Low noise Particle-In-Cell simulations of laser-plasma accelerator 10 GeV stages

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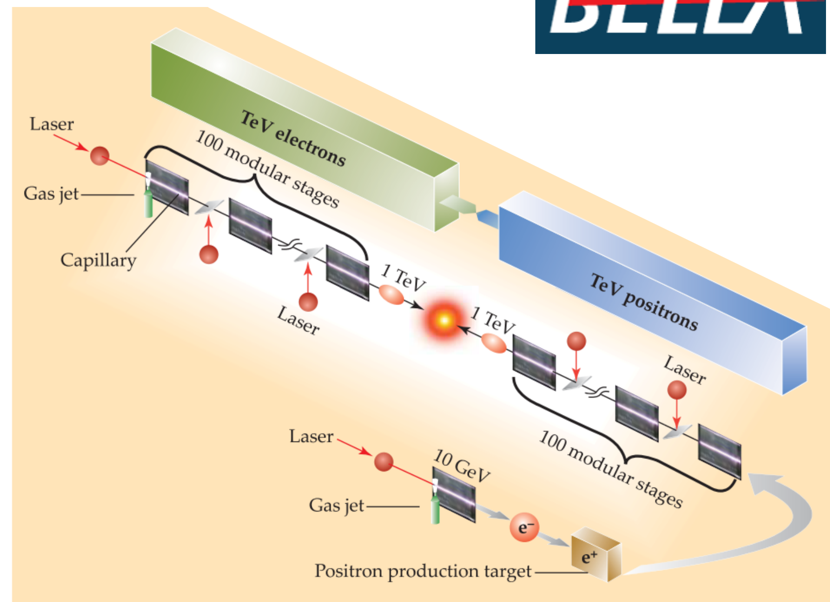
New generation of LPA experiments are challenging to simulate

- Hight accelerating gradient in laser-plasma accelerators allow compact devices
- For a linear collider, optimal succession of 10 GeV - 1m long stages†
- Involves disparate scale length:
  - laser wavelength ~10^{-6} m
  - plasma wavelength ~10^{-4} m
  - plasma length ~1 m
  - Use reduced model: envelope, boosted frame



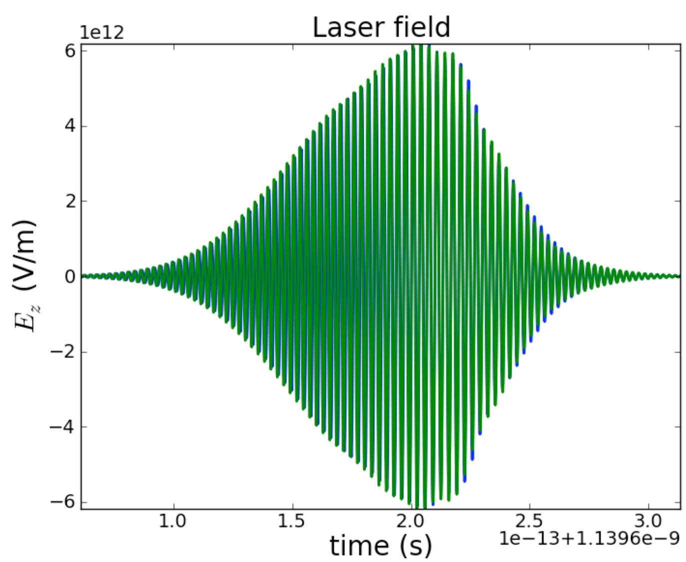
Leemans & Esarey, Physics Today (2009)

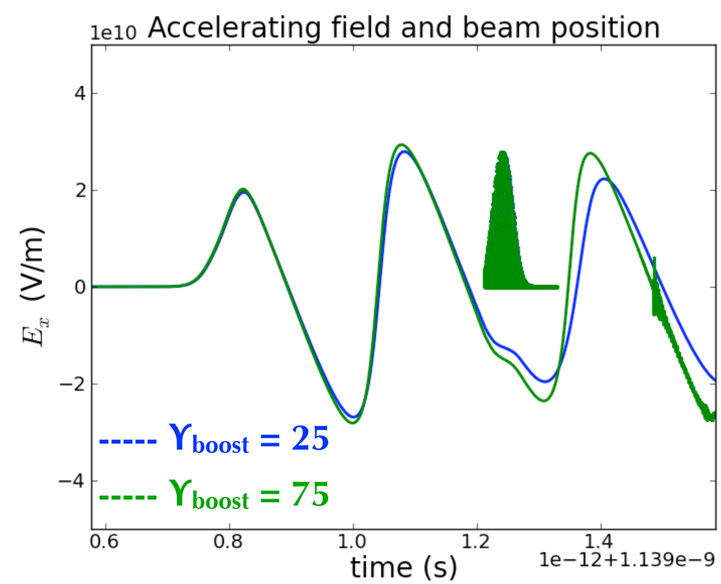
- Boosted frame allows modeling at full scale
- Calculating beam self-fields with a Poisson solve in the beam frame (BFPS) allows reduce noise for low energy spread, low emittance beams

† C.B. Schroeder *et al.*, PRSTAB **13**, 101301 (2010).

