

STORAGE RING EDM SIMULATION: METHODS AND RESULTS

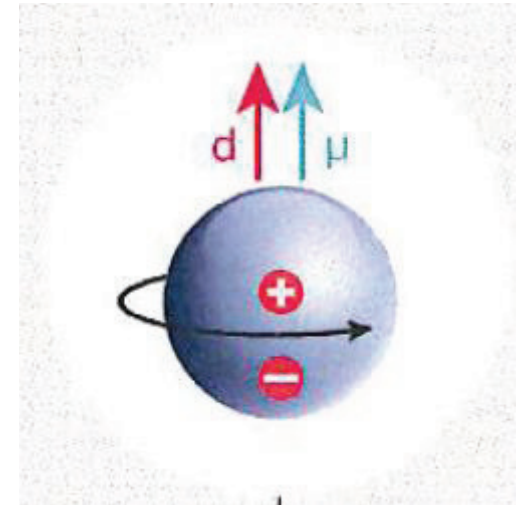
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on behalf of the JEDI Collaboration*

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Electric Dipole Moment and Standard Model

In frame of SM among the not yet understood phenomena are the reasons for the violation of the **CP symmetry**.

But CP violation is the only known mechanism that could explain the matter-antimatter asymmetry found in Universe.



The electric dipole moments (EDM) of fundamental particles are excellent probes of physics beyond the standard model (SM), e.g. SUSY, since they allow for values within experimental reach whereas the SM predictions are several orders below them.

“Tomas-Bargmann, Michel, Telegdi” equation with EDM term

The **spin is a quantum value**, but in the classical physics representation the “spin” means an expectation value of a quantum mechanical spin operator:

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m} \left\{ G\vec{B} + \left(\frac{1}{\gamma^2 - 1} - G \right) (\vec{\beta} \times \vec{E}) + \frac{\eta}{2} (\vec{E} + \vec{\beta} \times \vec{B}) \right\}$$

$$G = \frac{g - 2}{2},$$

$d = \eta e \hbar / 4mc$

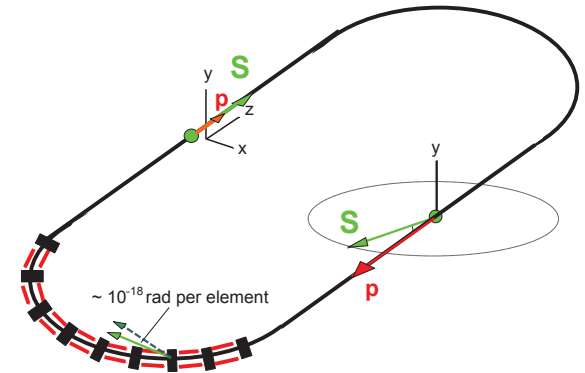
EDM

↑

$$\eta = \frac{4d \cdot mc^2}{e \hbar c} = \frac{4 \cdot [10^{-31} e \cdot m] \cdot [938.272 \text{ MeV}]}{e \cdot [6.582 \cdot 10^{-22} \text{ MeV} \cdot \text{sec}] \cdot [2.9979 \text{ m/sec}]} \approx 2 \cdot 10^{-15}$$

Main problems of EDM simulation

- the arithmetic coprocessor has a mantissa length of 52 bits (minimum recorded figure $\sim 2 \cdot 10^{-16}$) and can make a mistake in calculating spin projections after i -th element if $S_{i+1} - S_i < 10^{-16}$.
- the required revolution time simulation is $\sim 10^9$
- high accuracy of simulation



Therefore we take an approach, where the EDM signal is not implemented, and only the induced error signal is studied.

Methods of spin-orbital motion investigation

- COSY Infinity program;
- Symplectic Runge-Kutta integrating;
- Direct numerical integration of T-BMT differential equations;
- Analytical approach.

COSY Infinity program (M. Berz)

COSY Infinity is a program for the simulation, analysis and design of particle optical systems, based on differential algebraic methods.

In the EDM search, it is the only program which allows the spin-orbit motion of millions of particles to be simulated over a real time scale experiment during $n \times 1000$ seconds.

At present, we use the MPI (Message Passing Interface) version of the COSY Infinity program installed on a supercomputer with $3 \cdot 10^5$ processors.

At the initial stage of EDM research, we use COSY Infinity to study the behaviour of the spin aberrations for a large number of particles and a long-time calculation.

Symplectic Runge-Kutta integrating (A. Ivanov)

The integrating Runge-Kutta program is intended to model the spin-orbital motion with fringe fields in elements and including the EDM signal directly in the simulation.

