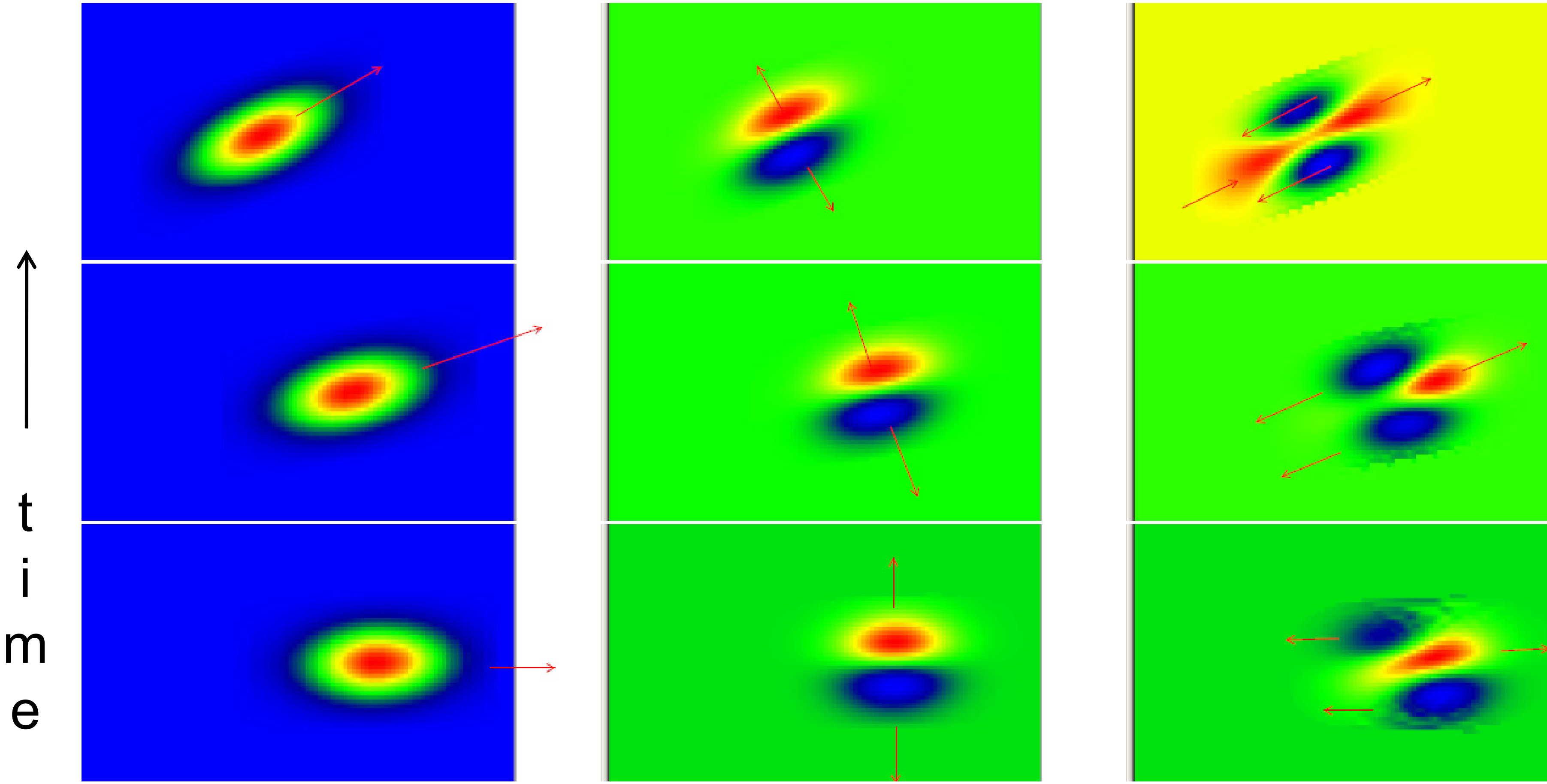
Bunch shape

transverse force

$$\vec{F}_{\perp} = \frac{\vec{J}}{|\vec{J}|} \times \left[\vec{E} \times \vec{J} \right]$$

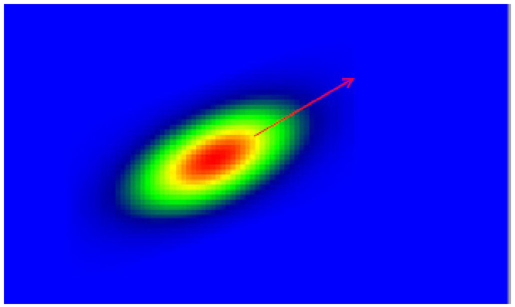
collinear force

$$F_{\parallel}^{e} = \left(\vec{J} \cdot \vec{E} \right)$$



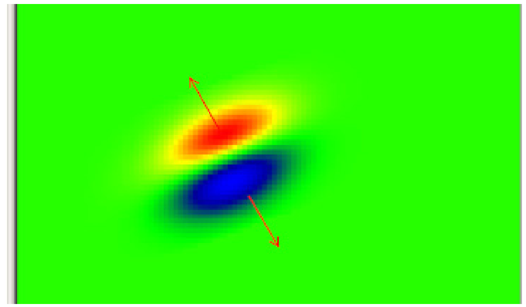
Electrical forces inside a bunch

Bunch shape



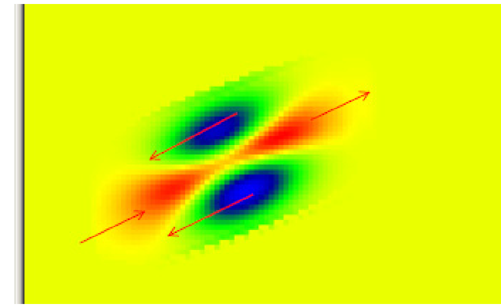
transverse force

$$\vec{F}_{\perp} = \frac{\vec{J}}{|\vec{J}|} \times \left[\vec{E} \times \vec{J} \right]$$



