ICAP2012, 23 August, Warnemünde

Calculation of Longitudinal Instability Threshold Currents for Single Bunches

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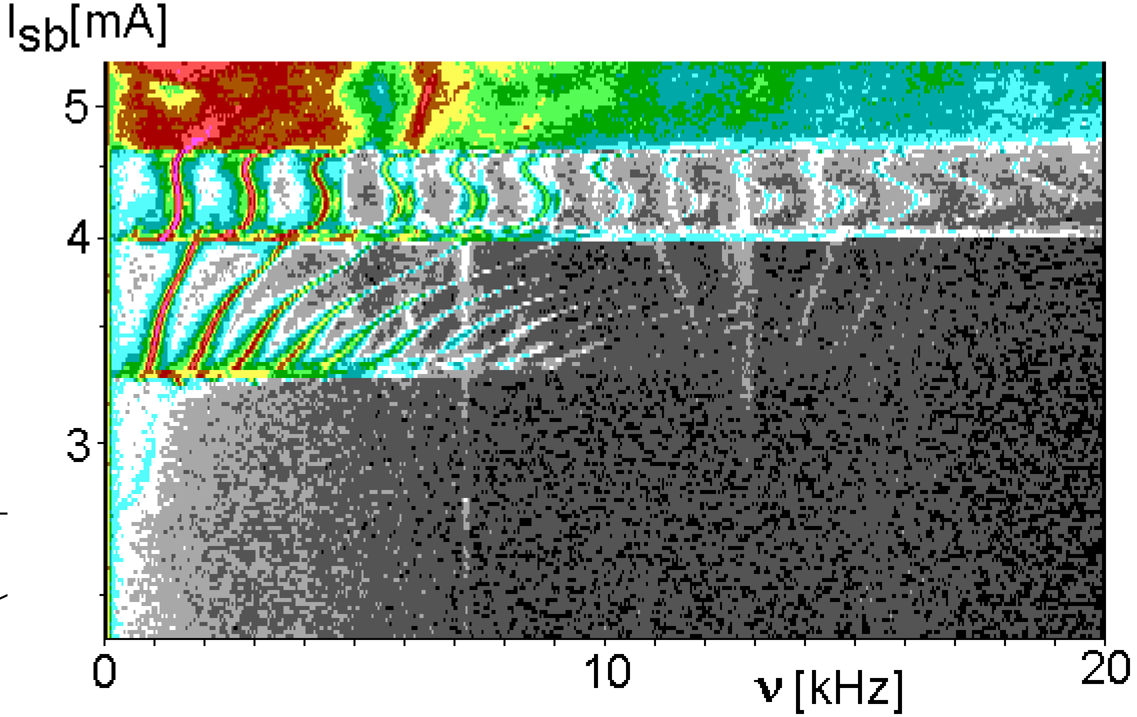
Content of the Talk

I. Observations at BESSY II

time dependent CSR-signal
observed in frequency domain:

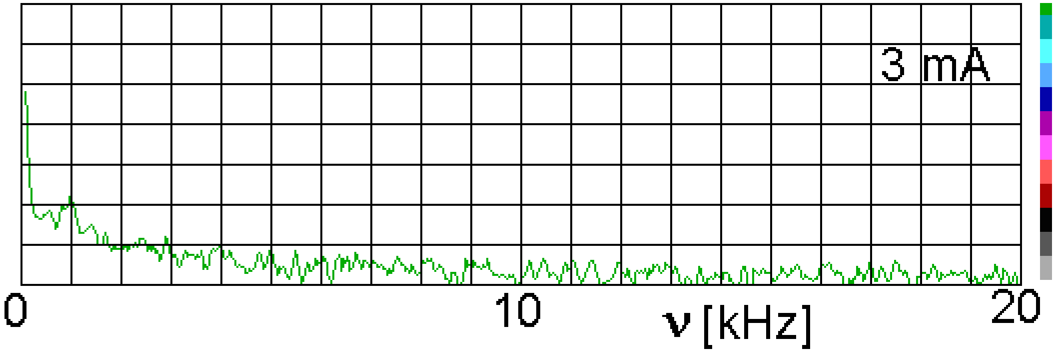
σ_0=14 ps, nom. optics, with 7T-WLS



CSR-bursting threshold

Stable, time independent CSR

Spectrum of the CSR-signal:



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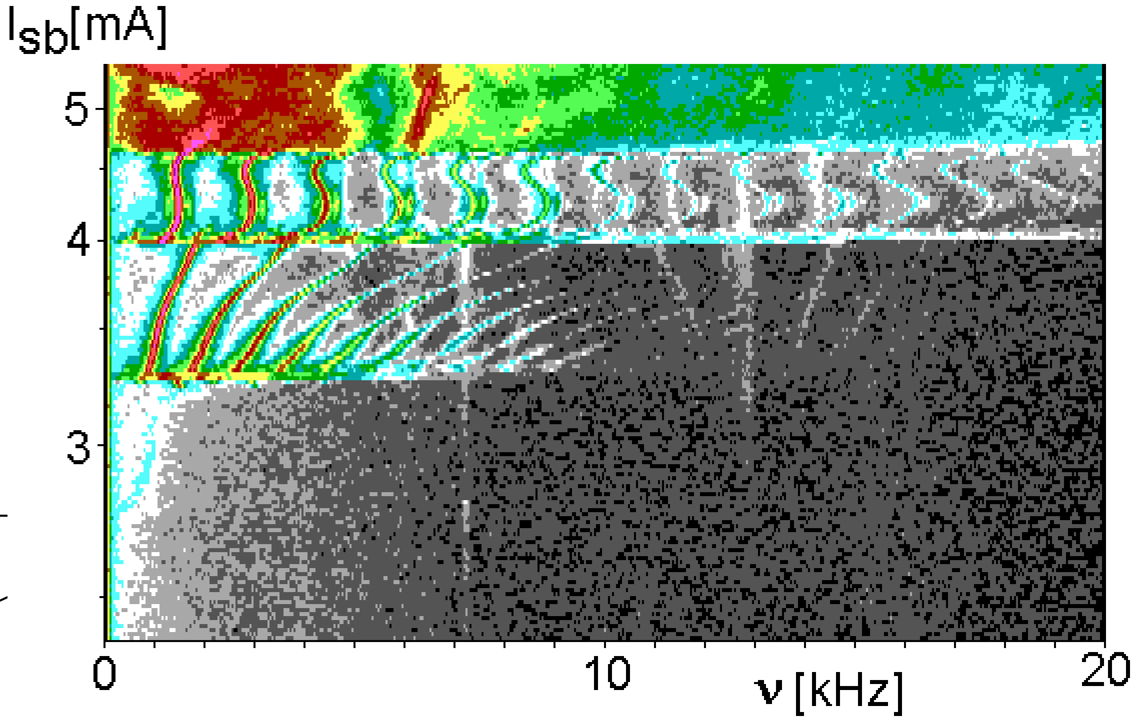

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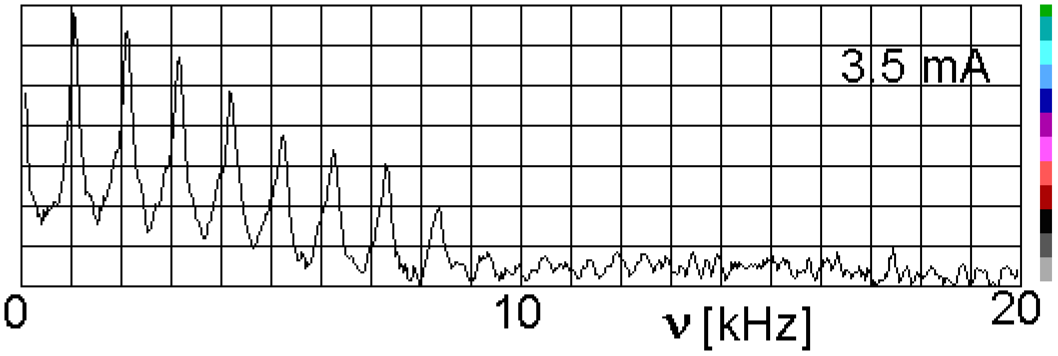