The algorithm used in the program is not as fast as COSY Infinity by several orders of magnitude. Therefore, we use it mostly to investigate a short-time phenomenon for single particle that does not require long calculation periods.

As a basic method for the tracking program, a symplectic Runge-Kutta scheme was implemented from W. Oevel, M. Sofroniou,

(Symplectic Runge- Kutta Schemes II: Classification of Symmetric Methods, preprint, University of Paderborn, Germany, 1996.)

The EDM search methods in Storage Ring:

1. Resonant method with initial spin orientation in ring

$$\mathbf{S} \parallel \mathbf{B}; \mathbf{S} = \{0, S_y, 0\} \text{ and } \mathbf{B} = \{0, B_y, 0\}$$

2. “Magic” method with initial spin orientation in ring

$$\mathbf{S} \parallel \mathbf{p}; \mathbf{S} \perp \mathbf{E}; \mathbf{S} = \{0, 0, S_z\} \text{ and } \mathbf{E} = \{E_x, 0, 0\}$$

In **resonant method*** the spin frequency is parameterized :

$$\vec{\Omega} = -\frac{e}{m} \left\{ G\vec{B} + \left(\frac{1}{\gamma^2 - 1} - G \right) (\vec{\beta} \times \vec{E}) + \frac{\eta}{2} (\vec{E} + \vec{\beta} \times \vec{B}) \right\}$$

using RF flipper $E \sim e^{i(f_{rf} \pm k f_{rev})t}$

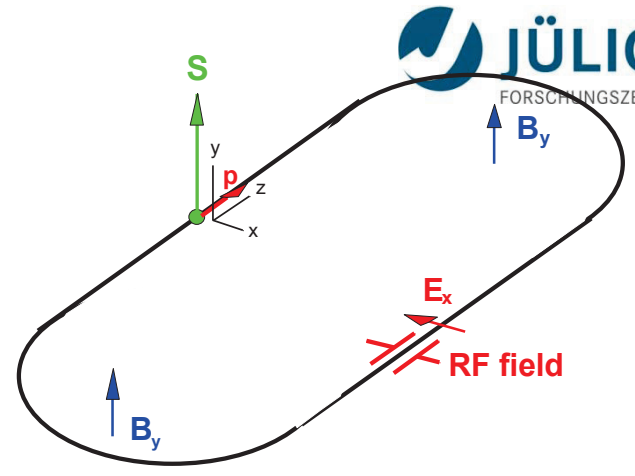
In case of parametric resonance when $\frac{f_{rf}}{f_{rev}} \pm k = \gamma G; k = 0,1,2,\dots$ we shall observe the resonant build up:

$$S_z(n) = \frac{(2\nu_e \nu_s + h \nu_s) \cdot \sin 2\pi \nu_s n}{2\nu_s^2} \rightarrow h\pi n \cdot \cos 2\pi \nu_s n$$

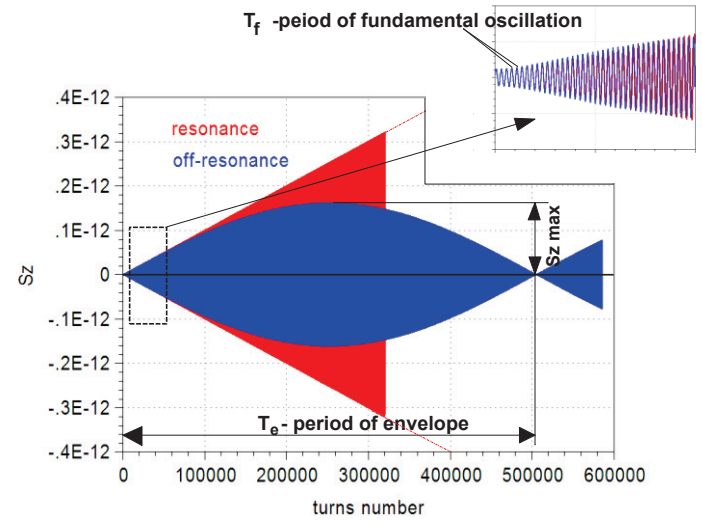
$$h = \frac{\eta}{2} \cdot \gamma \cdot \frac{E_{rf} l_{rf}}{B_y L_{cir}} \sim \text{EDM signal} \quad \nu_s = \gamma G$$