collinear force

$$F_{\parallel}^{e} = \left(\vec{J} \cdot \vec{E} \right)$$



The transverse force is the well known space-charge force, which probably is compensated by a magnetic force in the ultra-relativistic case.

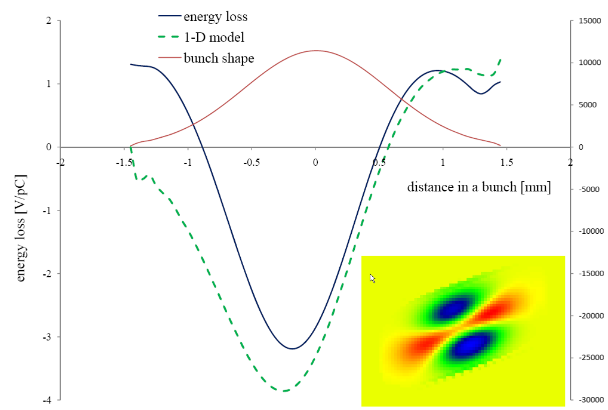
The collinear force is responsible for an energy gain or an energy loss. The particles, which are in the center, in front and at the end of the bunch are accelerating, whereas the particles at the boundaries are decelerating.

The total effect is deceleration and the bunch loses energy, however the bunch gets an additional energy spread in the transverse direction.

A new effect

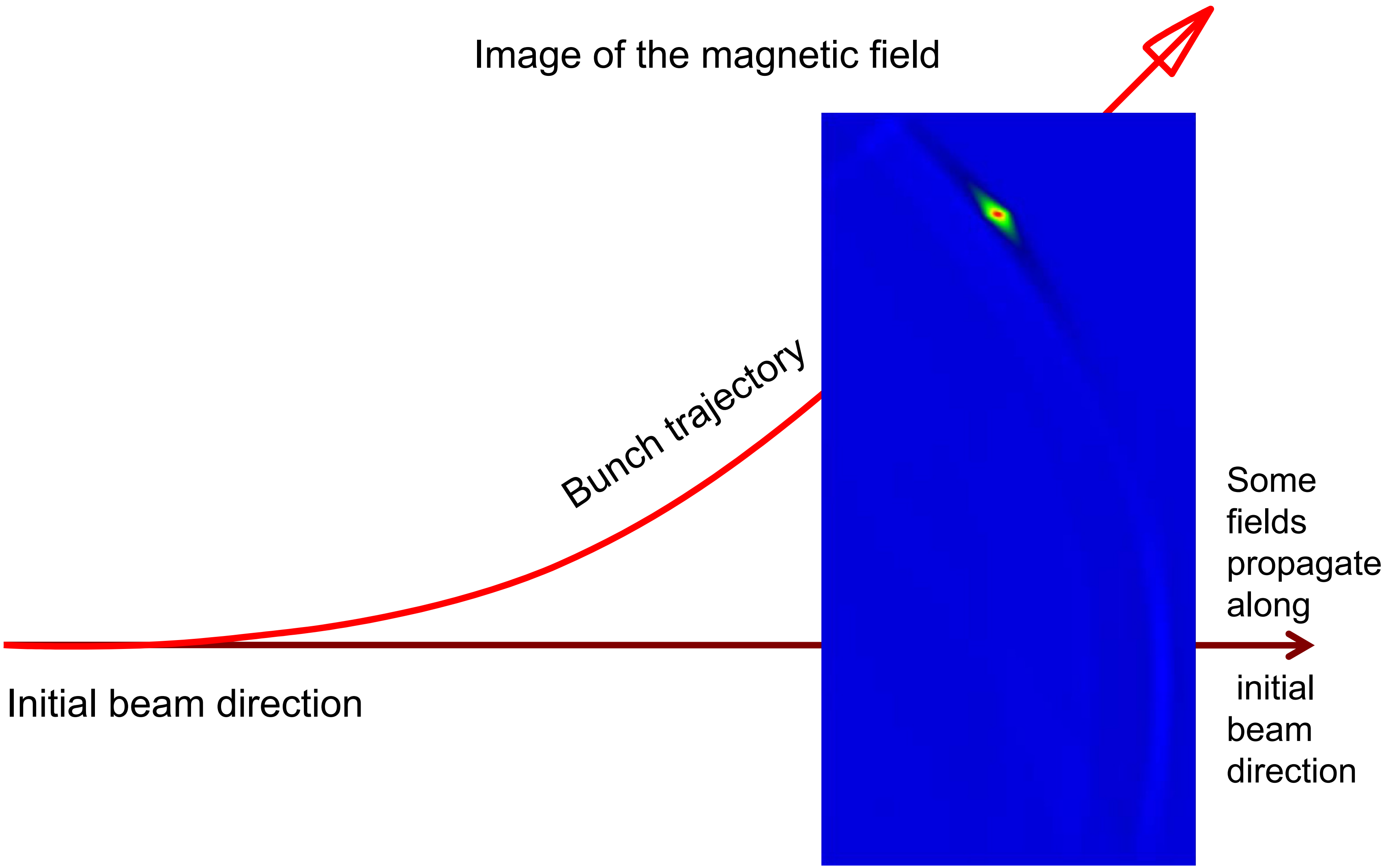
- This complicated structure of the collinear field is very important.
- A bunch will get an additional transverse energy spread, which may not be compensated.
- This energy spread in the magnetic field immediately generates emittance growth.