Boosted frame simulations quantitatively reproduce lab frame simulations

- Laser field is launched from a moving plane (fixed in the lab)†
- Fields and particle information are recorded on a fixed diagnostic plane in the lab frame (moving in the boosted frame)†
- BELLA relevant stage with beam loading and linear plasma taper (similar to Cormier-Michel *et al.* AAC 2008) at nominal density $n_0=10^{17}$ cm^{-3}
- Compare fields at Υ_{boost} = 25 (blue) and Υ_{boost} = 75 (green)
- max Υ_{boost} limited by density at the end of the plasma



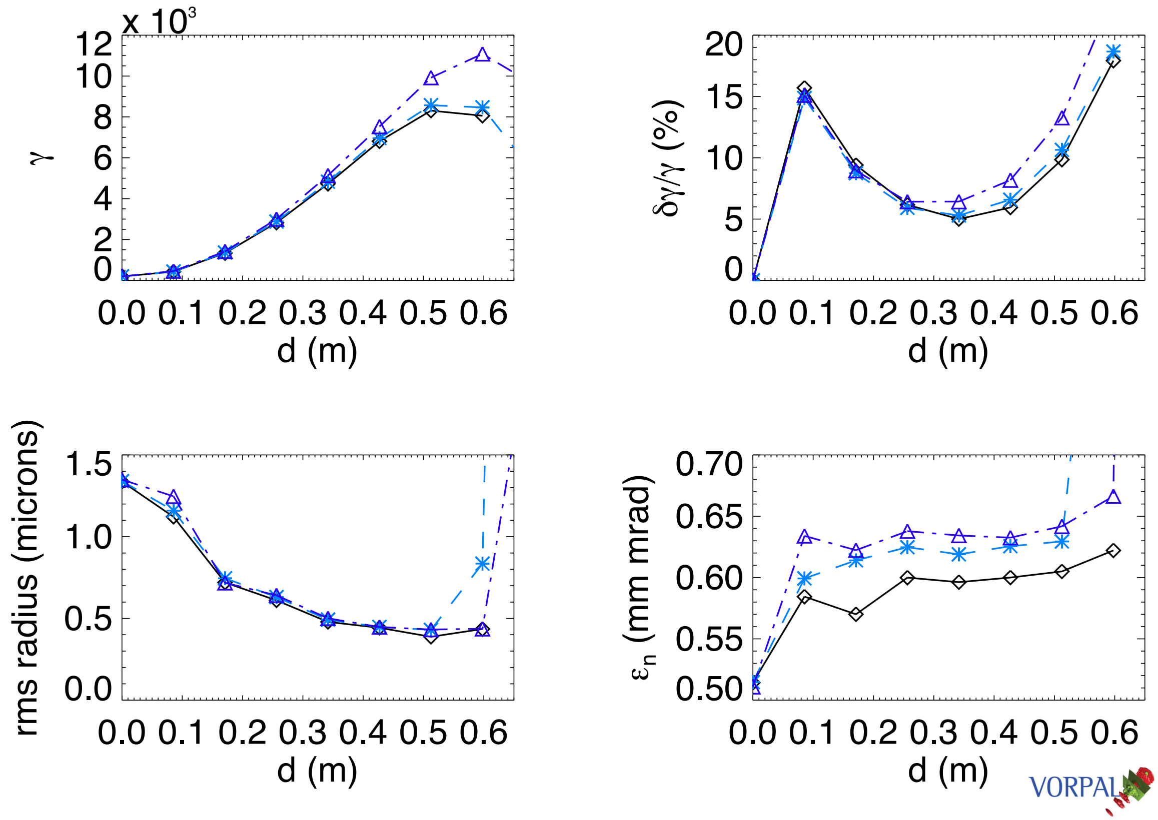


†J.-L. Vay *et al.* PoP **18**, 123103 (2011)

Evolution of an externally injected e- beam is the same for different values of Υ_{boost}

- BELLA relevant stage with beam loading and linear plasma taper (similar to Cormier-Michel *et al*. AAC 2008) at nominal density $n_0 = 10^{17}$ cm^{-3}
- Beam initial conditions: $k_pL = 0.26$; $k_p\sigma_r = 0.08$; $\Upsilon_{beam} = 195$ (100 MeV); $\varepsilon_n = 0.5$ mm mrad, Q = 2pC

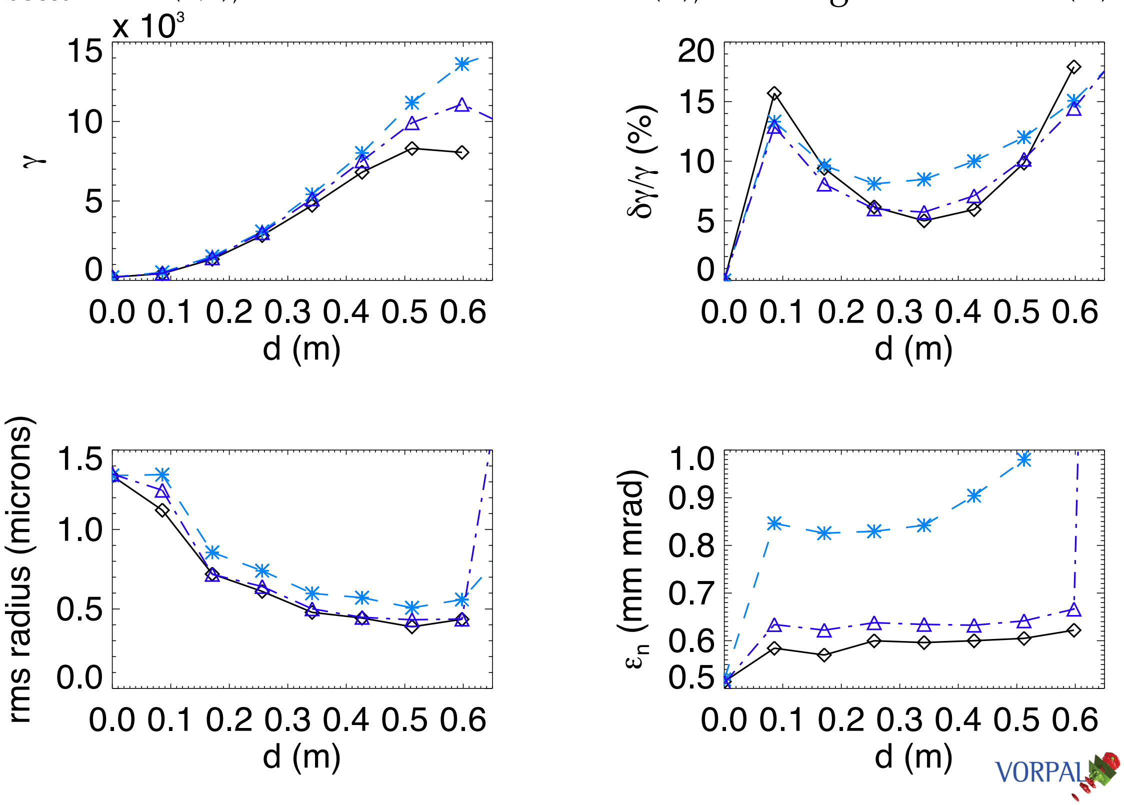
Υ_{boost} = 25 (◇), 50 (∗), 75 (Δ)