Advantage: the method can be realized in COSY ring

Disadvantage: the high requirement to stability of f_{rf}



$$\frac{f_{rf}}{f_{rev}} \pm k = \gamma G; k = 0,1,2,\dots$$



“Magic” method for purely electrostatic ring

In the “**magic**” method the beam is injected in the electrostatic ring with the spin directed along momentum $\mathbf{S} \parallel \mathbf{p}$ and $\mathbf{S} \perp \mathbf{E}$; $\mathbf{S} = \{0, 0, S_z\}$ and $\mathbf{E} = \{E_x, 0, 0\}$

at “magic” energy : $\frac{1}{\gamma_{mag}^2} - G = 0$

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m} \left\{ G\vec{B} + \left(\frac{1}{\gamma^2} - G \right) (\vec{\beta} \times \vec{E}) + \frac{\eta}{2} (\vec{E} + \vec{\beta} \times \vec{B}) \right\}$$

$$G = \frac{g-2}{2},$$

External fields

EDM

“Magic” method for purely electrostatic ring

In the “magic” method the beam is injected in the electrostatic ring with the spin directed along momentum $\mathbf{S} \parallel \mathbf{p}$ and $\mathbf{S} \perp \mathbf{E}$; $\mathbf{S} = \{0, 0, S_z\}$ and $\mathbf{E} = \{E_x, 0, 0\}$

at “magic” energy : $\frac{1}{\gamma_{mag}^2 - 1} - G = 0$

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m} \left\{ \text{External fields} + \frac{\eta}{2} (\vec{E} + \text{EDM}) \right\}$$

