A Fast Integrated Green Function Method for Computing 1D CSR Wakefields Including Upstream Transients

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Overview

- Longitudinal CSR models: a brief survey
- Integrated Green Function methods and implementation
- Error analysis and noise sensitivity
- Application to a Next Generation Light Source

1-D CSR models and upstream bend effects

Simulations using a 3D Lienard-Wiechert code with 6.24 billion particles indicate*
1-D models for the longitudinal CSR wakefield are robust when $\sigma_\perp << R(\sigma_z/R)^{2/3}$

Typical assumptions of 1-D models for the longitudinal CSR wakefield:

- Assume all bends and drifts are coplanar.
- Longitudinal bunch shape is unchanged during the radiation transit time; retardation effects occur only due to the motion of the bunch centroid.
- Small-angle and $1/\gamma$ approximations.

J. Murphy et al, Particle Accelerators, **57**, pp. 9-64 (1997). **Steady-state**

E. L. Saldin et al., Nucl. Instrum. Methods Phys. Res. A **398**, 373 (1997). **Bend entry and exit transients**

M. Dohlus and T. Limberg, Nucl. Instrum. Methods Phys. Res. A 393, 494 (1997). **General trajectories**

G. Stupakov and P. Emma, Proc. EPAC 2002, Paris, France, 1479 (2002). **UR approximation to Saldin**

D. Sagan et al., Phys. Rev. ST – Accel. Beams **12**, 040703 (2009). **Upstream bend effects**

C. Mayes and G. Hoffstaetter, Phys. Rev. ST – Accel. Beams **12**, 024401 (2009). **Upstream bend effects**

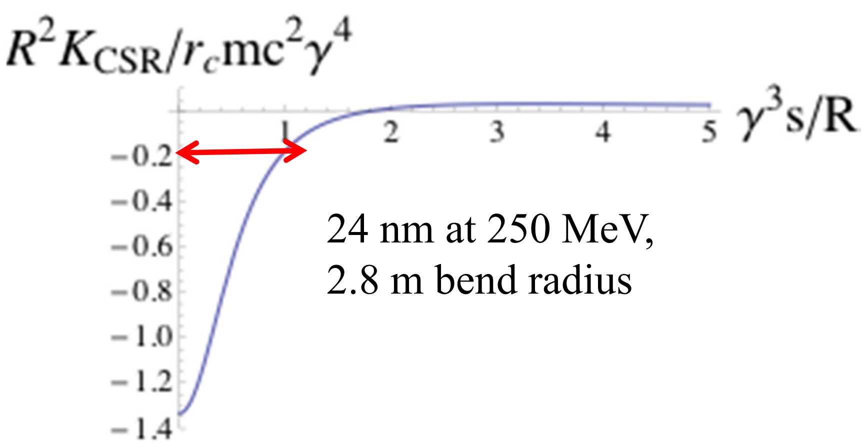
* R. D. Ryne et al,, Proc. IPAC 2012, New Orleans, Louisiana, 1689 (2012).

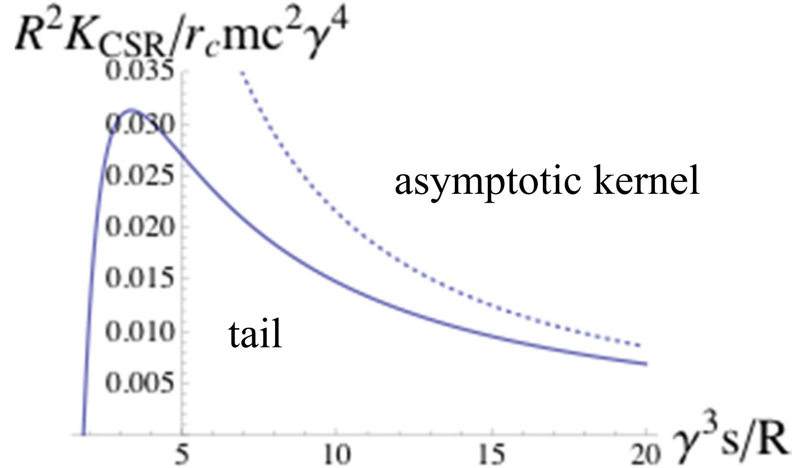
1-D CSR models and upstream bend effects

The longitudinal wakefield takes the form:

$$W(z) = \int_{-\infty}^{z} K_{CSR}(z - z_t)\lambda(z_t)dz_t \qquad K_{CSR}(s, s_t) = qn(s)\cdot\{E_{LW}[s, s'(s_t)] - E_{SC}[s, s'(s_t)]\}$$

Motivation: Accurately and efficiently model short-range behavior of the CSR wake interaction.





- Difference in scale between bunch length and short-range CSR interaction: difficult to resolve.

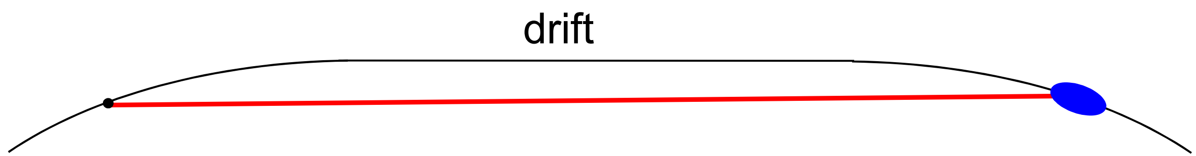
- Common 1-D CSR calculation: $$W(z) = \int_{-\infty}^{z} I_{CSR}(z - z_t)\frac{d\lambda}{dz}dz_t$$

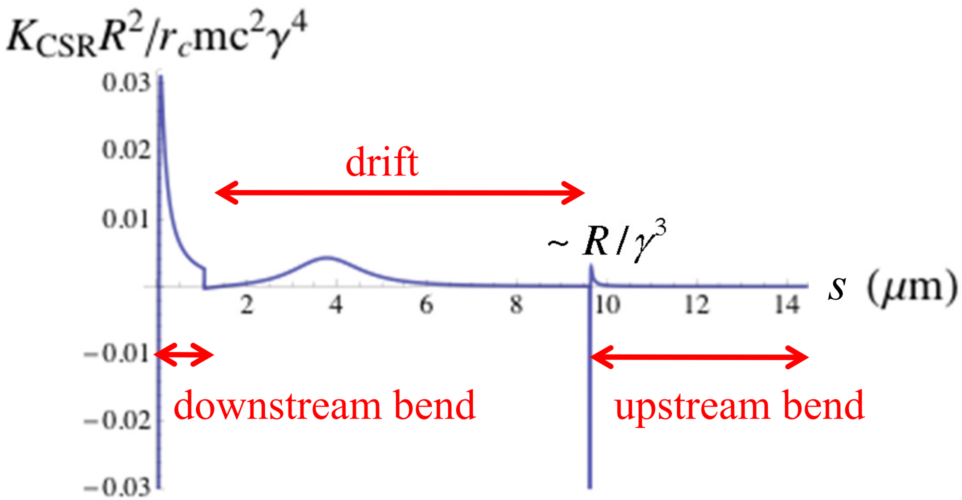
asymptotic kernel → I_{CSR}; numerical evaluation of derivative → $\frac{d\lambda}{dz}$