Large γ_{boost} needs higher resolution to reduce noise, decreases effective speed-up

- Simulation at γ_{boost} = 1 (full scale-stage): estimated 2.5 × 10^6 proc.h
- γ_{boost} = 75: 706 proc.h ➜ ×3,500 speed-up
- Higher γ_{boost} requires higher resolution to reduce noise and artificially high emittance, effective speed-up × 550 (4,500 proc.h)

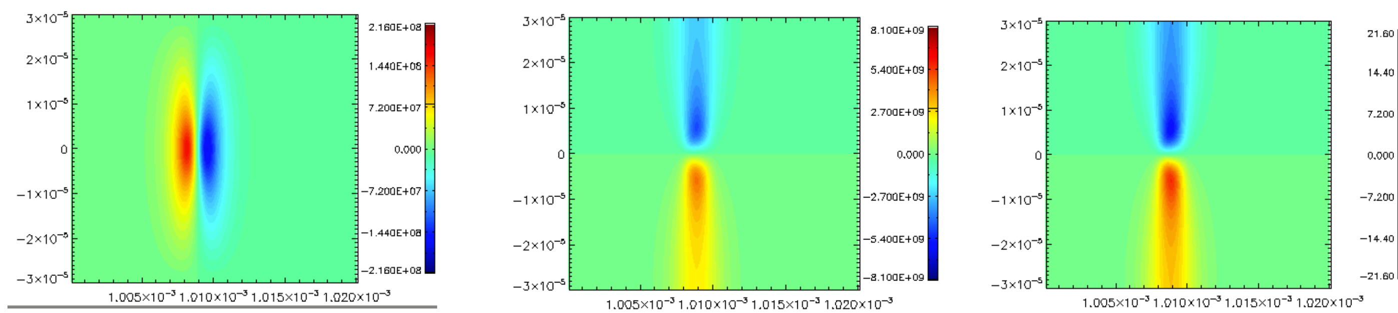
γ_{boost} = 25 (◇), 75 - normal resolution (∗), 75 - high resolution (Δ)



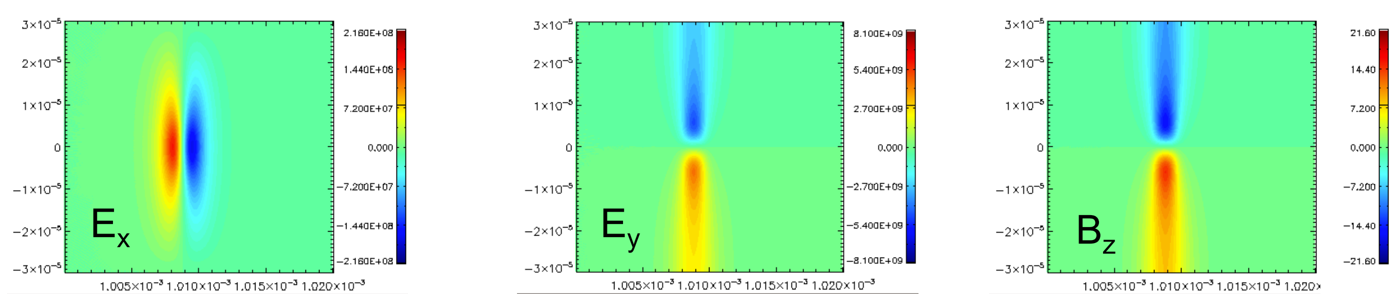
Self-fields on the e- bunch can be found from a Poisson solve in the beam frame

- Very similar to what is done in tracking codes
- The beam self-fields are calculated at each time step using a Poisson solver in the frame of the moving beam
- Works for low-emittance, low divergence bunches:
  - relative motion must be non-relativistic in the beam frame
  - we refer to this as a “beam-frame Poisson solve” **BFPS** algorithm
- After 1 mm of propagation fields are consistent with the self-consistent PIC fields.

BFPS fields at 1mm:



Self consistent electro-magnetic fields at 1mm:



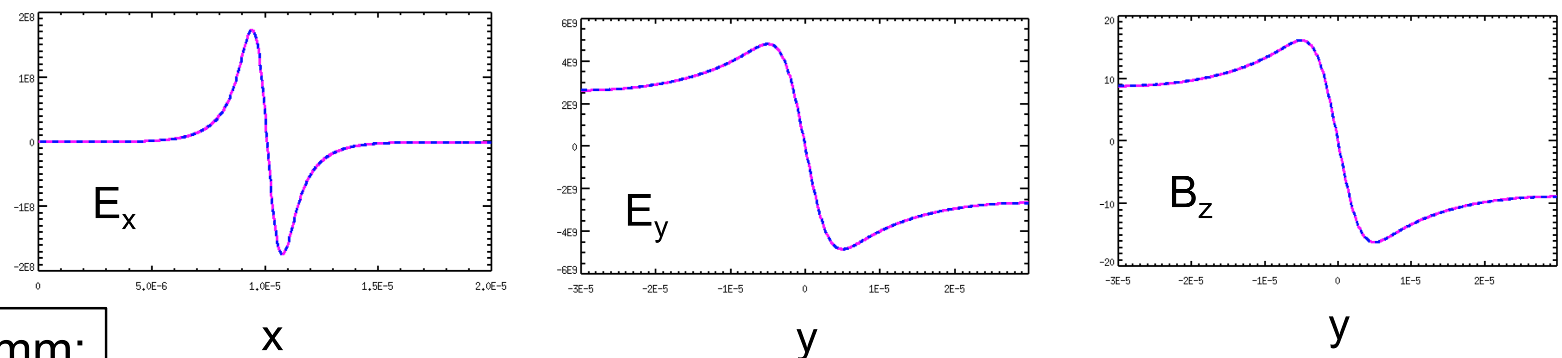
VORPAL

Agreement between BFPS & EM PIC for an ideal bunch

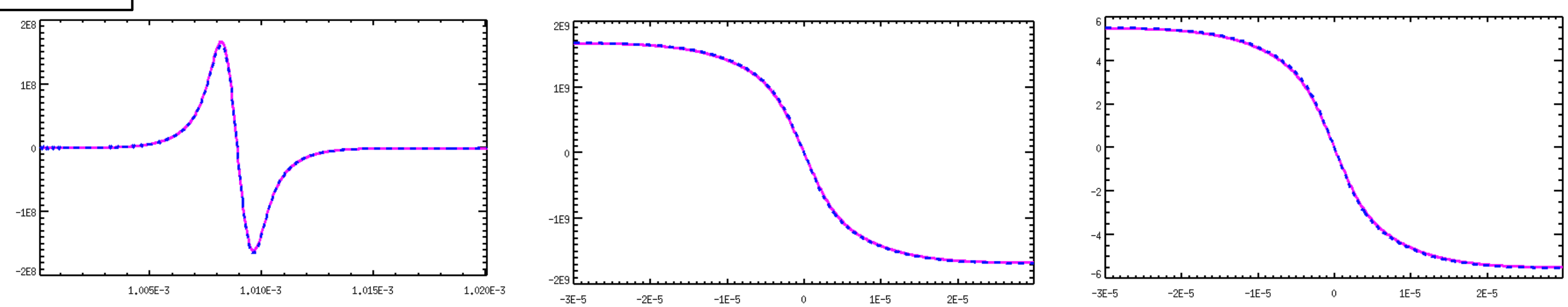
$r = 2.5 \times 10^{-6}$ m
$\varepsilon = 2.5 \times 10^{-8}$ m-R
$l = 0.5 \times 10^{-6}$ m
$\gamma = 20$
$\langle p_y^2 \rangle^{1/2} = c/100$

--- BFPS fields
--- Self-consistent

initial conditions:



after 1mm:



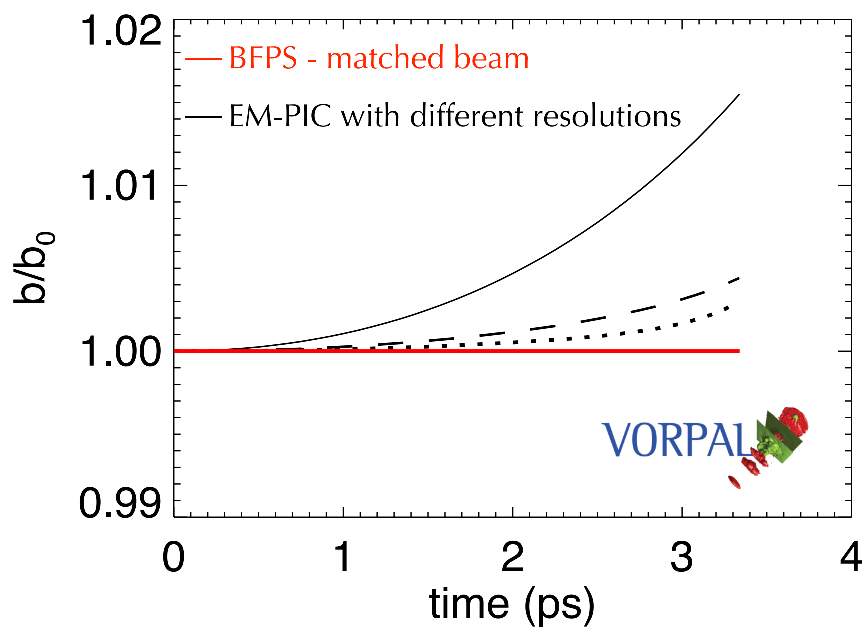
- For ideal bunch: matched beam and low transverse momentum spread

BFPS treatment of the e- beam self-fields enables correct modeling of transverse forces

- The uniformly-filled beam envelope equation* has been modified in 2D slab geometry: here *a* is the beam half-length, *b* the radius, *y* the transverse coordinate, k_y the focusing wavenumber, ε_y the transverse geometric emittance.

$$y'' + k_y^2 y - \frac{q^2 N / L}{m\beta^2 c^2 \gamma^2 \pi \varepsilon_0 b(\gamma a + b)} y - \frac{\varepsilon_y^2}{y^3} = 0$$