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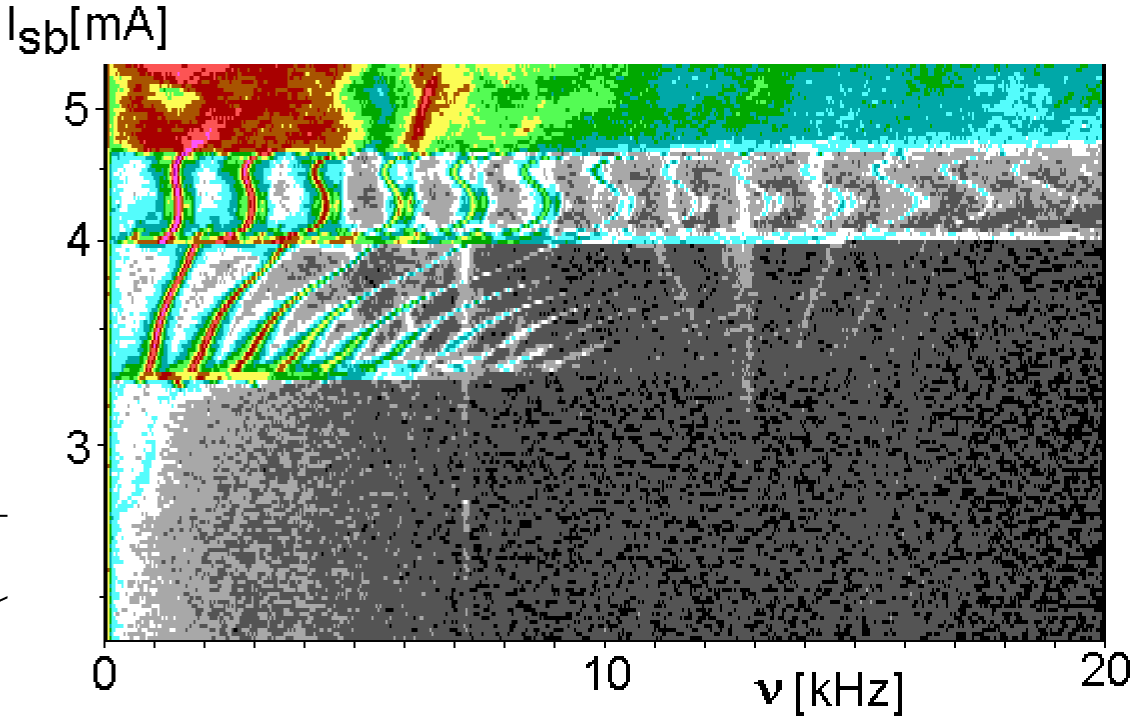

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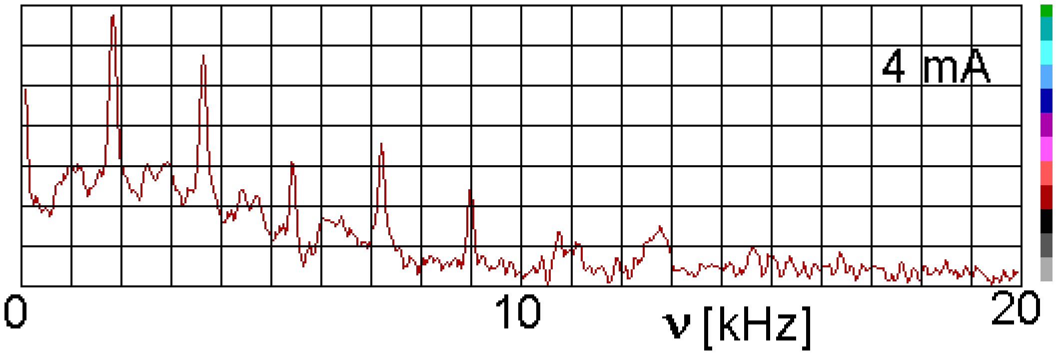