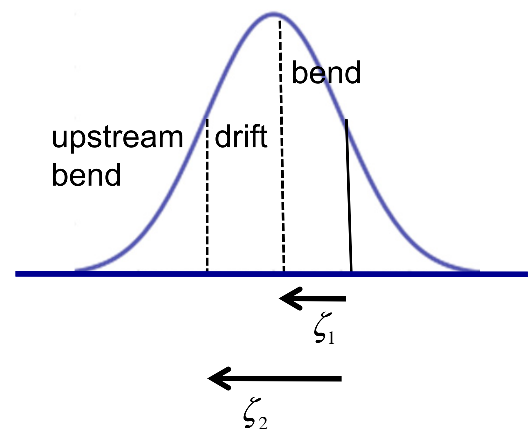
1-D CSR models and upstream bend effects

The propagation of radiation downstream across multiple lattice elements can be included:

- The kernel K_{CSR} depends on both the location of the observation point and the location of the source point within the lattice.
- The retardation condition must in general be inverted numerically.







Difference in scales between variations in bunch and kernel: IGF needed

Integrated Green Function Method and Implementation

The longitudinal wakefield as a function of bunch coordinate takes the form:

$$W(z) = \int_{-\infty}^{z} \lambda(z')K_{CSR}(z,z')dz' \qquad \lambda(z) \approx \sum_{j=1}^{N} \lambda_j P_j(z)$$

Approximate the charge density using a basis of piecewise polynomials P_{j}.

Then we have:

$$W(z_k) \approx \sum_{j=1}^{N} \lambda_j \int_{-\infty}^{z} P_j(z')K_{CSR}(z_k,z')dz' = h\sum_{j=1}^{N} \lambda_j w_{k,j}^{igj} \quad \text{where} \quad w_{k,j}^{igf} = \int_{-\infty}^{z_k} P_j(z')K_{CSR}(z_k,z')dz.$$

K_{CSR} is a rational function of retarded variables: each basis integral can be evaluated *exactly* using partial fraction decomposition in terms of rational functions, log, arctan in retarded variables.

In general a bracketed Newton's method search is used for root-finding to solve for the retarded position corresponding to a given path separation.

Evaluate the convolution sum *using an FFT*.

R. D. Ryne et al., http://arxiv.org/abs/1202.2409 (2012).
Ji Qiang et al., Nucl. Instrum. and Meth. in Phys. Res. A **682**, 49-53 (2012).

Integrated Green Function Method and Implementation

Example: piecewise constant basis (implemented in IMPACT)

Case A

$$\frac{RI_{CSR}}{\gamma r_c mc^2} = -\frac{2(\hat{\phi}+\hat{y})+\hat{\phi}^3}{(\hat{\phi}+\hat{y})^2+\hat{\phi}^4/4}+\frac{1}{\hat{s}}, \quad \text{where} \quad \hat{s} = \frac{\hat{\phi}+\hat{y}}{2}+\frac{\hat{\phi}^3}{24}\frac{\hat{\phi}+4\hat{y}}{\hat{\phi}+\hat{y}}.$$

Case B

$$\frac{RI_{CSR}}{\gamma r_c mc^2} = -\frac{4\hat{u}(\hat{u}^2+8)}{(\hat{u}^2+4)(\hat{u}^2+12)}, \quad \text{where} \quad \hat{s} = \frac{\hat{u}^3}{24}+\frac{\hat{u}}{2}.$$

Case C

$$\frac{RI_{CSR}}{\gamma r_c mc^2} = -\frac{2(\hat{\phi}_m+\hat{x}+\hat{y}+\hat{\phi}_m^3/2+\hat{\phi}_m^2\hat{x})}{(\hat{x}+\hat{y}+\hat{\phi}_m)^2+(\hat{\phi}\hat{x}+\hat{\phi}_m^2/2)^2}+\frac{1}{\hat{s}}, \quad \text{where}$$

$$\hat{s} = \frac{\hat{\phi}+\hat{x}+\hat{y}}{2}+\frac{\hat{\phi}_m^2}{24}\frac{\hat{\phi}_m^2+4\hat{\phi}_m(\hat{x}+\hat{y})+12\hat{x}\hat{y}}{\hat{\phi}+\hat{x}+\hat{y}}.$$