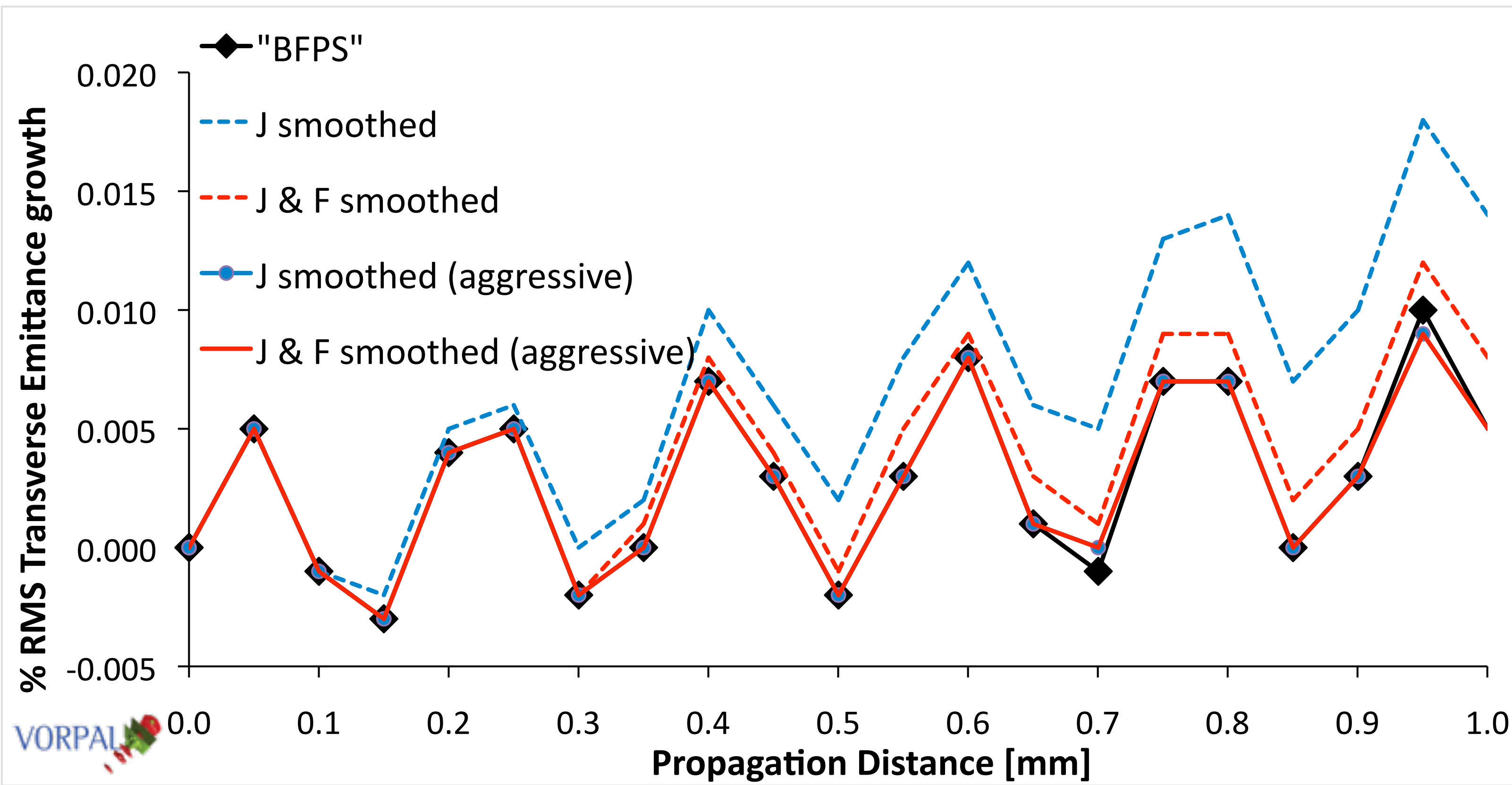
- The equation assumes correct cancellation of transverse forces $\Upsilon >> 1$
- Evolution of theoretically matched beam [from equation above] in a linear focusing field
  - BFPS shows constant radius over 1mm
  - EM PIC suffers from interpolation errors, although higher resolution helps (2nd order convergence)
  - Shown: $dx = a/60$, $a/120$, $a/180$; $dy = 4dx$; $a = b = 2\mu m$



*M. Reiser, *Theory and Design of Charged Particle Beams* (Wiley & Sons, 1994).