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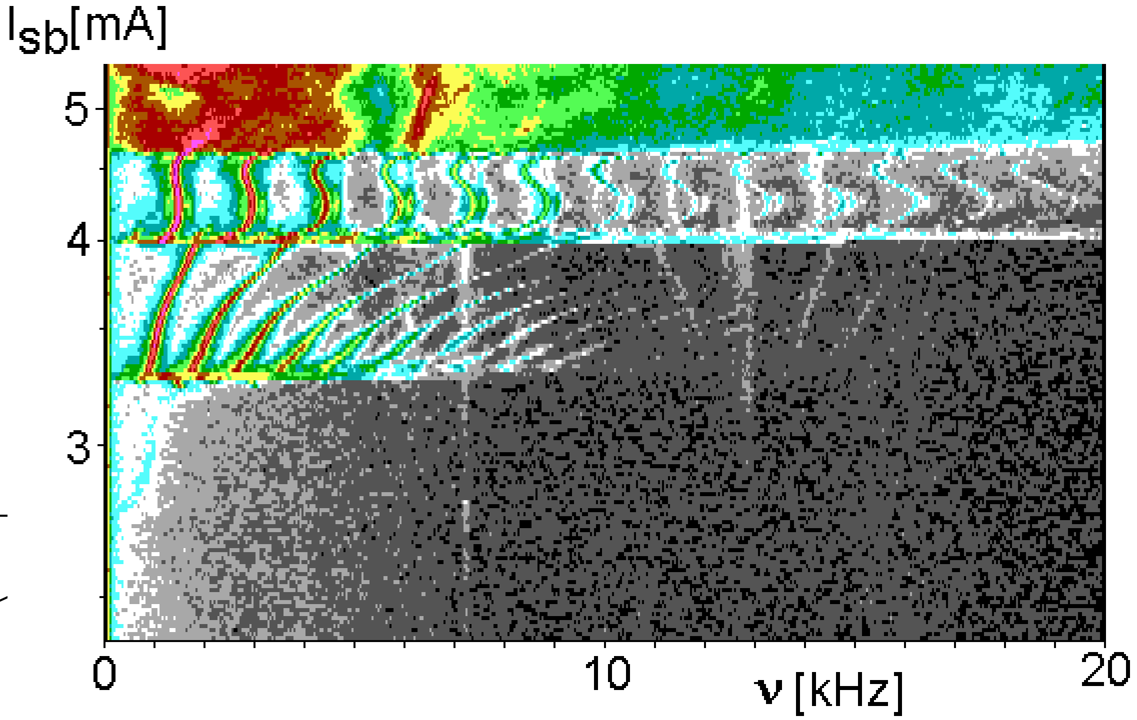

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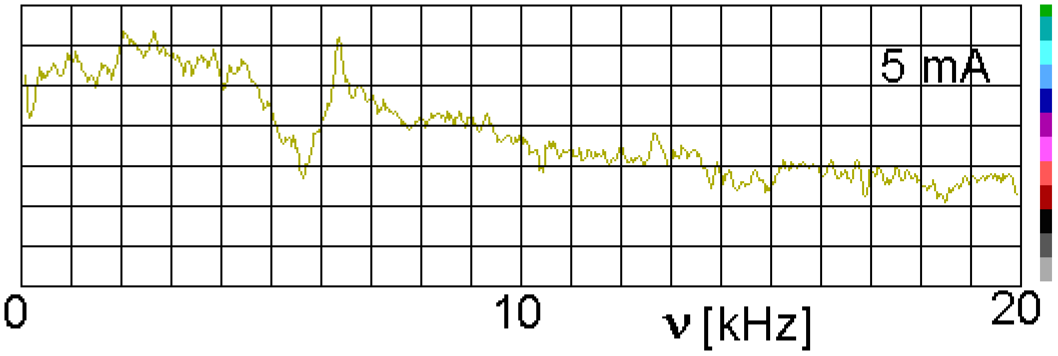