Case D

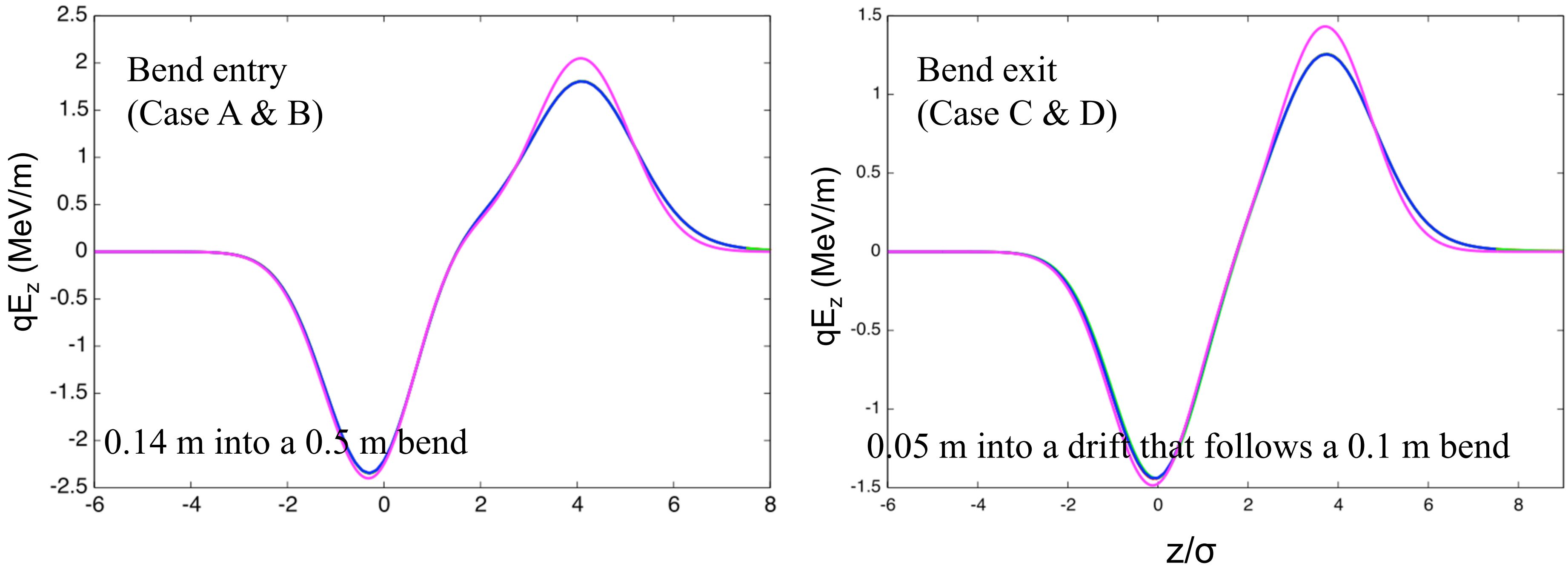
$$\frac{RI_{CSR}}{\gamma r_c mc^2} = -\frac{2(\hat{\psi}+\hat{x}+\hat{\psi}^3/2+\hat{\psi}^2\hat{x})}{(\hat{x}+\hat{\psi})^2+(\hat{\psi}\hat{x}+\hat{\psi}^2/2)^2}+\frac{1}{\hat{s}}, \quad \text{where} \quad \hat{s} = \frac{\hat{\psi}+\hat{x}}{2}+\frac{\hat{\psi}^2}{24}\frac{\hat{\psi}^2+4\hat{x}\hat{\psi}}{\hat{\psi}+\hat{x}}.$$

Integrated Green function $$w_{k,k'}^{igf} = \frac{1}{h}\left[I_{CSR}\left(h(k-k')+\frac{h}{2}\right)-I_{CSR}\left(h(k-k')-\frac{h}{2}\right)\right] \qquad k'<k$$

E. L. Saldin et al., Nucl. Instrum. Methods Phys. Res. A **398**, 373 (1997).

Comparison of 1-D models

1nC, 50 µm Gaussian bunch at 150 MeV; bend with radius R = 1.5 m*



IGF method obtains the same accuracy as direct integration with a factor of 100 fewer sample points

IGF 1024 points
Non-IGF 104312 points
Stupakov and Emma* ($\gamma \rightarrow \infty$)

*G. Stupakov and P. Emma, Proc. EPAC 2002, Paris, France, 1479 (2002).

Error Analysis and Noise Sensitivity

For simplicity, define f and g by: $f(z')=\lambda(z'),\quad g(z')=K_{CSR}(z,z')$.

Compare local error in $\int_{z_j}^{z_{j+1}} f(z')g(z')dz'$ for several related algorithms.

1) Direct integration: $E_{direct}=\frac{h^3}{12}\left[f''(z_j)g(z_j)+2f'(z_j)g'(z_j)+f(z_j)g''(z_j)\right]+O(h^4)$

2) IGF method – constant basis: $E_{const}^{igf}=\frac{h^3}{12}\left[f''(z_j)g(z_j)+\frac{1}{2}f'(z_j)g'(z_j)\right]+O(h^4)$

3) IGF method – linear basis: $E_{lin}^{igf}=\frac{h^3}{12}\left[f''(z_j)g(z_j)-f'(z_j)g'(z_j)\right]+O(h^4)$

4) Direct integration by parts: $E_{lin}^{igf}=\frac{h^3}{12}\left[f^{(3)}(z_j)G(z_j)-2f''(z_j)g(z_j)-f'(z_j)g'(z_j)\right]+O(h^4)$

1) Integration by parts with finite differences: $E_{ibp}^{appx}=\frac{h^3}{12}\left[f''(z_j)g(z_j)-f'(z_j)g'(z_j)\right]+O(h^4)$

Derivatives of the CSR kernel:

$$\frac{d^n g}{dz^n}\sim\left(\frac{\gamma^{\mathbf{4+3n}}}{R^{\mathbf{2+n}}}\right)r_c mc^2$$

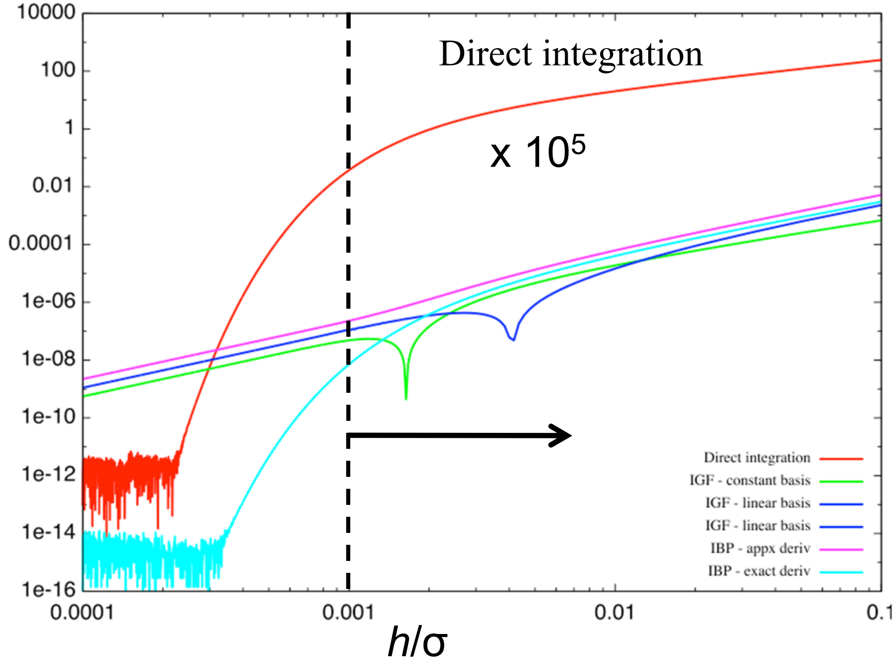
Integral of the CSR kernel:

$$G(z)=-\int_{-\infty}^{z} g(z')dz'\sim\left(\frac{\gamma}{R}\right)r_c mc^2$$

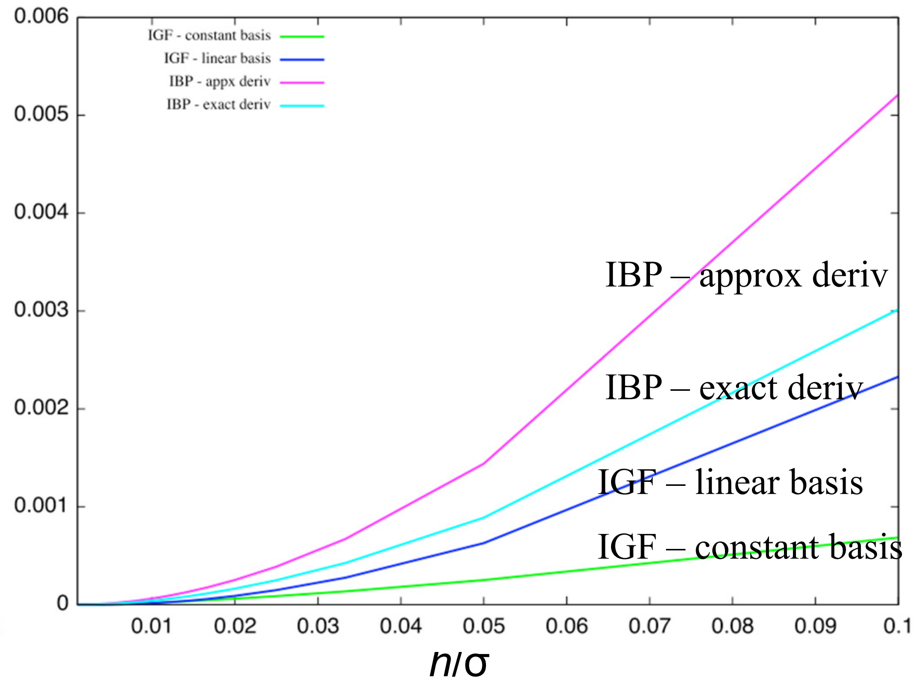
Error Analysis and Noise Sensitivity

Relative error in the longitudinal CSR wake as a function of stepsize at the centroid of a Gaussian bunch ($z = 0$) with E=200 MeV, R=10 m, σ=0.1 mm.

log scale

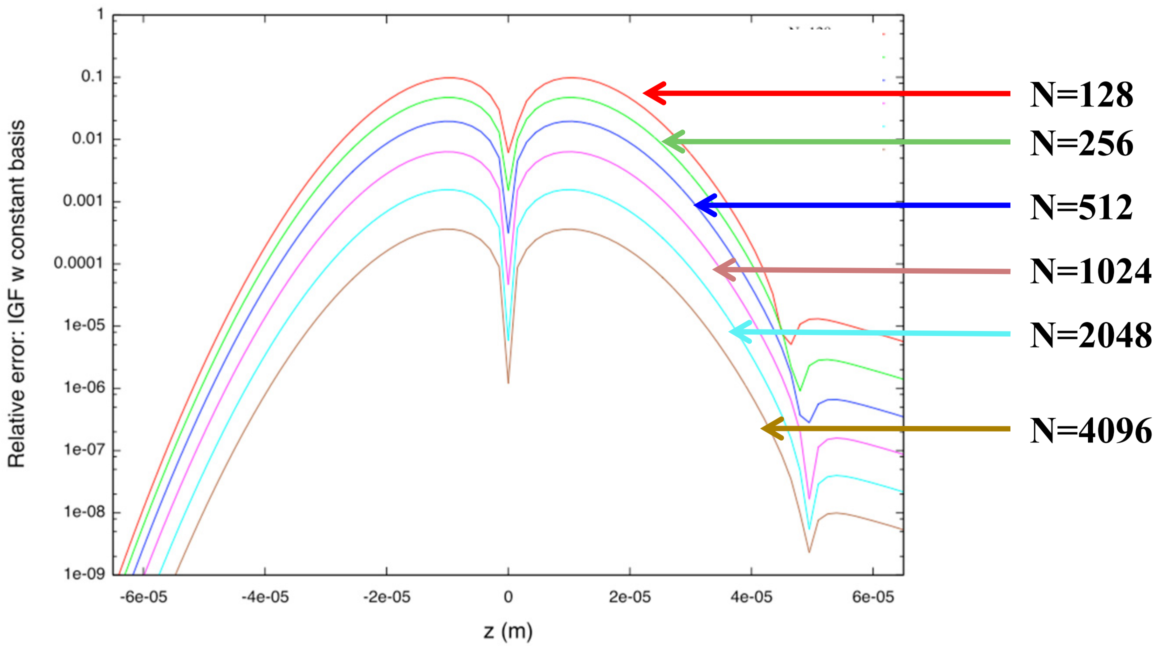


linear scale



Error Analysis and Noise Sensitivity

Relative error in the longitudinal CSR wake along a Gaussian bunch for various stepsizes, with E=100 MeV, R=1 m, σ=10 μm, computed using an IGF method with piecewise constant basis.



Global error: $$E^{igf}_{const,global} = \frac{h^2}{24}\left[\lambda'(z)K_{CSR}(z,z) - \lambda'(z_{\min})K_{CSR}(z,z_{\min}) + \int_{z_{\min}}^{z} \lambda''(z')K_{CSR}(z,z')dz'\right] + O(h^3)$$

Error Analysis and Noise Sensitivity

Shot noise model (N_p particles into N bins):