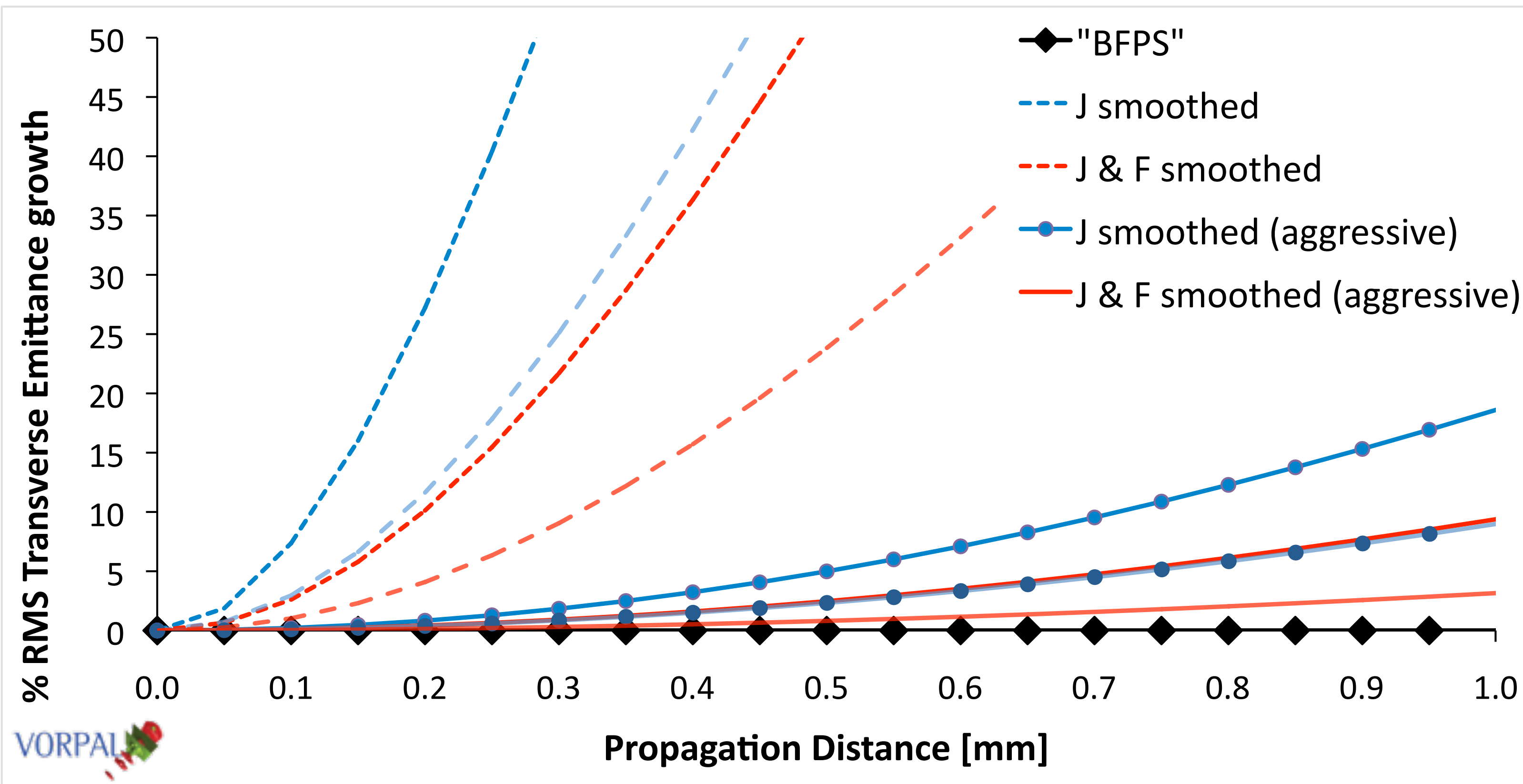
High-quality 100 MeV e- beam in 2D: aggressive smoothing is required to converge to

- Theoretically matched beam in linear focusing field
- % transverse normalized emittance growth
- Beam parameters characteristic of gas jet LPA experiments:
  - 10 pC, 100 MeV gaussian e- beam, $\varepsilon_{\perp,rms}$ = 10 mm-mrad, $\delta\Upsilon/\Upsilon$ = 1%



High-quality 1 GeV e- beam in 2D: aggressive smoothing not enough for EM to converge to BFPS results

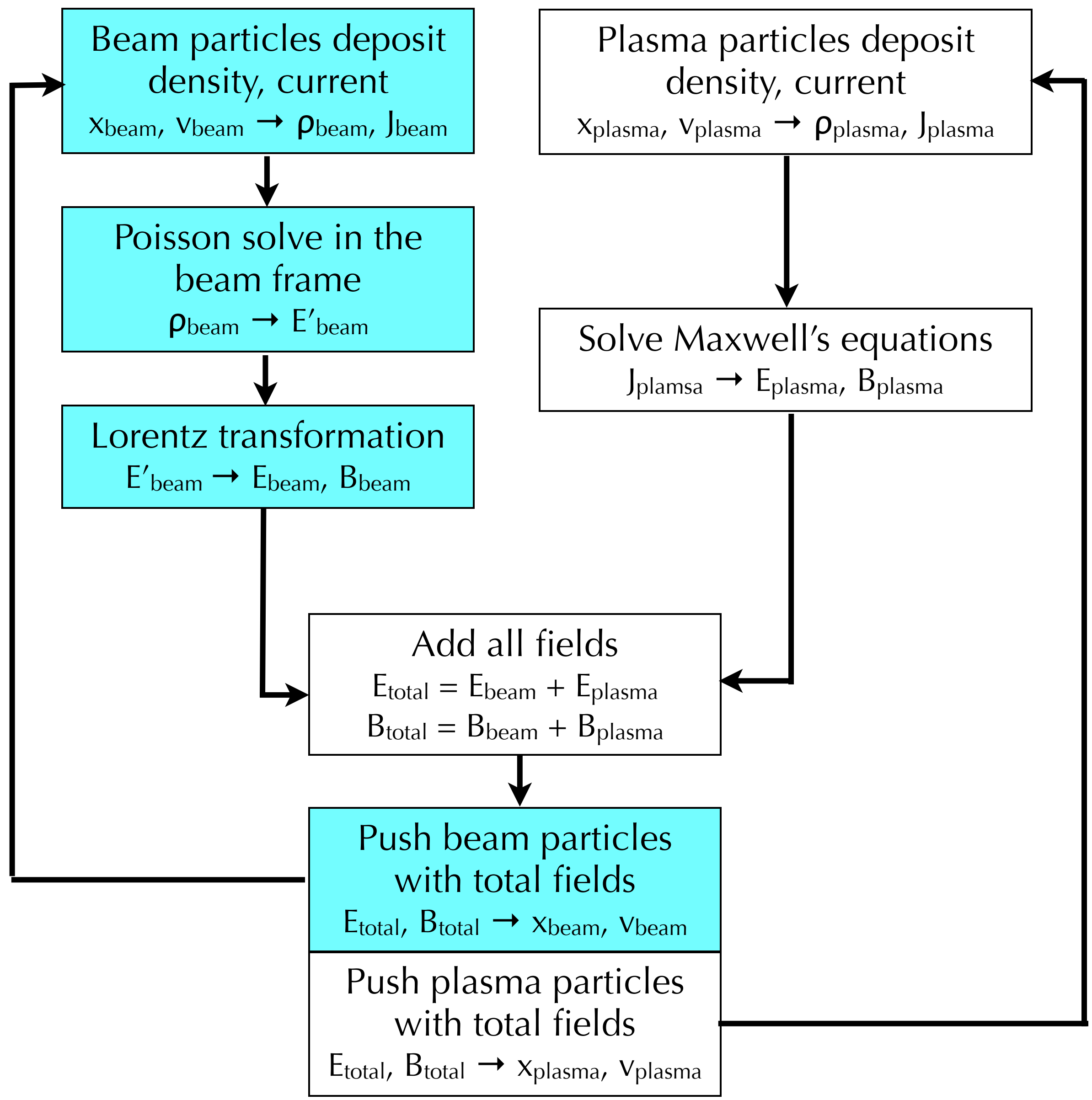
- Theoretically matched beam in linear focusing field
- % transverse normalized emittance growth
- Beam parameters characteristic of m-scale LPA stage for collider design:
  - 300 pC, 1 GeV gaussian e- beam, $\varepsilon_{\perp,rms}$ = 0.01 mm-mrad, $\delta\Upsilon/\Upsilon$ = 1%