Integrated in X

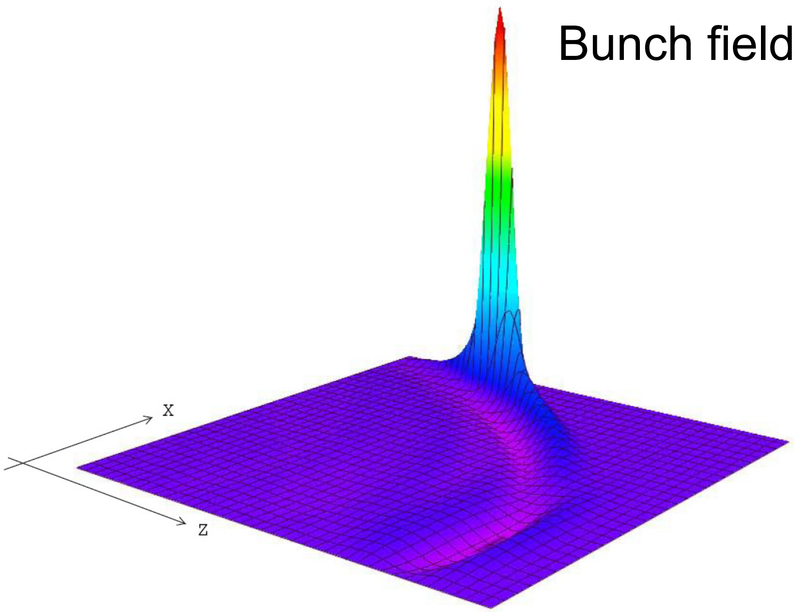


- Transverse energy spread may increase decoherence .
- This effect can limit the efficiency of the magnetic bunch compressors and as a result the efficiency of FELs.

Coherent edge radiation?