$$\lambda_j^r = \lambda_j(1+\varepsilon_j), \qquad j = 1,\ldots,N$$

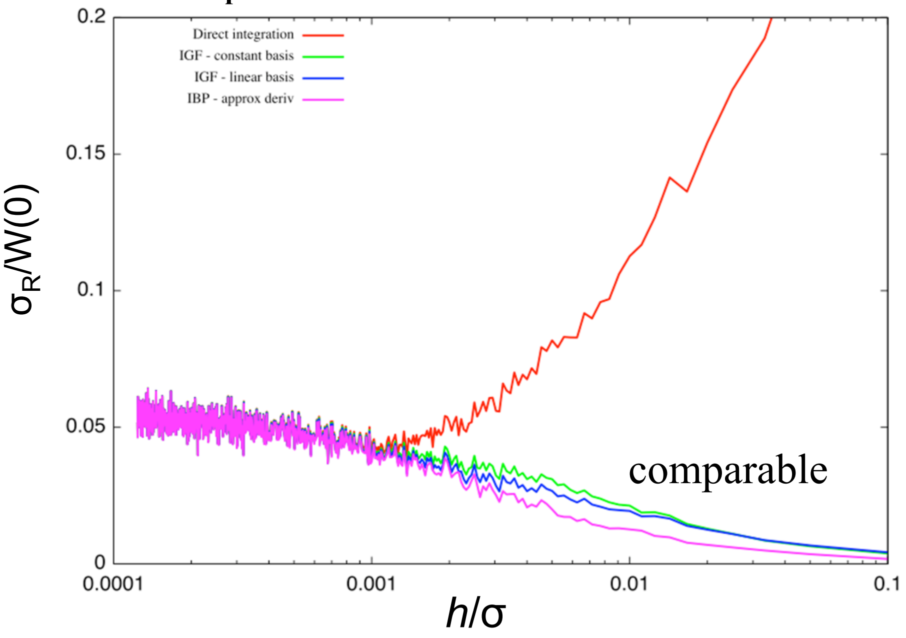
$$\langle \varepsilon_j \rangle = 0, \quad \langle \varepsilon_j^2 \rangle = \frac{N}{N_p} = \frac{L}{hN_p}$$

Limiting value of rms noise at centroid:

$$\lim_{h\to 0} \sigma_R(z) = \sqrt{\frac{L}{N_p}} ||\lambda K_{CSR}(z,\cdot)||$$

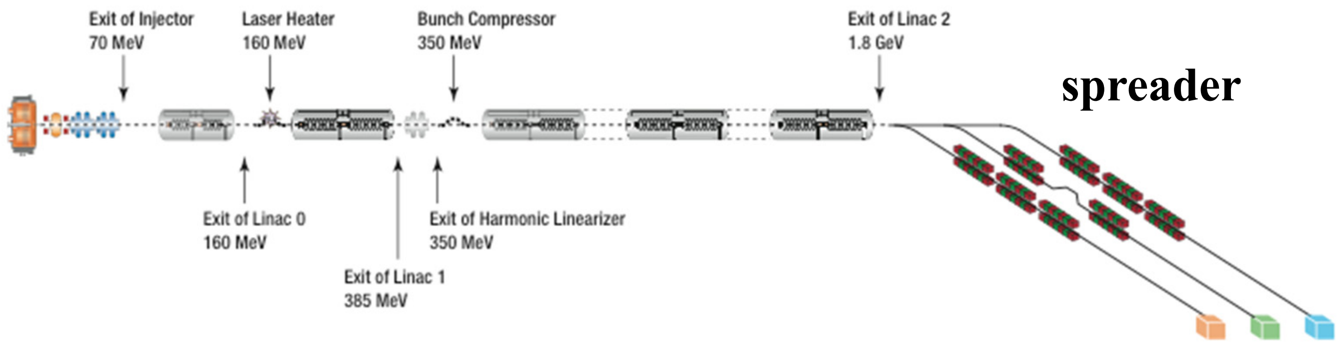
$$||\lambda K_{CSR}(z,\cdot)||^2 = \int_{-z_m}^{z} |\lambda(z')K_{CSR}(z,z')|^2 dz'$$

$N_p=10^6$, L=20σ, 100 random seeds



How do we maintain sensitivity to microbunching structure, while limiting sensitivity to random noise?

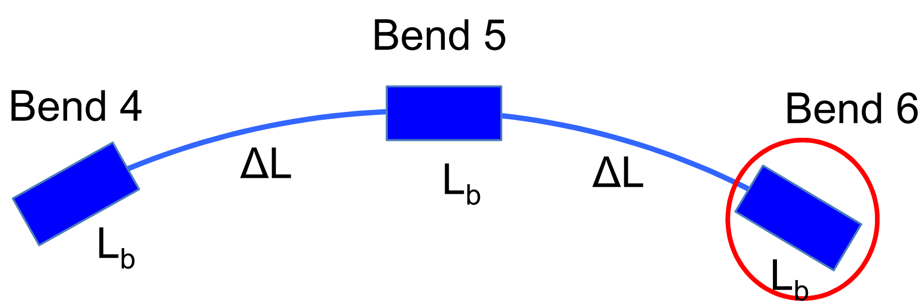
Application to a Next Generation Light Source



Final 3 spreader dipoles: Consider the propagation of CSR downstream in a single extraction line.

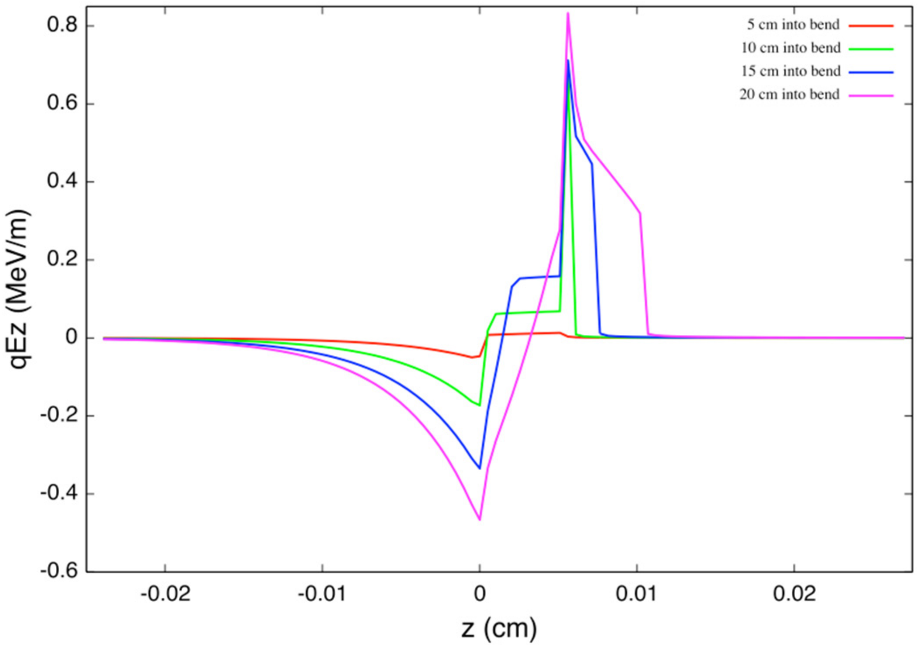
Parameters

Beam energy: 2.4 GeV
Peak current: 1.01 kA
Bunch charge: 300 pC
Drift length (ΔL): 6.735 m
Bend arc length (L_b): 0.902 m
Bend angle: 0.176437 rad