BFPS is also valid *inside* the plasma

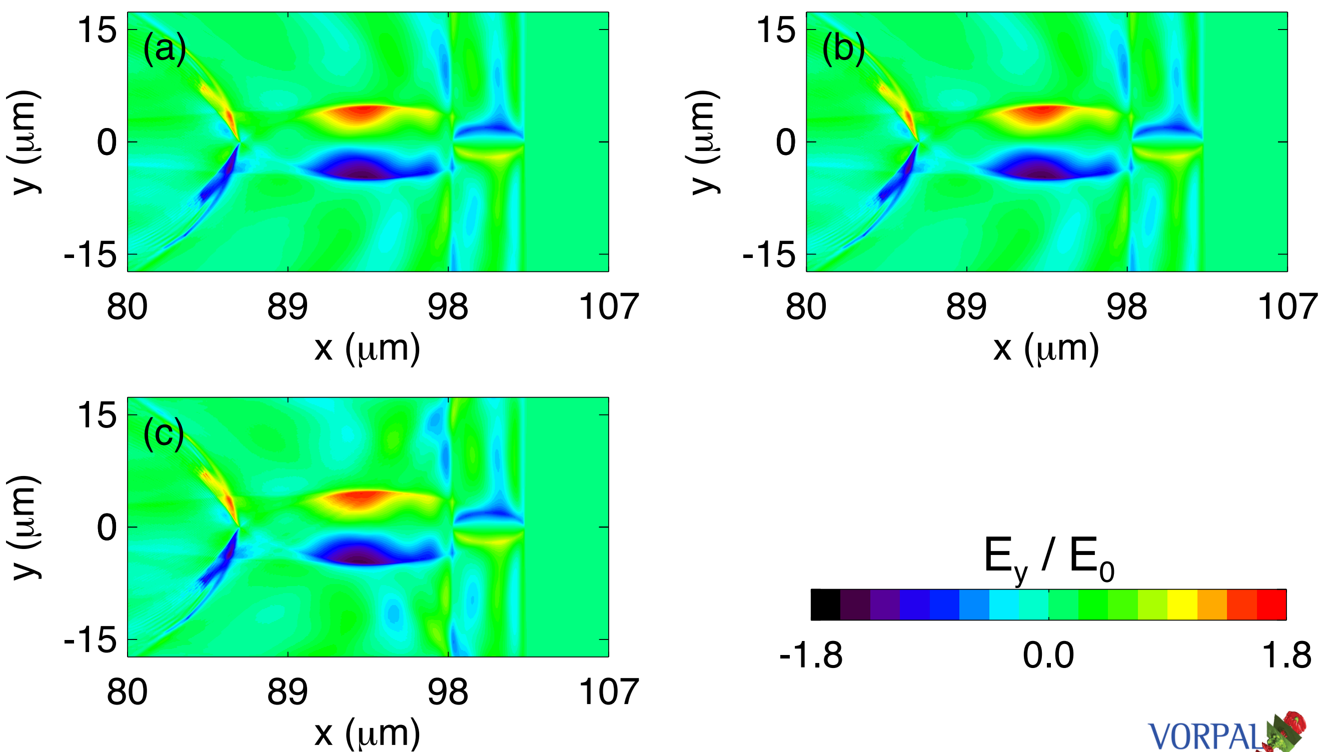
- Linearity of Maxwell's equations allows separate treatment of the beam in the plasma:
  - beam and plasma must be separated at time zero
  - all particles must respond to the combined fields

- Algorithm made possible by generality of Vorpal's structure, controlled from the input file