$$G = \frac{g-2}{2},$$

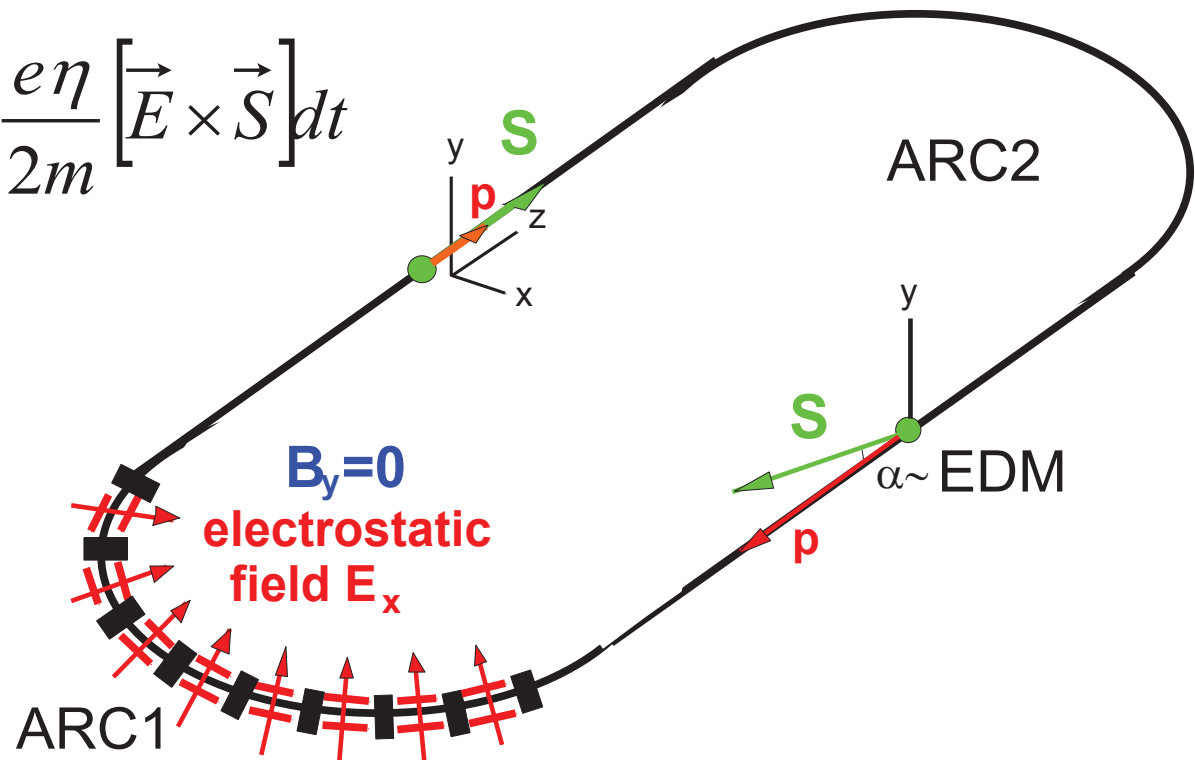
External fields

EDM

“Magic” method in purely electrostatic ring

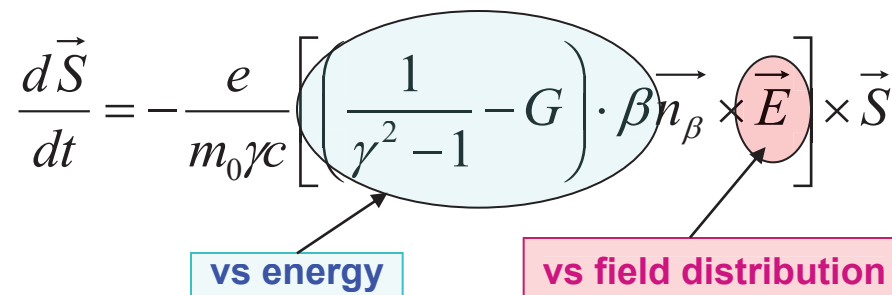
In purely electrostatic ring the spin of particle with “magic energy” rotates with the same angular frequency as the momentum and it tilts up in the YZ plane due to the EDM with angular rate

$$d\vec{S} = -\frac{e\eta}{2m} [\vec{E} \times \vec{S}] dt$$

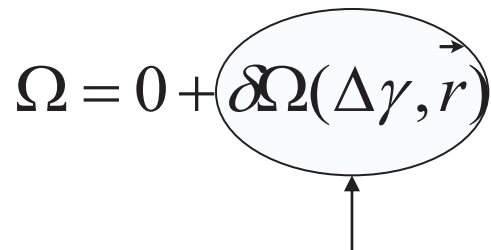


Spin tune aberration in purely electrostatic ring

In reality the beam has energy spread $\gamma_{\text{mag}} \Delta\gamma$ and all particles move in different external field. Therefore the spin tune has the aberrations dependent on energy γ and trajectory $\mathbf{r}(\mathbf{t})$ of particles.

$$\frac{d\vec{S}}{dt} = -\frac{e}{m_0\gamma c} \left[\left(\frac{1}{\gamma^2 - 1} - G \right) \cdot \beta \vec{n}_\beta \times \vec{E} \right] \times \vec{S}$$


At “magic” energy it is no precession of spin. For no “magic” energy $\gamma_{\text{mag}} \Delta\gamma$

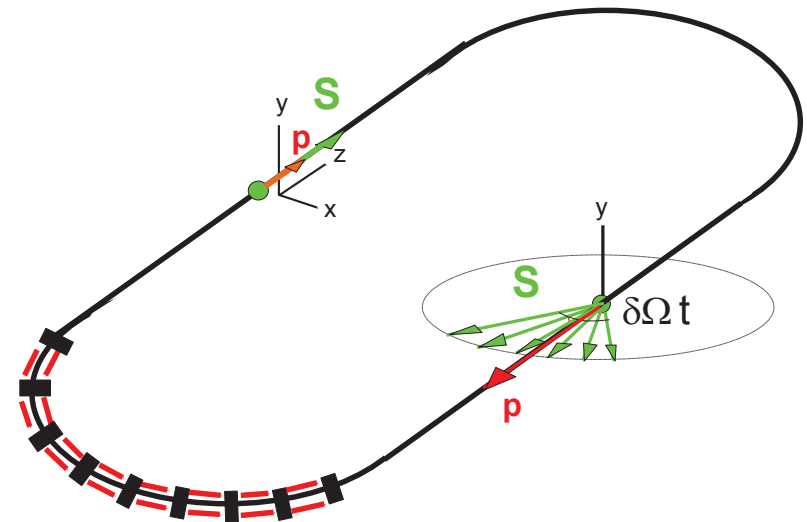
$$\Omega = 0 + \delta\Omega(\Delta\gamma, \vec{r})$$


Spin tune aberration

Spin Coherence Time is time when RMS spin orientation of the bunch particles reaches one radian (YS,BNL), and it has to be $> 1000\text{sec}$.

During **SCT** each particle performs $\sim 10^9$ turns in the storage ring moving with different trajectories through the optics elements.

At such conditions the spin-rotation aberrations associated with various types of space and the time dependent nonlinearities start to play a crucial role.