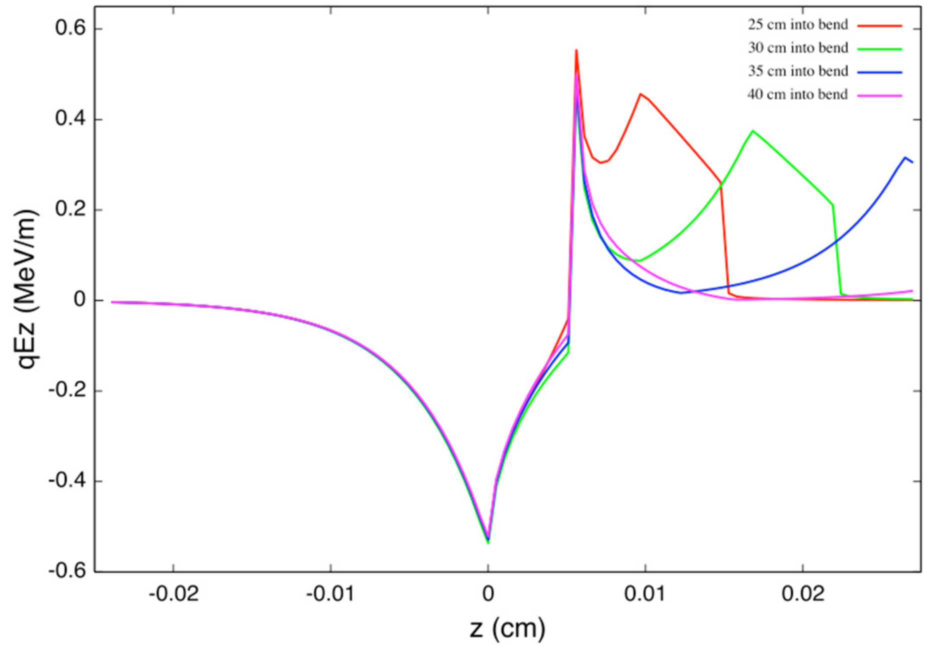


Examination of the effect of CSR from bends 4 & 5 on the wake in bend 6.

CSR Wake Inside NGLS Spreader Bend 6, Including Upstream CSR from Bends 4 and 5 and Associated Drifts