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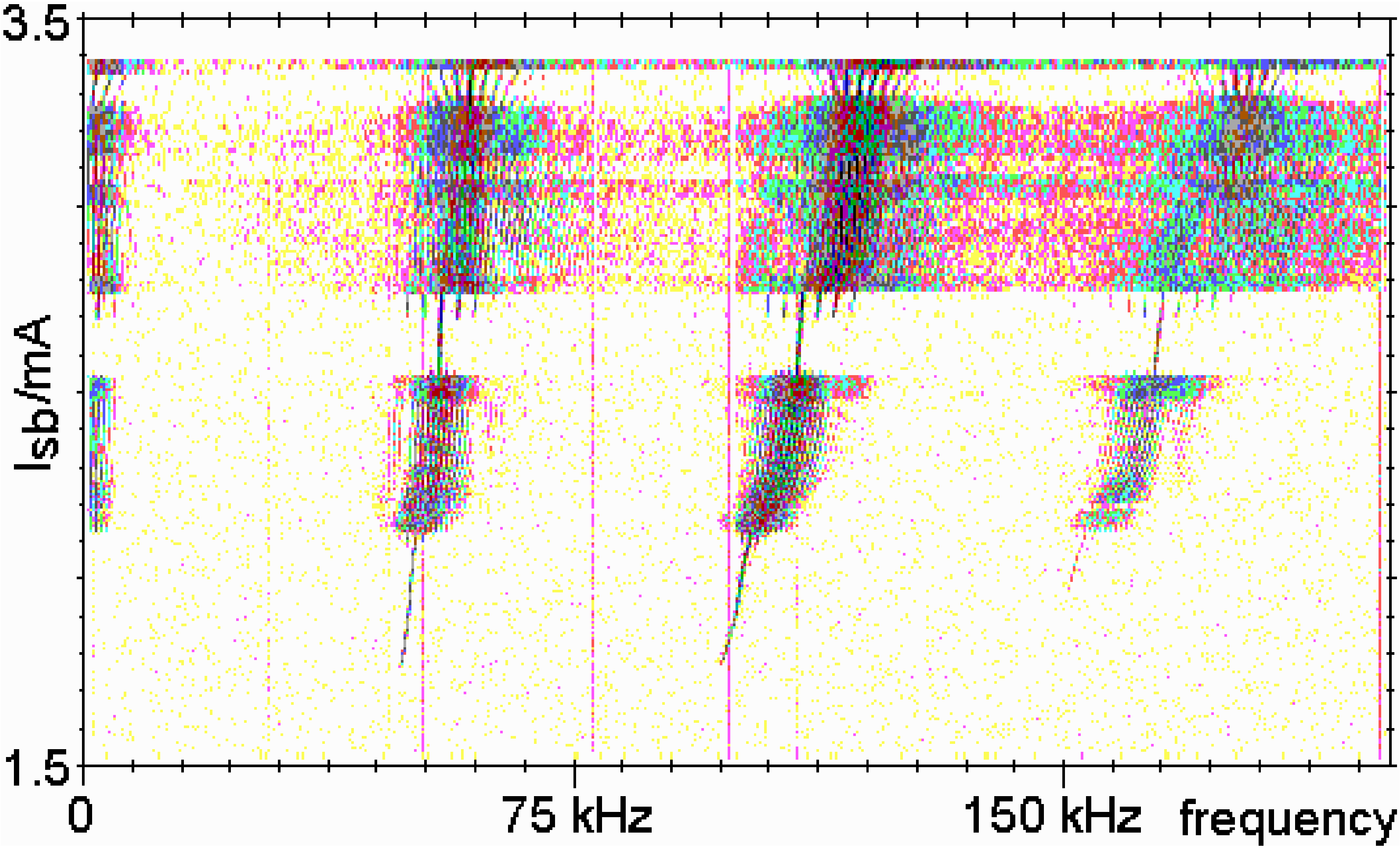


II. Vlasov-Fokker-Planck-Equation (VFP)

N>10^9 electrons per bunch → smooth distribution in phase space

→ distribution function:

$$f(q, p, \tau)$$

$$q = z / \sigma_z$$

$$p = -\Delta E / \sigma_E$$

$$\tau = \omega_s t$$

$$\frac{\partial f}{\partial \tau} + p \frac{\partial f}{\partial q} - [q + F_c(q, \tau, f)] \frac{\partial f}{\partial p} = \frac{2}{\omega_s t_l} \frac{\partial}{\partial p} \left(pf + \frac{\partial f}{\partial p} \right)$$

(M. Venturini)

RF focusing Collective Force Damping Quantum Excitation

→ Numerical solution based on
M. Venturini, et al., Phys. Rev. ST-AB 8, 014202 (2005)
Other numerical solutions:
R.L. Warnock, J.A. Ellison, SLAC-PUB-8404, March 2000
S. Novokhatski, EPAC 2000 and SLAC-PUB-11251, May 2005

II.1 Vlasov-Fokker-Planck-Equation (VFP) „Wave Function“ Approach

original VFP-equation:

$$\frac{\partial f}{\partial \tau}+p\frac{\partial f}{\partial q}-\left[q+F_c(q,\tau,f)\right]\frac{\partial f}{\partial p}=\frac{2}{\omega_s t_l}\frac{\partial}{\partial p}\left(pf+\frac{\partial f}{\partial p}\right)$$