Spin tune aberration due to energy spread

Longitudinal component

$$\frac{dS_x}{d\tau} = -\frac{eL_{orb}E_x}{2\pi mc^2} \left(\frac{1}{\gamma^2 - 1} - G \right) \cdot S_z$$

$$\frac{dS_z}{d\tau} = +\frac{eL_{orb}E_x}{2\pi mc^2} \left(\frac{1}{\gamma^2 - 1} - G \right) \cdot S_x \quad \longrightarrow \quad \delta\nu_s = \frac{eL_{orb}E_x}{2\pi m_0 c^2} \cdot \delta \left(\frac{1}{\gamma^2 - 1} - G \right)$$

$$d\tau = 2\pi dt / T_{rev}$$

$$\delta \left(\frac{1}{\gamma^2 - 1} - G \right) = -2G \frac{\Delta p}{p} + \frac{1 + 3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p} \right)^2 + \dots$$

RF cavity as first step to increase SCT

RF off:

$$\frac{d^2 S_z}{d\varphi^2} + \left(\frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \cdot 2G \frac{\Delta p}{p} \right)^2 S_z = 0$$

for $\Delta p/p = 10^{-4}$ SCT=6300 turns, which is **~ 1 msec.**

RF on: $(\Delta p / p) = (\Delta p / p)_{\max} \cdot \cos(\nu_z \cdot \varphi)$

Idea of using the RF cavity was expressed long time ago by many authors, for instance [A.P. Lysenko et al., Part.Accel. 18, 215 (1986)].

$$\frac{d^2 S_z}{d\varphi^2} + \left[\frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \cdot 2G \left(\frac{\Delta p}{p} \right)_{\max} \cdot \cos(\nu_z \cdot \varphi) \right]^2 S_z = 0$$

The spin swing in a rapidly oscillating field with RF frequency and it is bounded within a very narrow angle $\sim 10^{-6}$ dependent on $\Phi_{\max} \sim (\nu_s / \nu_z)^2$

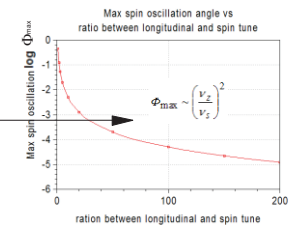
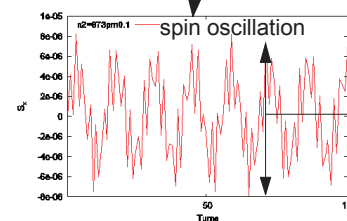
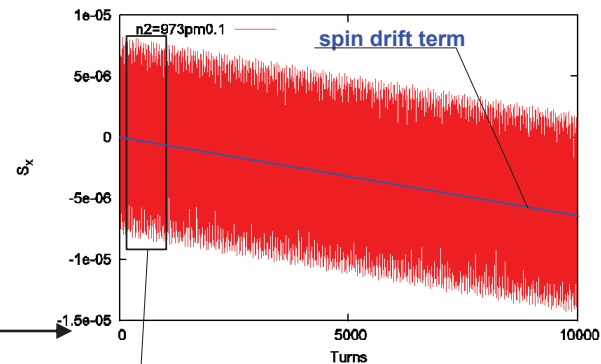
RF on: Second order approach of spin tune versus $\Delta p/p$

However, in the second approach versus momentum the average tune spin is not zero

$$\bar{\nu}_{sz} = \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \cdot \left[2G \frac{\Delta p}{p} \cdot \cos(\nu_z \cdot \varphi) - \frac{1+3\gamma^2}{\gamma^2} \cdot G \cdot \left(\frac{\Delta p}{p} \right)_{\max}^2 \cdot \cos^2(\nu_z \cdot \varphi) \right] = \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \cdot \frac{1+3\gamma^2}{\gamma^2} \cdot \frac{G}{2} \cdot \left(\frac{\Delta p}{p} \right)_{\max}^2$$

At $(\Delta p/p)_{\max} = 10^{-4}$ and an axial particle the number of turns for SCT is $\sim 6 \cdot 10^7$ turns, that is **~180 sec.**

The code COSY infinity simulation



Off-axial particle: Longitudinal-transverse coupling in electrostatic storage ring