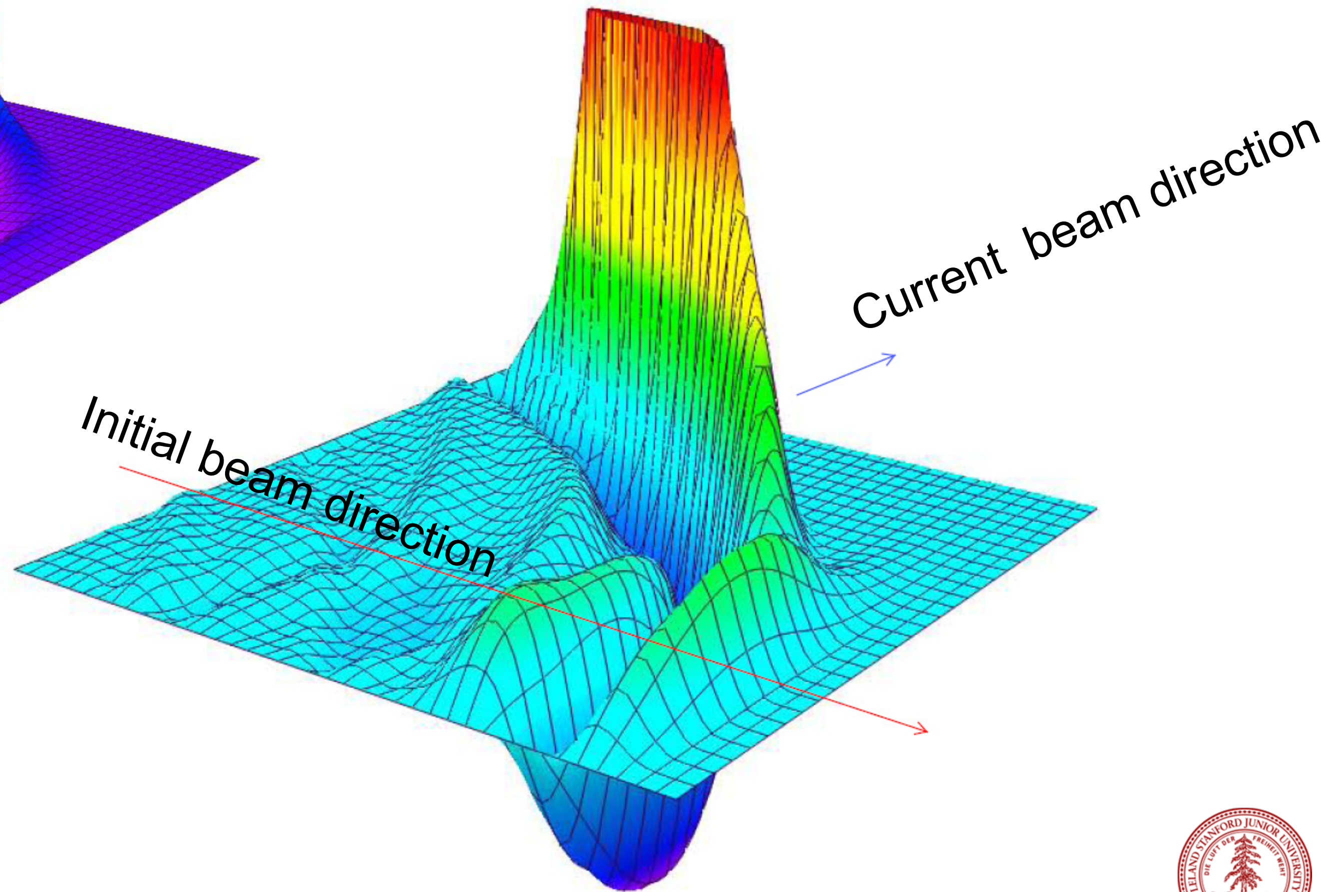


Magnetic field plots



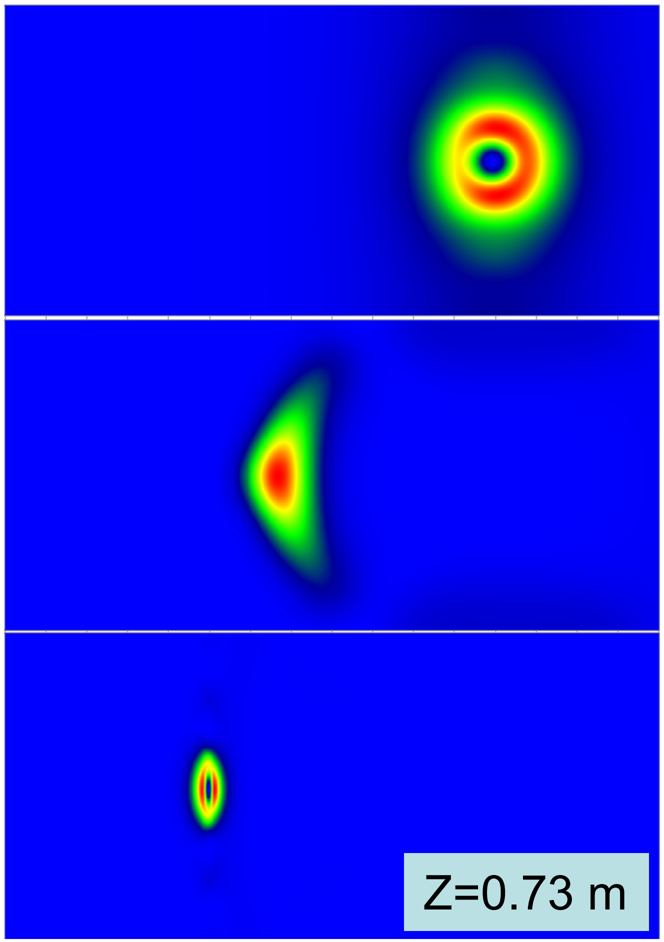
Bunch field

Magnified



Images of radiation (transverse magnetic field)

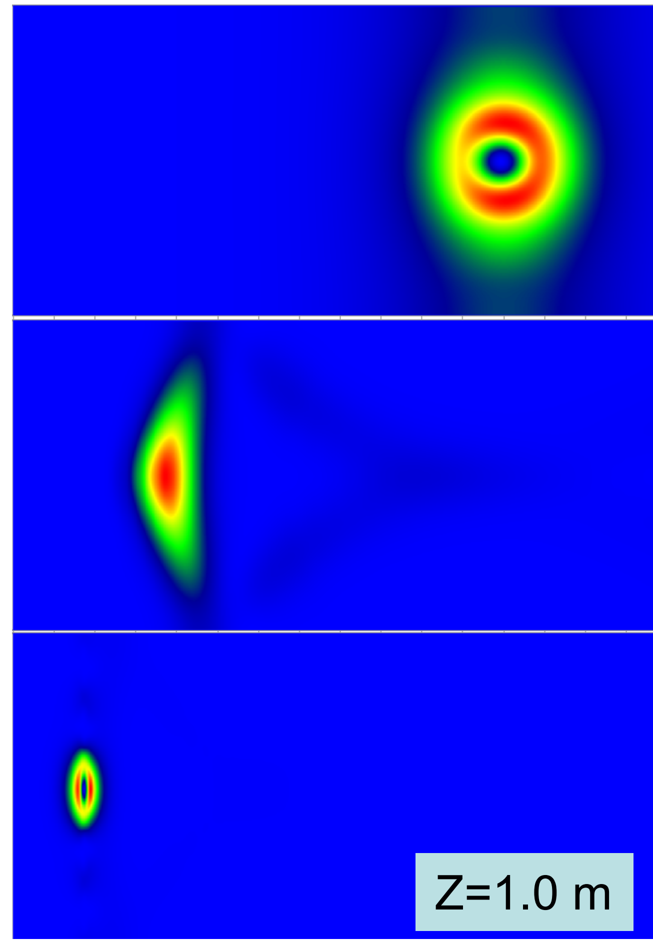
very similar to the images, which we have seen on the YAG screen after the dump magnets, which bend down the beam at LCLS.