Ansatz – “wave function” approach: Distribution function, f, expressed as product of amplitude function, g:

$$f=g\cdot g$$

$$\frac{\partial g}{\partial \tau}+p\frac{\partial g}{\partial q}-\left[q+F_c(q,\tau,g^2)\right]\frac{\partial g}{\partial p}=$$

$$\frac{2}{\omega_s t_l}\left(\frac{g}{2}+p\frac{\partial g}{\partial p}+\frac{1}{g}\left(\frac{\partial g}{\partial p}\right)^2+\frac{\partial^2 g}{\partial p^2}\right)$$

f >= 0 and solutions numerically more stable

III. Numerical Solution of the VFP Equation

$$\frac{\partial g}{\partial \tau} = \frac{g(q,p,\tau+\Delta\tau)-g(q,p,\tau)}{\Delta\tau} = -p\frac{\partial g}{\partial q} + [q+F_c]\frac{\partial g}{\partial p} + \frac{2}{\omega_s\tau_l}\left(\frac{g}{2} + p\frac{\partial g}{\partial p} + \frac{1}{g}\left(\frac{\partial g}{\partial p}\right)^2 + \frac{\partial^2 g}{\partial p^2}\right)$$

$$g(q,p,\tau+\Delta\tau) = g(q,p,\tau) - p\Delta\tau\frac{\partial g}{\partial q} + [q+F_c]\Delta\tau\frac{\partial g}{\partial p} + \frac{2}{\omega_s\tau_l}\left(\frac{g}{2} + p\frac{\partial g}{\partial p} + \frac{1}{g}\left(\frac{\partial g}{\partial p}\right)^2 + \frac{\partial^2 g}{\partial p^2}\right)\Delta\tau$$

r.h.s split into 4 steps:

$$g_1 = g_0(q - p\Delta\tau/2, p, \tau)$$

$$g_2 = g_1(q, p + [q+F_c]\Delta\tau)$$

$$g_3 = g_2(q - p\Delta\tau/2, p, \tau)$$

$$g_4 = g_3 + \frac{2}{\omega_s\tau_l}\left(\frac{g_3}{2} + p\frac{\partial g_3}{\partial p} + \frac{1}{g_3}\left(\frac{\partial g_3}{\partial p}\right)^2 + \frac{\partial^2 g_3}{\partial p^2}\right)\Delta\tau$$

III. BASIC Code for the First Step

Interpolation with 4th order polynomial:

```
For iq = -iqmax To iqmax: For ip = -ipmax To ipmax
  g0 = gold(iq, ip)
  If Abs(g0) > .000001 Then
    dQ = - ip *deltaP/deltaQ* dtau / 2
    gmm = gold(iq - 2, ip): gm = gold(iq - 1, ip)
    gp = gold(iq + 1, ip): gpp = gold(iq + 2, ip)
    a1=(gmm-8*gm+8*gp-gpp) / 12*dQ
    a2=(-gmm+16*gm-30*g0+16*gp-gpp)/24*dQ^2
    a3=(-gmm+2*gm-2*gp+gpp)/12*dQ^3
    a4=(gmm-4*gm+6*g0-4*gp+gpp)/24*dQ^4
    gnew(iq, ip) = (g0 + a1 + a2 + a3 + a4)
  End If
Next ip: Next iq
```

III. Final Step

Divided differences for the Fokker-Planck-term:

```
For iq = -iqmax To iqmax: For ip = -ipmax To ipmax
  g0 = gold(iq, ip)
  If Abs(g0) > .000001 Then
    gp = gold(iq, ip + 1): gm = gold(iq, ip - 1)
    g1= (4*gp-6*g0+4*gm-2*gp*gm/g0)/deltaP ^ 2
    g1=g1+ ip*(gp-gm) + (g0+gp)/2
    gnew(iq, ip)= g0+g1*dtau/Omega_syn/Tau_long
  End If
Next ip: Next iq
```

Step size: $\frac{\Delta q \cdot \Delta p}{\Delta \tau^2} = const$, to be determined numerically