The electrical deflector has the central field symmetry:

$$E \propto \frac{\alpha}{r^n}$$

where angular momentum $M = mv_{\theta}r = \text{const}$

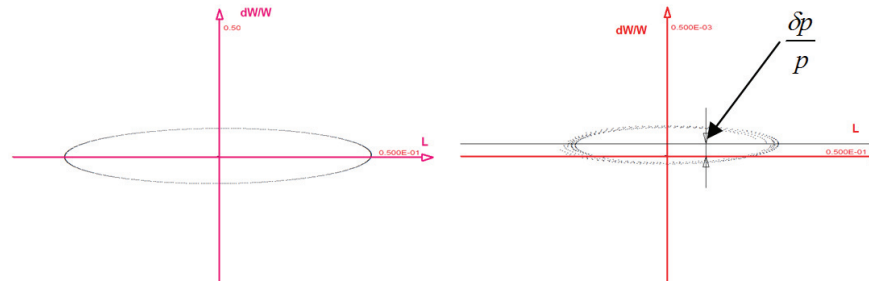
Due to this fact the total energy can be represented as the function of the coordinates :

$$W(r, \dot{r}) = \frac{m}{2} \dot{r}^2 + \frac{M^2}{2mr^2} + \varphi(r)$$

The radial motion is one dimensional motion in the field with the effective potential, and the equilibrium radius: $R_{eq} \propto M^n$

Off-axial particle: influence on spin motion

The particle with different momentum oscillates with respect to different energy levels:



COSY infinity tracking results
for initial coordinates $x=0, y=0$ and $x=3\text{mm}, y=0$.

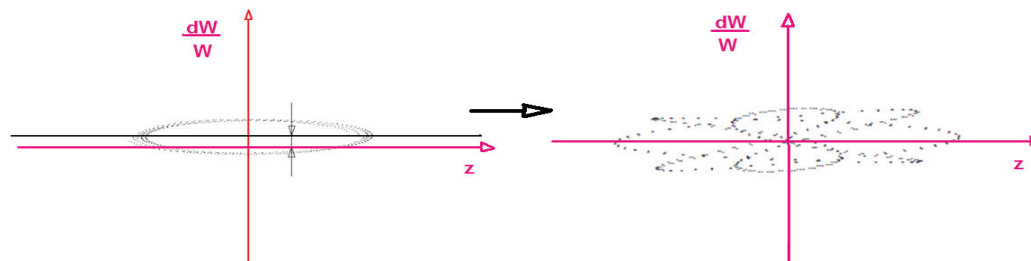
Thus, RF cavity will not be able to reduce the oscillation of the spin for off-axial particles, since:

$$v_{sz} = \frac{\overline{eE_x L_{cir}}}{\pi m_0 c^2 \gamma} \cdot G \frac{\delta p}{p}$$

Equilibrium energy level modulation as method to increase SCT

Following physical logic the only solution to increase SCT is the modulation of the energy level itself relative of the magic level.

For this purpose, we have increase coupling between longitudinal and transverse motion that is, approach frequencies as close as possible to each other. In result we have got **SCT=400 sec**



COSY infinity tracking results
 for initial coordinates $x=0, y=0$ and $x=3\text{mm}, y=0$.

Spin tune aberration vs momentum and axial deviation

Assuming violation “magic” condition for non-reference particle the spin tune aberration is defined:

$$\delta\nu_s = \frac{e}{2\pi mc^2} \cdot \delta\left(\frac{1}{\gamma^2 - 1} - G\right) \cdot L_{orb} E_x \left[1 + \frac{\delta L_{orb}}{L_{orb}} + \frac{\delta E_x}{E_x} \right]$$

with

$$\delta\left(\frac{1}{\gamma^2 - 1} - G\right) = -2G \frac{\Delta p}{p} + \frac{1 + 3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p}\right)^2 + \dots$$

$$\frac{\delta L_{orb}}{L_{orb}} = \alpha_1 \cdot \frac{\Delta p}{p} + \alpha_2 \cdot \left(\frac{\Delta p}{p}\right)^2 + \dots$$

$$\frac{\delta E_x}{E_x} = -k_1 \frac{x}{R} + k_2 \left(\frac{x}{R}\right)^2 + \dots$$

Spin tune aberration vs momentum and axial deviation

Grouping the coefficients of powers up to second order, we obtain an equation having a parabolic form :

$$\delta\nu_s = \frac{eL_{orb}E_xG}{2\pi m_0\gamma c^2} \cdot \left[F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right) \cdot \left(\frac{\Delta p}{p}\right)^2 + 2F_1\left(k_1, k_2, \frac{x}{R}\right) \cdot \frac{\Delta p}{p} \right]$$