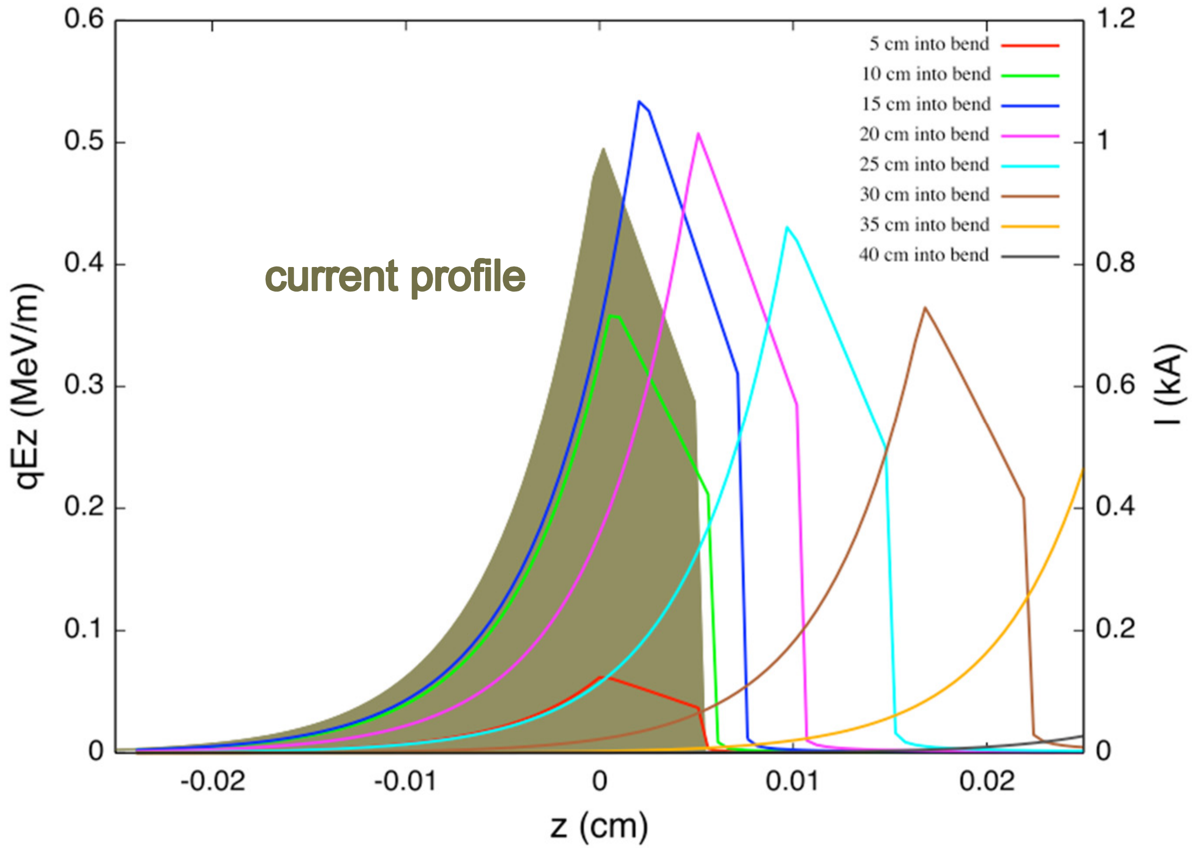
5-20 cm into Bend 6



25-40 cm into Bend 6

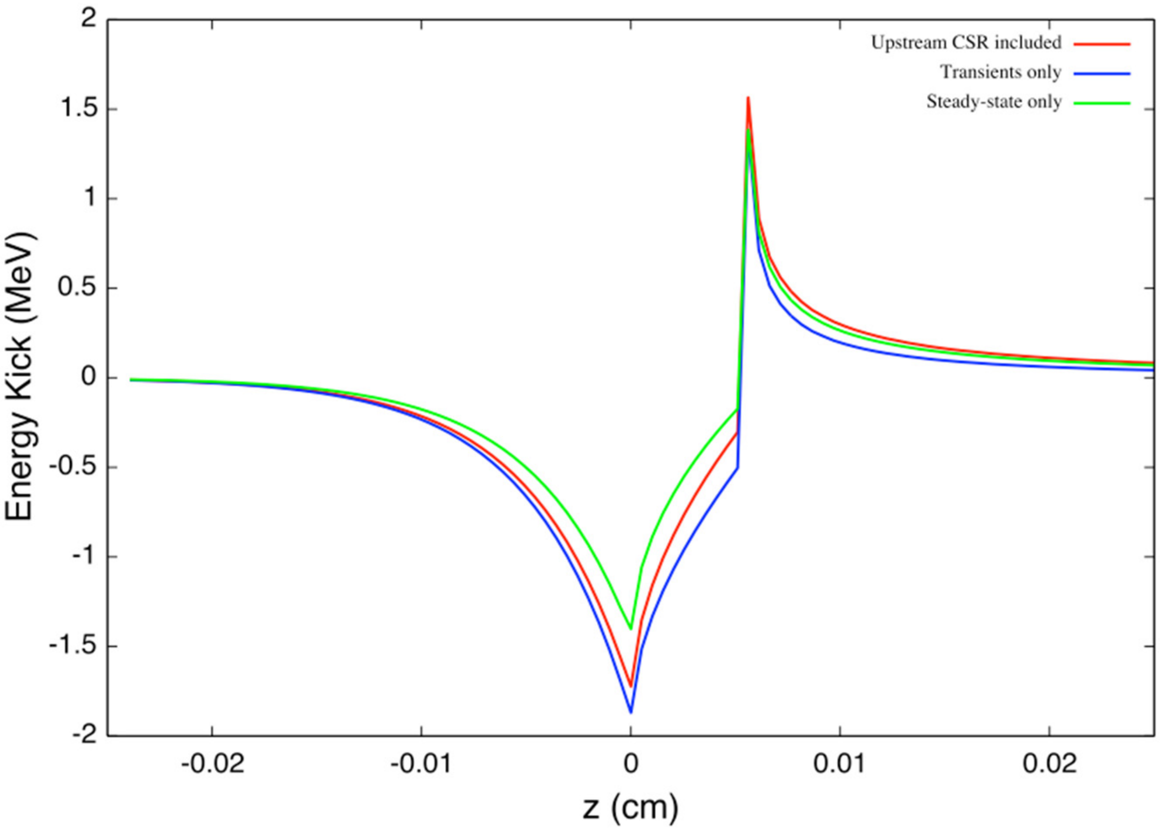
Steady-state is approached near 45 cm into the bend.

Upstream Contribution from Bends 4, 5, and Associated Drifts to CSR Wake Inside NGLS Spreader Bend 6



- The location of the upstream CSR wake shifts slowly with respect to the bunch centroid within the bend
- The behavior of the wake peak is understood analytically, with maximum occurring 16 cm into the bend.
- Beyond 40 cm, CSR from upstream bends is decoupled from the bunch.

Net Energy Kick Through NGLS Spreader Region Defined by Bends 4-6 with and without the Effect of CSR from Upstream Bends and associated Drifts



Upstream bends included
Transients only included
Steady-state only

12% effect from upstream bends

Summary

- IGF techniques have been implemented in IMPACT to treat 1) transient fields due to bend entry and exit, 2) CSR from multiple upstream bends and drifts using a 1-D model.
- Short-range behavior of the CSR wake is captured, and convergence is set by resolution of the charge density (not the CSR kernel). No numerical differentiation required.
- Error estimates reveal $O(h^3)$ convergence of local error, robust against shot noise.
- This method has been applied to NGLS spreader dipoles: upstream CSR contributes 12% of total energy kick experienced by particles in the NGLS spreader.
- Implementation of vertical shielding effects is ongoing.

Error Analysis and Noise Sensitivity

1) Direct integration: $\int_a^b f(z)g(z)dz \approx \frac{h}{2} f_1 g_1 + \frac{h}{2} f_N g_N + \sum_{j=2}^{N-1} h f_j g_j$

2) IGF method – constant basis: $\int_a^b f(z)g(z)dz \approx f_1[G(a) - G(a+h/2)] + f_N[G(b-h/2) - G(b)] + \sum_{j=2}^{N-1} f_j [G(z_j - h/2) - G(z_j + h/2)]$

3) IGF method – linear basis:
$$\int_a^b f(z)g(z)dz \approx f_1[G(a) + (H(a+h) - H(a))/h] + f_N[G(b) + (H(b) - H(b-h)/h]$$
$$+\frac{1}{h}\sum_{j=2}^{N-1} f_j [H(z_{j-1}) - 2H(z_j) + H(z_{j+1})]$$

4) Direct integration by parts: $\int_a^b f(z)g(z)dz \approx f_1 G(a) - f_N G(b) + \sum_{j=2}^{N-1} h f_j G(z_j) + \frac{h}{2} f_1' G(a) + \frac{h}{2} f_N' G(b)$

5) Integration by parts with finite differences: $\int_a^b f(z)g(z)dz \approx \frac{f_1}{2}[G(a) - G(a+h)] + \frac{f_N}{2}[G(b-h) - G(b)] + \frac{1}{2}\sum_{j=2}^{N-1} f_j [G(z_j - h) - G(z_j + h)]$

where $G(z) = -\int_{-\infty}^{z} g(z')dz'$, $\quad H(z) = -\int_{-\infty}^{z} G(z')dz'$

Error Analysis and Noise Sensitivity

1) Direct integration: $\int_{z_{j-1}}^{z_j} f(z)g(z) \approx \frac{1}{2}(f_j g(z_j) + f_{j-1}g(z_{j-1}))/h$

For the remaining methods, it is convenient to write

$$I = \int_{z_{j-1}}^{z_j} f(z)g(z)dz = f_{f-1}G(z_{j-1}) - f_j G(z_j) + \int_{z_{j-1}}^{z_j} f'(z)G(z)dz$$, with

2) IGF method – constant basis: $\int_{z_{j-1}}^{z_j} f'(z)G(z) \approx (f_j - f_{j-1})G(z_j - h/2)$

3) IGF method – linear basis: $\int_{z_{j-1}}^{z_j} f'(z)G(z) \approx (f_j - f_{j-1})(H(z_{j-1}) - H(z_j))/h$

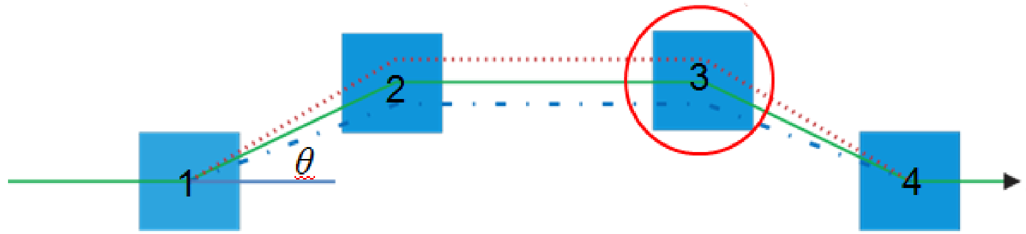
4) Direct integration by parts: $\int_{z_{j-1}}^{z_j} f'(z)G(z) \approx \frac{1}{2}(f'_{j-1}G(z_{j-1}) + f'_j G(z_j))/h$