Simulations for 6 – 10 damping times and as many synchrotron periods needed

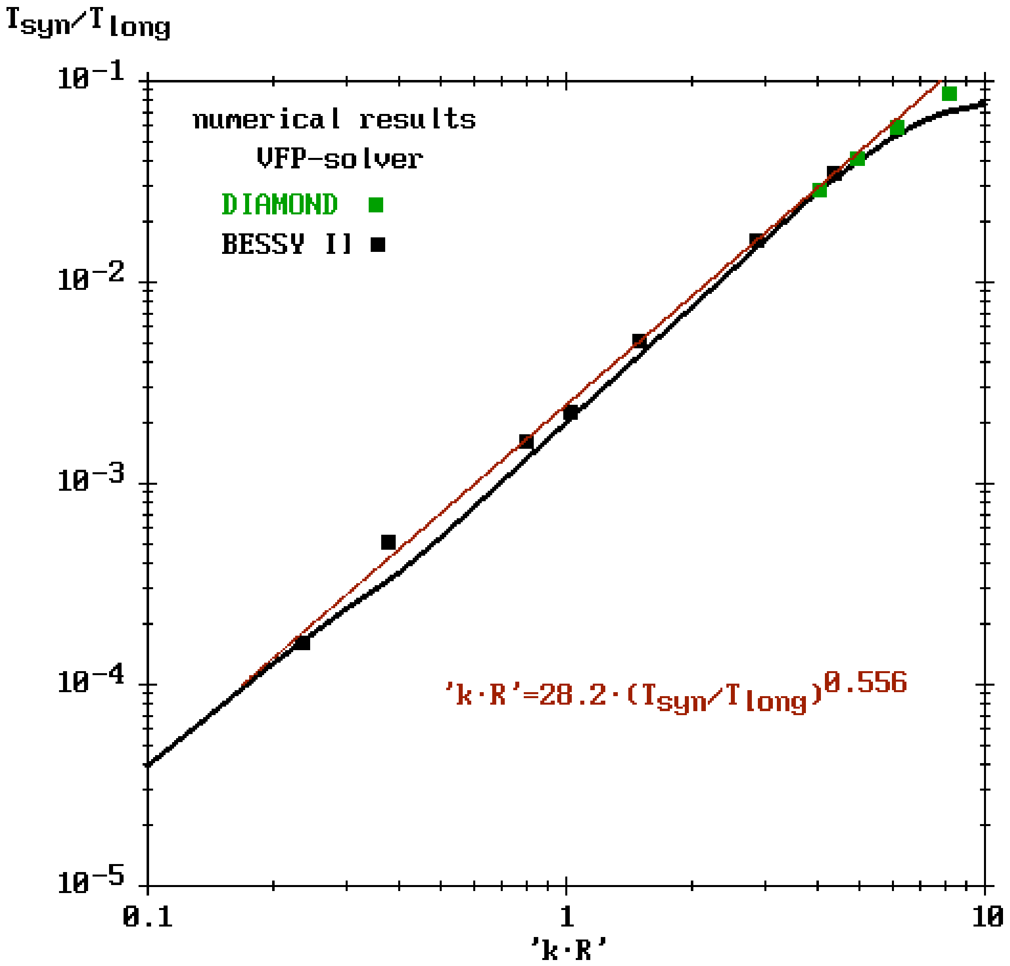
During the last 64 periods the line density, ρ(q), is stored 64 times per period for later analysis: FFT gives CSR-spectrum, and the integrated spectral power is to the instantaneous CSR signal. FFT of this signal corresponds to observed signal.

IV. Parameter Used in the Simulations

Parameter	**BESSY II**	**MLS**
Energy, **E_0/MeV**	1700	629
Bending radius, **ρ/m**	4.35	1.528
Momentum compaction, **α**	$7.3\ 10^{-4}$	$1.3\ 10^{-4}$
Cavity voltage, **V_{rf}/kV**	1400	330
Accelerating frequency, **ω_{rf}/MHz**	2π 500	2π 500
Revolution time, **T_0/ns**	800	160
Natural energy spread, **σ_E**	$7.0\ 10^{-4}$	$4.36\ 10^{-4}$
Zero current bunch length, **$\sigma_{0/}$ps**	10.53	1.549
Longitudinal damping time, **τ_l/ms**	8.0	11.1
Synchrotron frequency, **ω_s/kHz**	2π 7.7	2π 5.82
Height of the dipole chamber, ***2h/cm***	3.5	4.2

IV. Tests With Resistive Impedance

Instability thresholds: black solid line - weak instability theory by K. Oide, Part. Accel. **51**, 43 (1995), numerical results for the Diamond Light Source (DLS) and BESSY II



DLS: Vrf=2, 4, 8, and 16 MV

BESSY II: Vrf=0.35, ... 14000 MV.