with coefficients having a parabolic dependence on axial deviation

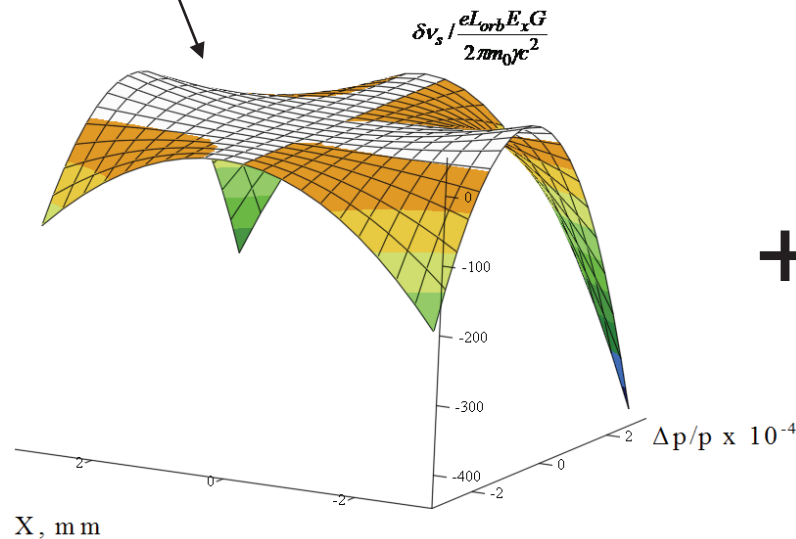
$$F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right) = \frac{1+3\gamma^2}{\gamma^2} k_2 \left(\frac{x}{R}\right)^2 - \frac{1+3\gamma^2}{\gamma^2} k_1 \frac{x}{R} + \frac{5\gamma^2-1}{\gamma^2} - 2\alpha_1$$

$$F_1\left(k_1, k_2, \frac{x}{R}\right) = -k_1 \frac{x}{R} + k_2 \left(\frac{x}{R}\right)^2$$

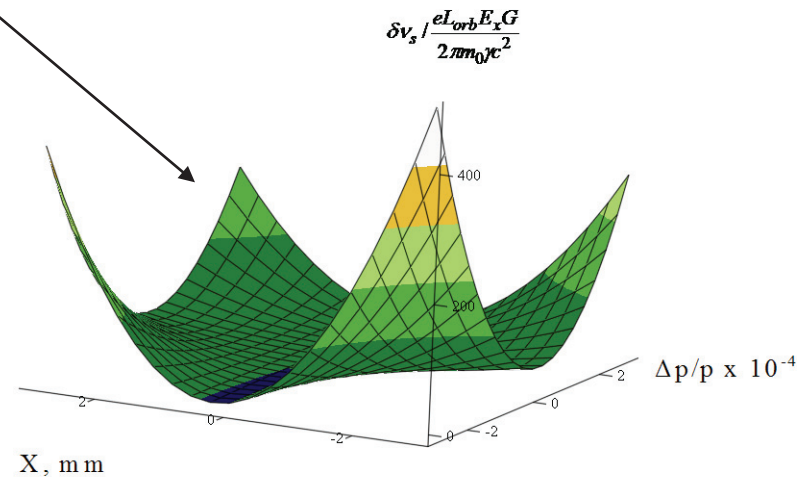
Spin tune aberration δv_s vs $\frac{\Delta p}{p}$ and x

Convex surface, or concave surface depends on the sign of $F_{1,2}$

$$F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right) \cdot \left(\frac{\Delta p}{p}\right)^2 + 2F_1\left(k_1, k_2, \frac{x}{R}\right) \cdot \frac{\Delta p}{p}$$



+



Two steps to minimize the spin aberrations

- The lattice with a compensation of the mutual influence of deflector parameters k_1, k_2 and lattice parameters α_1
- The lattice must provide the alternate change of the spin aberration surface from concave to convex and vice versa

First step

The maximum flatness of spin aberration surface is reached by choice of parameters of deflector k_1 , k_2 and α_1 momentum compaction factor.

After optimization: $k_1 = 0.94, k_2 = 0.96$ $\alpha = 0.6$

Electrostatic lattice consisting of electrostatic deflectors and electrostatic quadrupoles