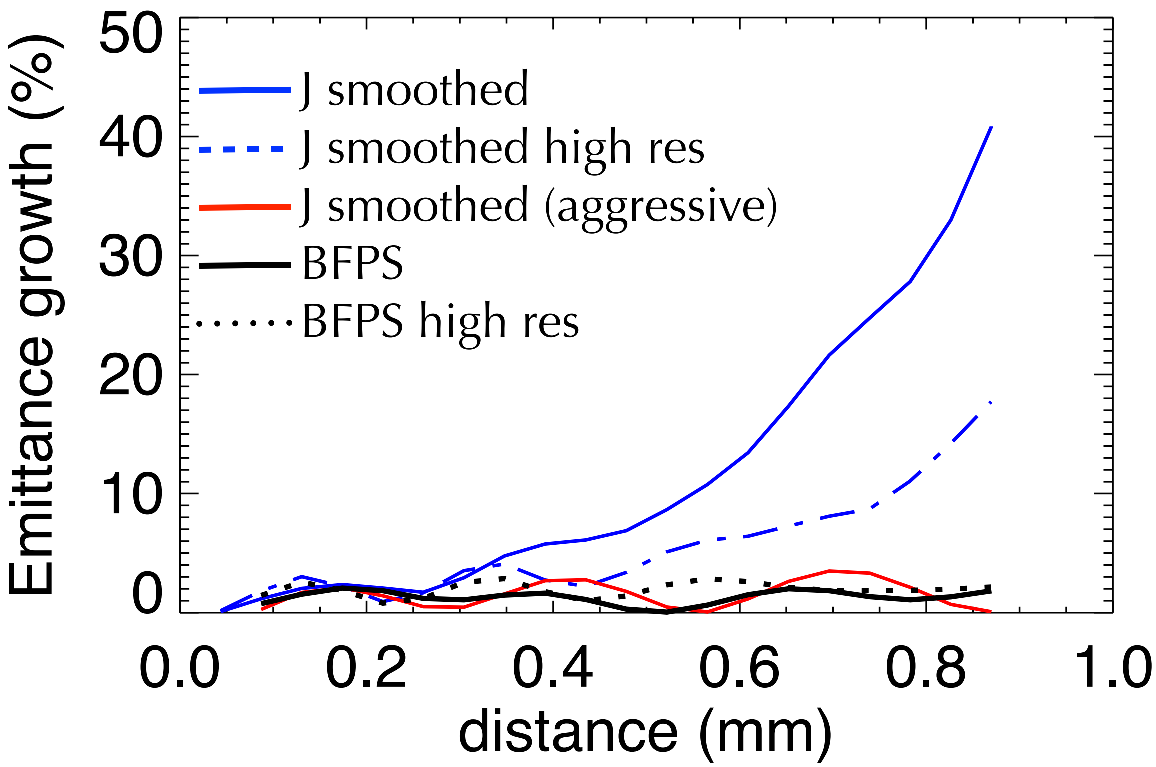
Separate beam/plasma with BFPS agrees with standard PIC

- Transverse fields when beam has propagated in the plasma
  - (a) fully self-consistent EM PIC
  - (b) self-consistent EM PIC with separate sequences of updates for the beam and the plasma
  - (c) beam self-fields calculated with the BFPS



BFPS inside plasma wakefield prevents beam artificial emittance growth

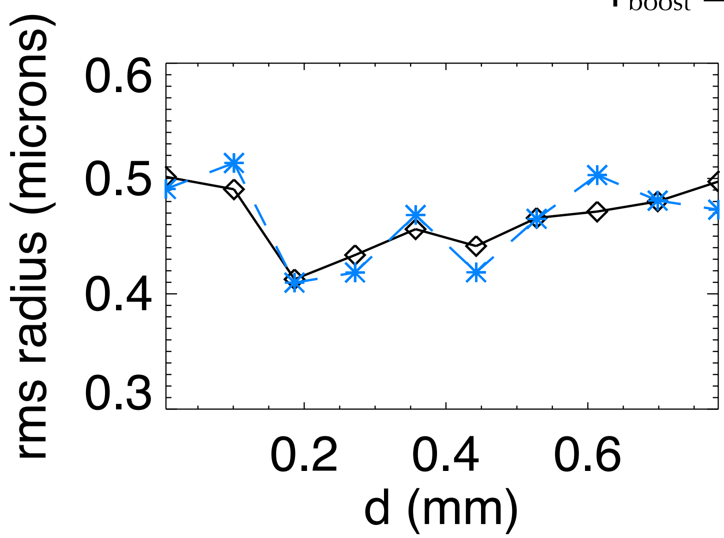
- $n_0 = 10^{19}$ cm^{-3} (100 MeV) stage
- 10 pC, $\varepsilon_{ny} = 0.5$ mm.mrad, Gaussian beam, matched to the wakefield focusing field
- acceleration turned off (not supported from the input file implementation of the BFPS)

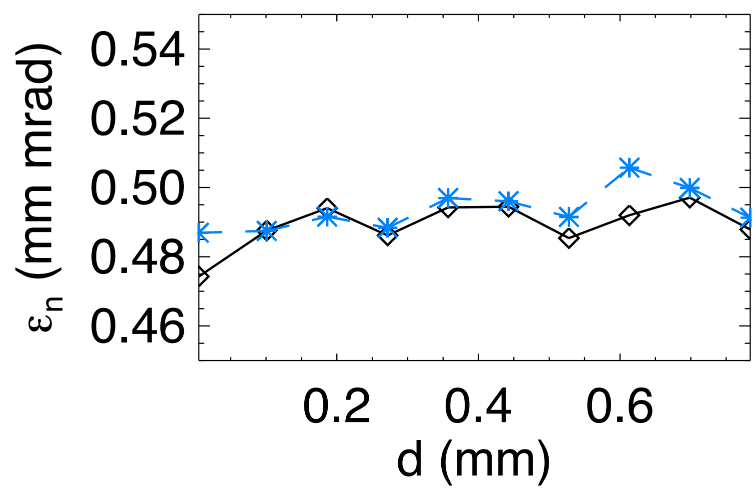


BFPS is used in the boosted frame

- $n_0 = 10^{19}$ cm^{-3} (100 MeV) stage
- will enable full-scale stage simulations with low particle noise

Υ_{boost} = 1 (◇), 13 (∗)





Summary

- VORPAL is modeling unscaled BELLA parameters with boosted frame simulations - 3500x speedup at γ=75.
- First implementation of a "tracking code space charge algorithm" in self-consistent LPA simulation.
- Very accurately model very low emittance spread bunches
    - correctly model cancelation of beam self-forces

- Implement BFPS for accelerating beam (not possible from the input file)
- Use fluid algorithm for plasma and further reduce noise