5) Integration by parts with finite differences: $\int_{z_{j-1}}^{z_j} f'(z)G(z) \approx (f_j - f_{j-1})(G(z_j) + G(z_{j-1}))$

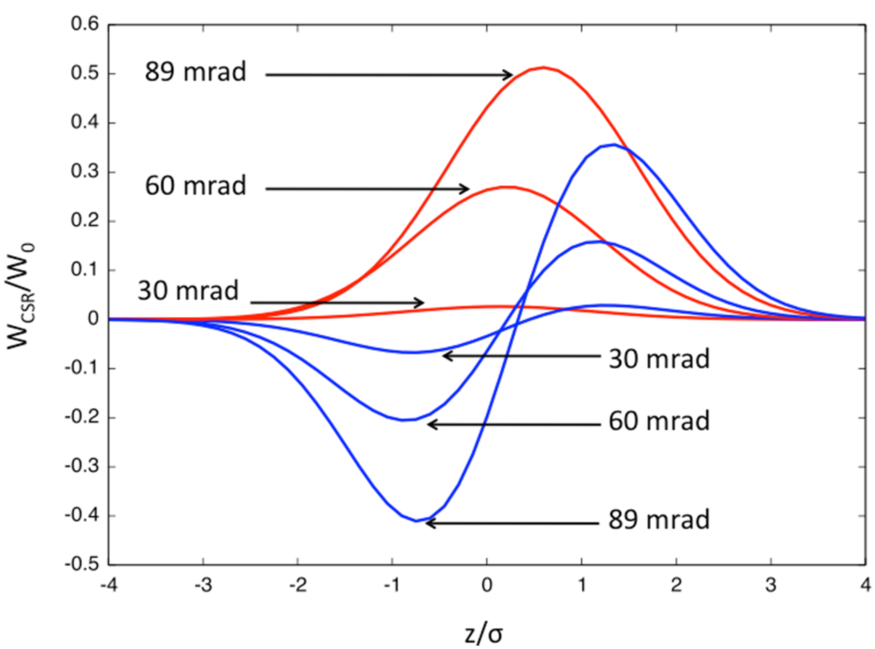
1-D CSR models and upstream bend effects

NGLS BC1 Chicane

Drift length (ΔL): 4.5 m
Bend length (L_b): 0.25 m
Bend angle: 90 mrad



CSR wake in Bend 3 for a 0.3 nC Gaussian bunch at 250 MeV with rms length 577 μm:



Blue – total CSR wake

Red – contribution from Bend 2

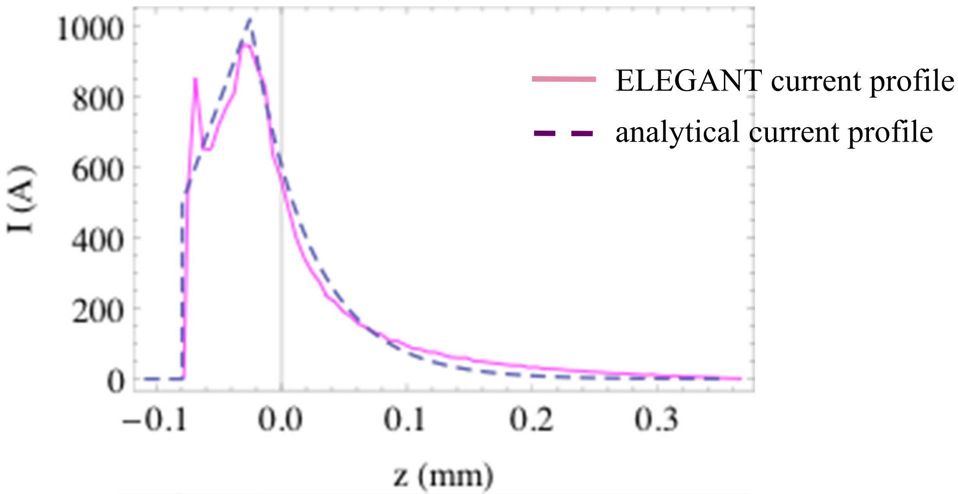
39%, 132%, and 126% of total

Scaling factor:

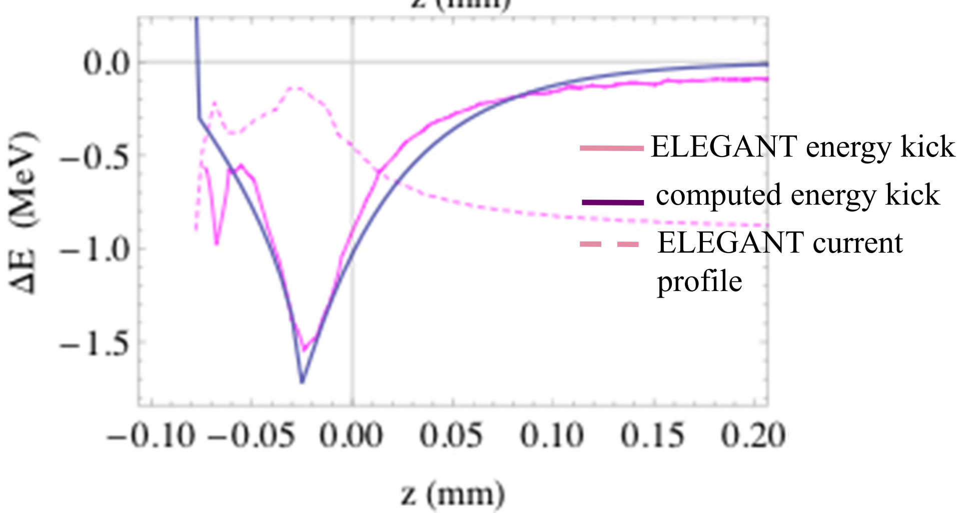
$$W_0 = N r_c m c^2 \frac{(\sigma/R)^{2/3}}{\sigma^2} = 28.2 \text{ keV/m}$$

Upstream CSR can dominate!

Comparison against ELEGANT



Current profile



CSR-induced E kick

- ELEGANT - cf longitudinal phase space just before Bend 4 with phase space at the end of Bend 6.
- Reasonable agreement without upstream CSR included.