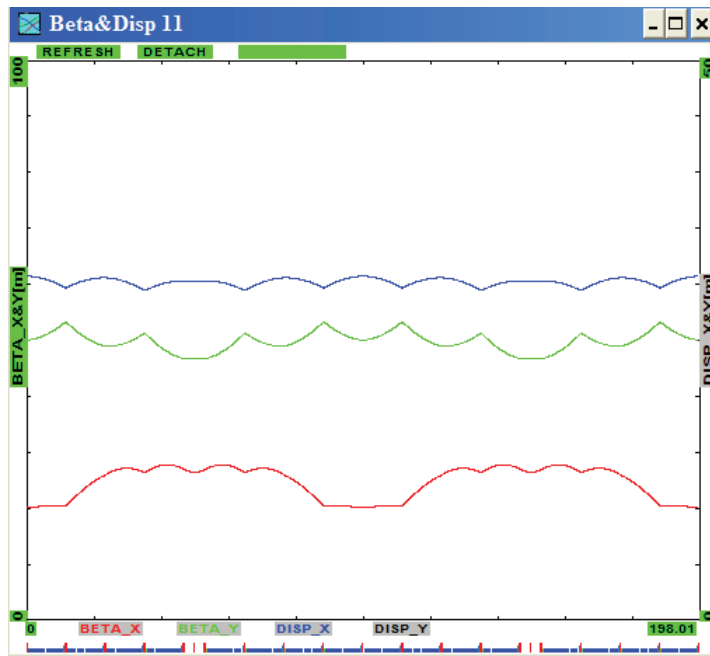


Figure: Twiss functions of electrostatic ring for ring and one cell

Method realization

Question is how to customize the required k_1 , k_2 ?

Second step: alternating spin aberration method

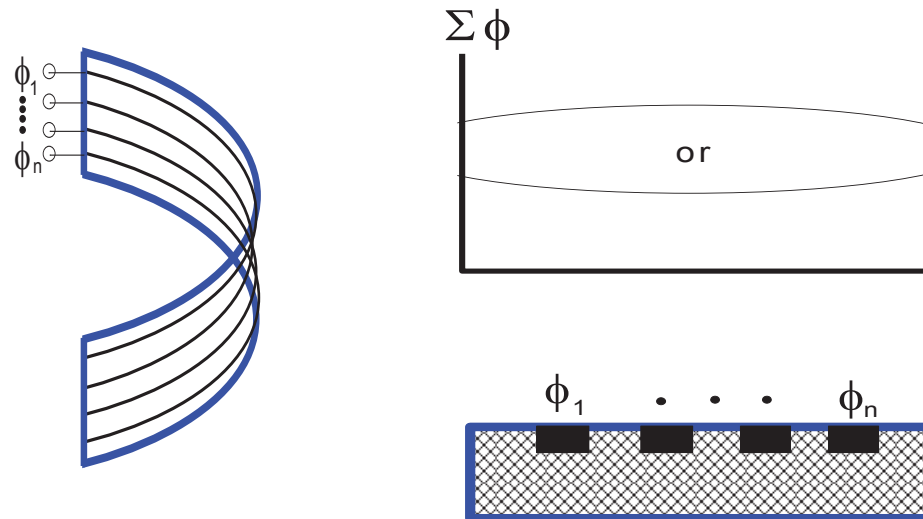
The ring is equipped with two types of deflector with

$k_1; k_2 = k_{av} + \delta k$ and $k_1; k_2 = k_{av} - \delta k$
changing from one deflector to another.

- In such optics is easier to achieve minimum spin aberration
- Raising the field strength between the plates in even deflectors and reducing in the odd deflectors it effectively adjusts the required coefficients k_1 and k_2 . It allows to adjust the spin of aberration to minimum.

Alternating spin aberration method

Another possibility is the creation of the required potential distribution due to potential changes in stripline deposited on the surface of the ceramic plates.



Such plates may be the additional corrective elements placed on the sides of the main deflector

The limit capabilities of alternating spin aberration method

The spread due to final $\Delta p/p$ it is impossible to remove completely using the correct k_1 and k_2 only. Nevertheless the total spread of spin deflection angle does not exceed ± 0.5 rad after billion turns, which one corresponds to a SCT about 5000 seconds.

Tracking results:

We used differential algebra methods realized in COSY Infinity and integrating program with symplectic Runge-Kutta methods.

1. Cylindrical deflector: after 10^6 turns $S_{x_{rms}} \approx 0.002$ that is SCT~500 sec
2. Alternating k_2 deflector: after 10^6 turns $S_{x_{rms}} \approx 0.0002$ that is SCT~5000 sec

Conclusion:

- we have studied the behavior of spin aberration in the structure and developed techniques to minimize it;
- one of the most effective methods is the alternating spin aberration;
- the analytical model allows finding the general solution of the retention of aberrations within the values allowed SCT to have about 5000 seconds confirmed by COSY-Infinity.

Nearest future plan:

- 3D shape deflector
- spin-orbital tracking in 3 D deflector
- including B field