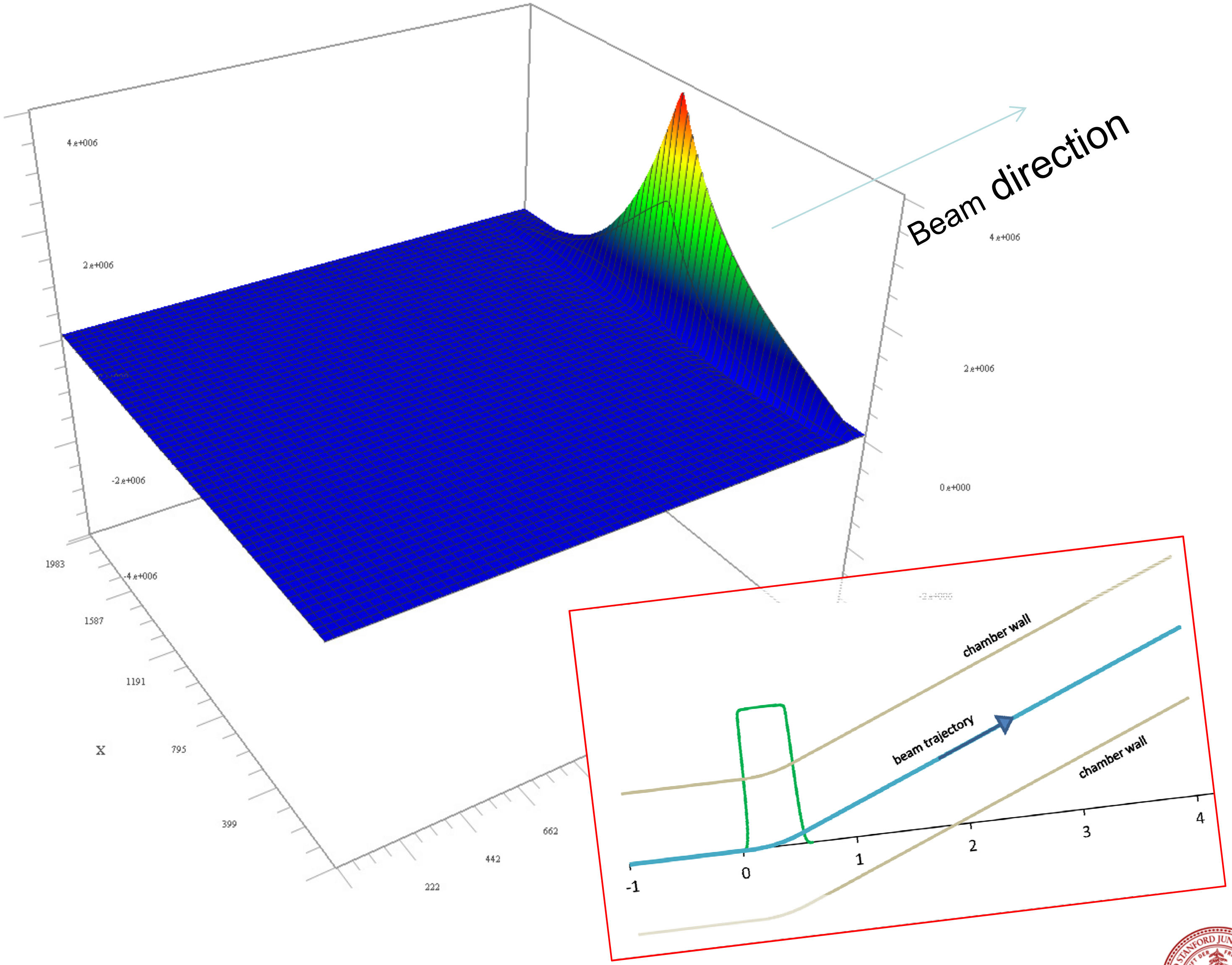
Edge radiation

Synchrotron radiation

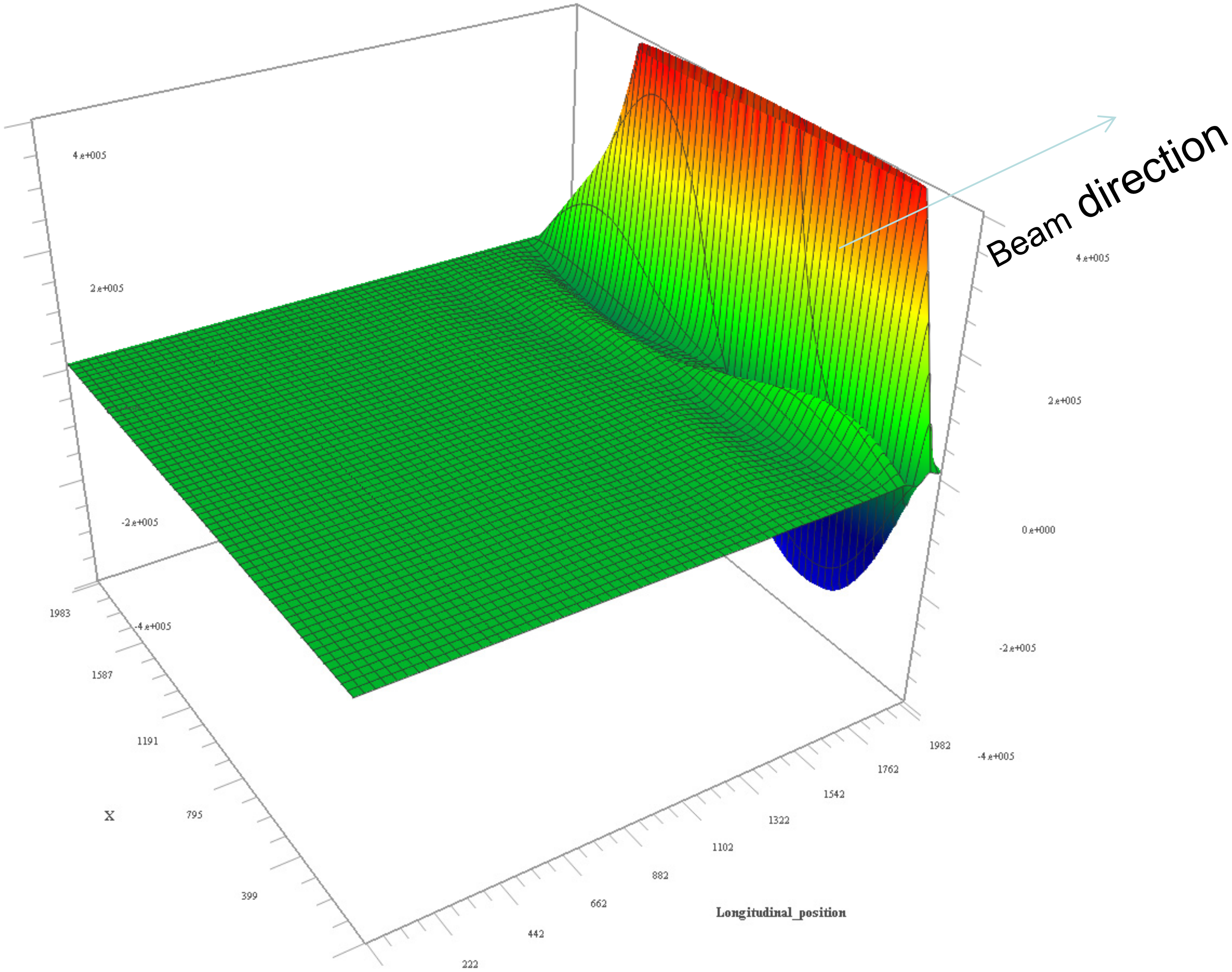
Bunch field



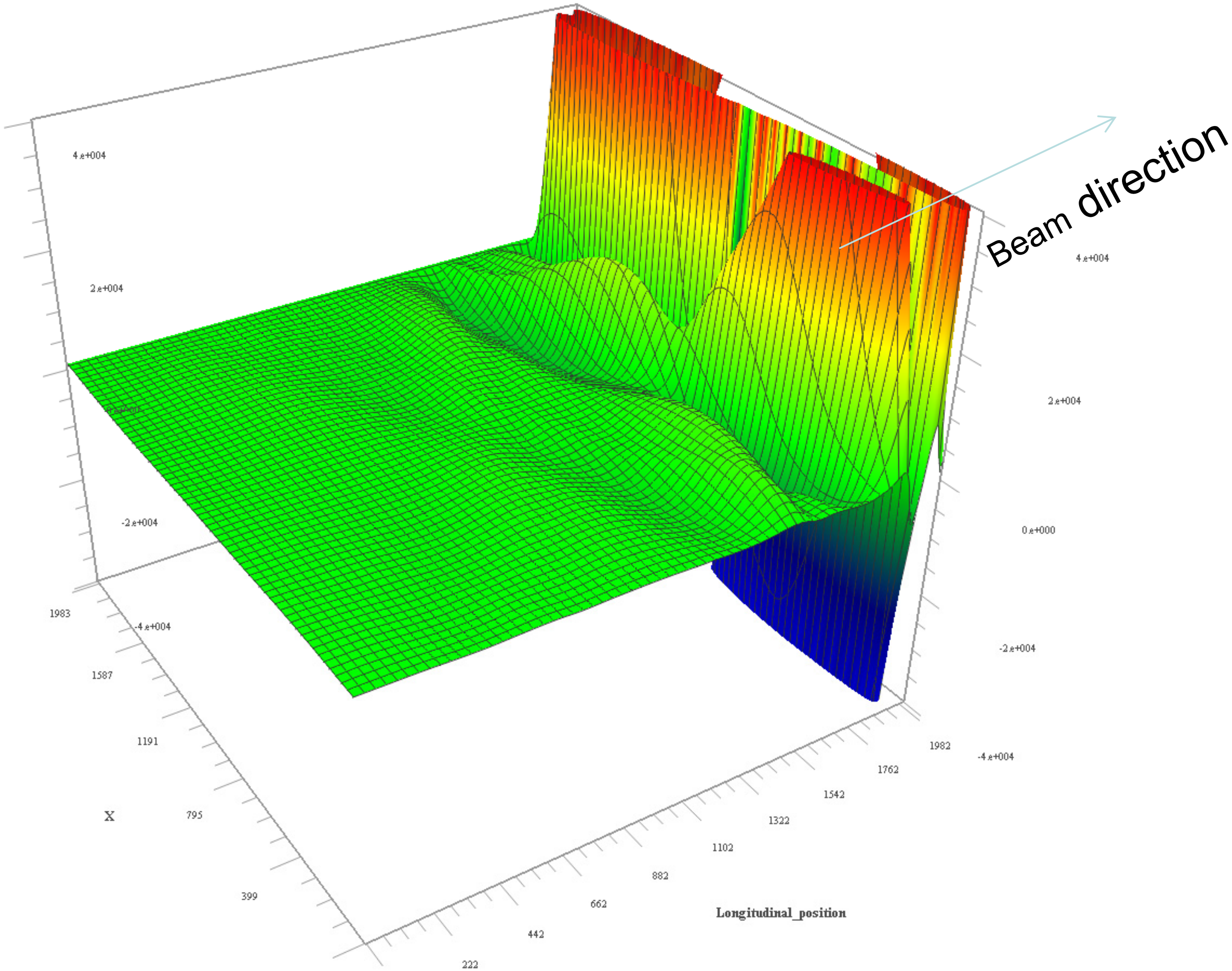
Fields in a beam chamber



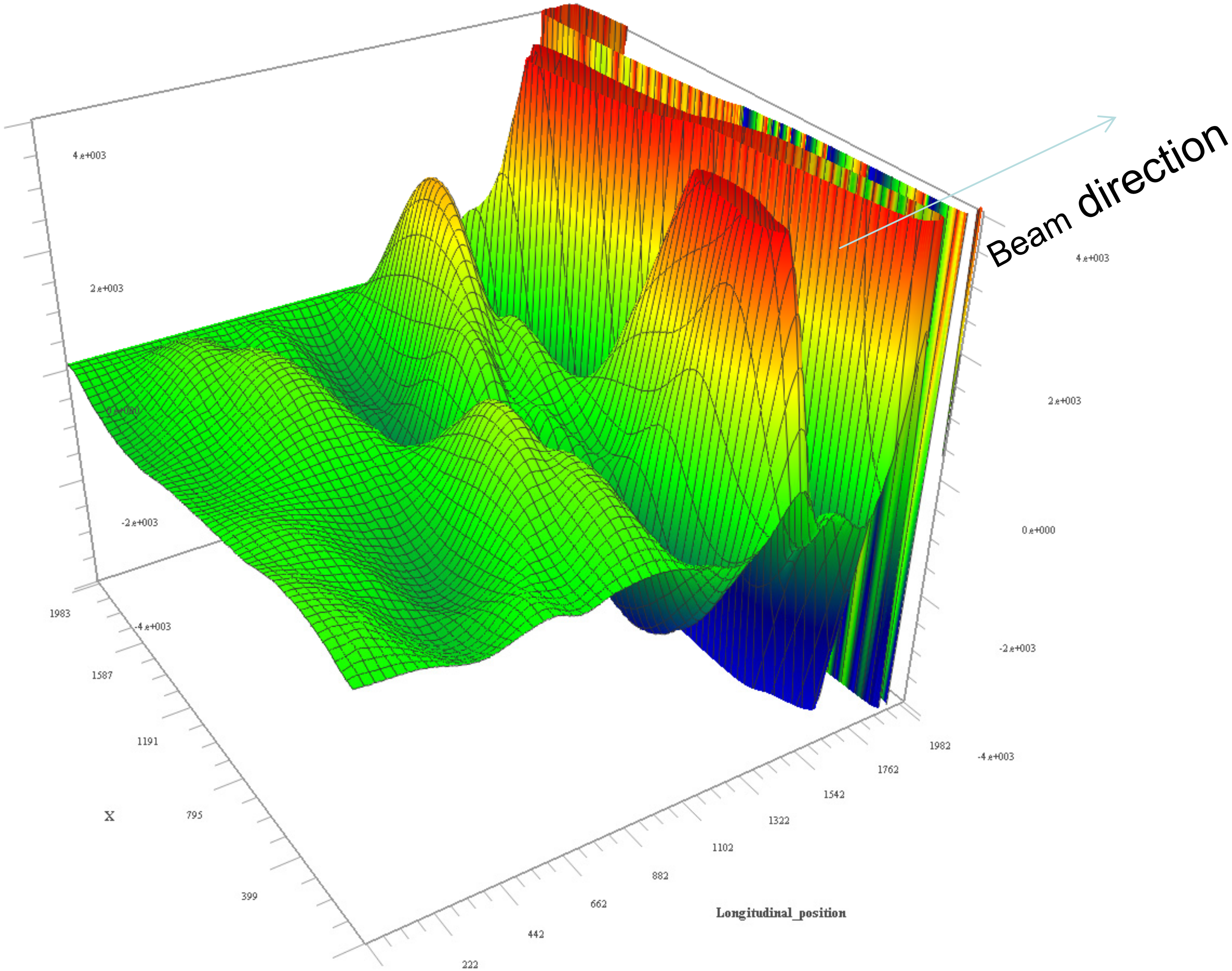
Magnified by 10 times



Magnified by 100 times

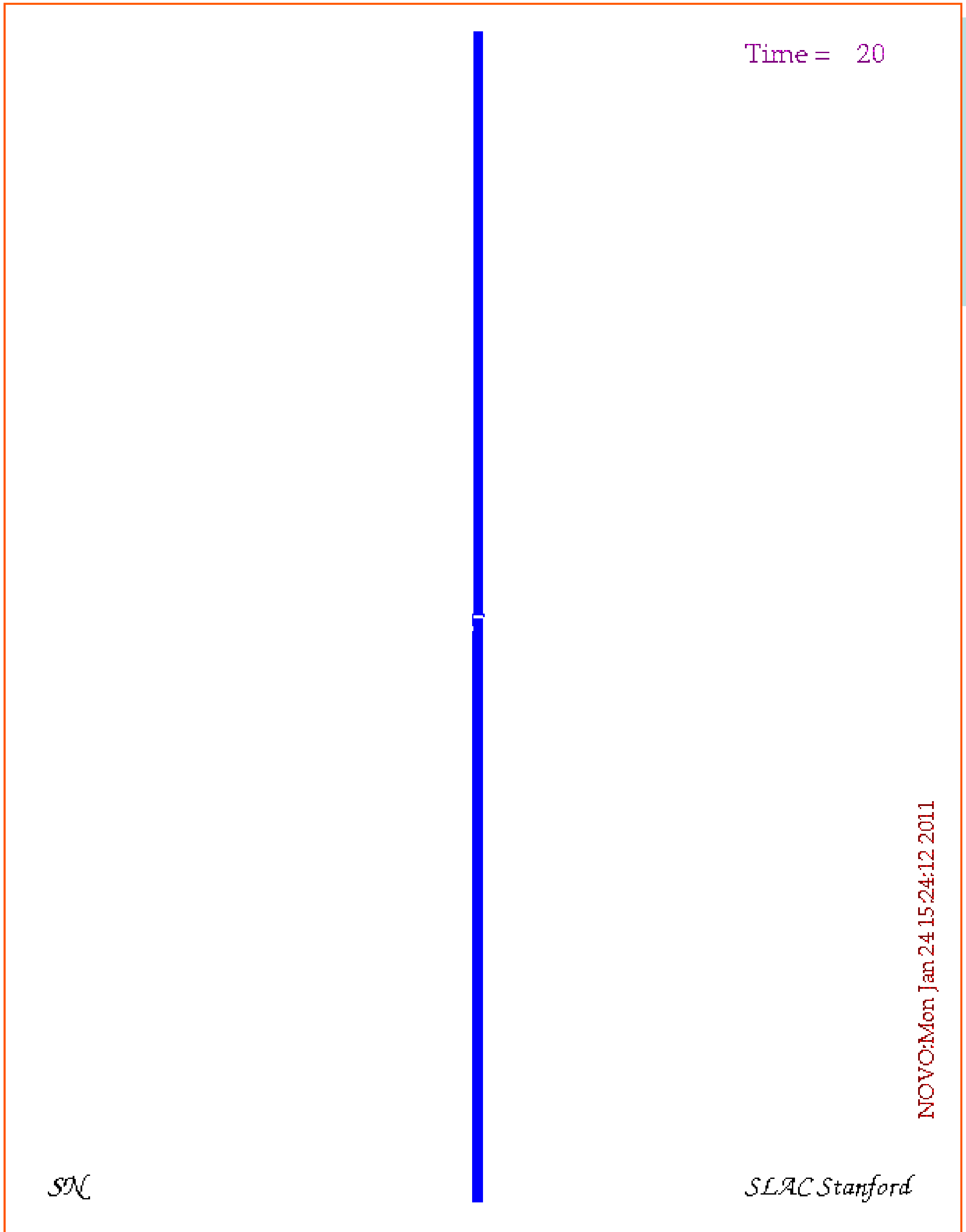


Magnified by 1000 times

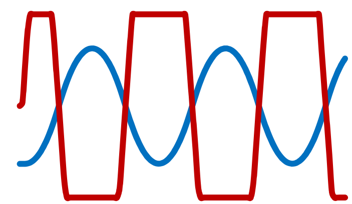


Summary

- We analyze the fine structure of coherent synchrotron fields, excited by a short bunch in a bending magnet using a new computer simulation code.
- We have found that there is much more interesting and detailed structure of the CSR fields, which have not been described by any previous study.
- A very important result is discovering the structure of the complicated collinear force. A bunch will get an additional energy spread in the transverse direction. This immediately leads to an emittance growth and decoherence that could limit FEL lasing for very short bunches.
- This effect may play same the role as the effect of quantum fluctuations of synchrotron radiation in damping rings. It can limit the minimum achievable emittance in the synchrotron light sources for very short bunches.



Movie show: retarding and undulator radiation



Undulator

Please click inside the image to start the movie

Sasha Novokhatski August 21, 2012

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