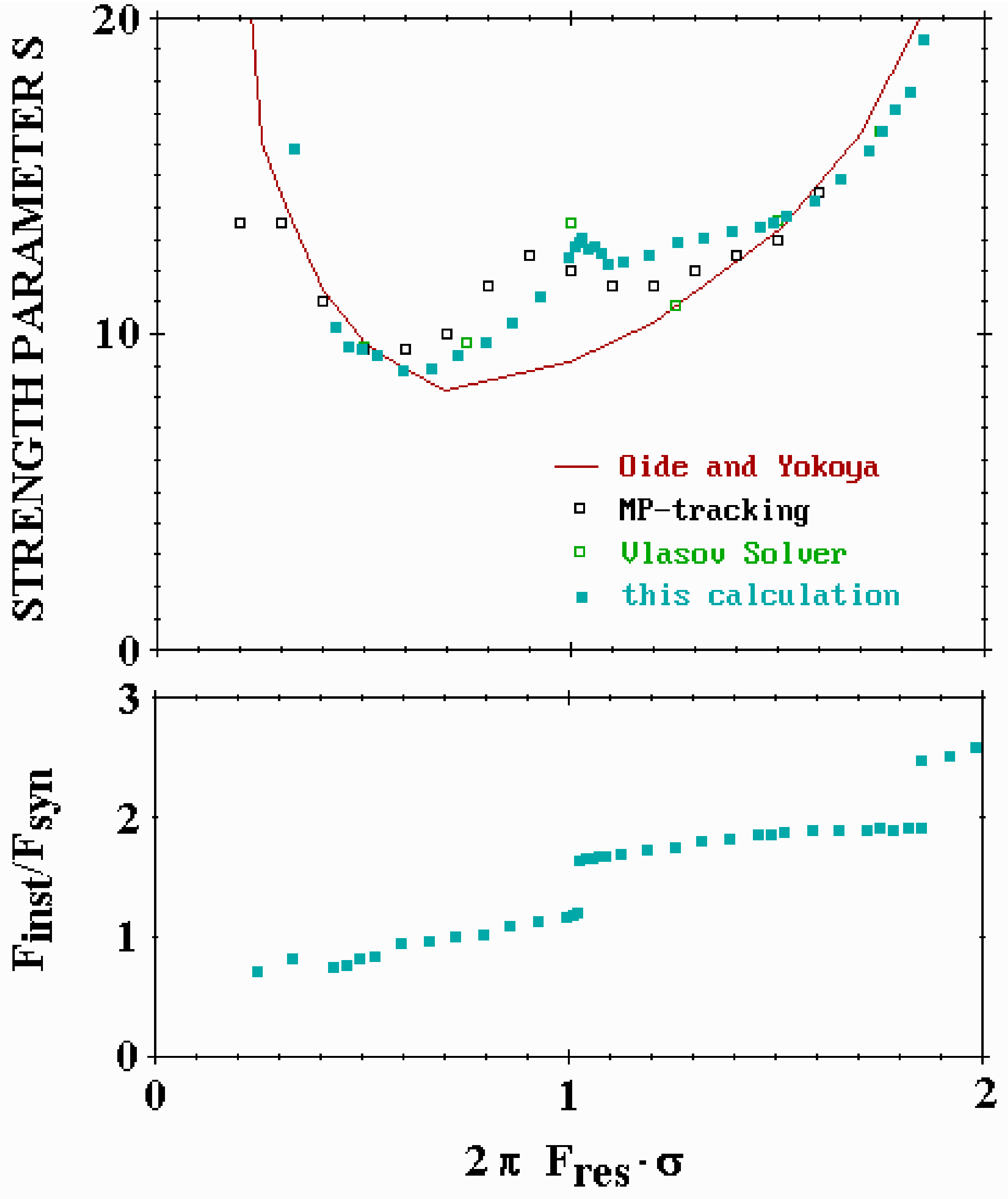
Oide's dimensionless parameter:

$$'k\cdot R' = \frac{I_{threshold}[A]\cdot R[\Omega]\cdot T_0[s]}{\dot{V}_{rf}[V/s]\cdot\sigma_0^2[s]}$$

weak instability –
damping time matters

$$I_{threshold}(\sigma_0 = const) \propto \sqrt{\dot{V}_{rf}}$$

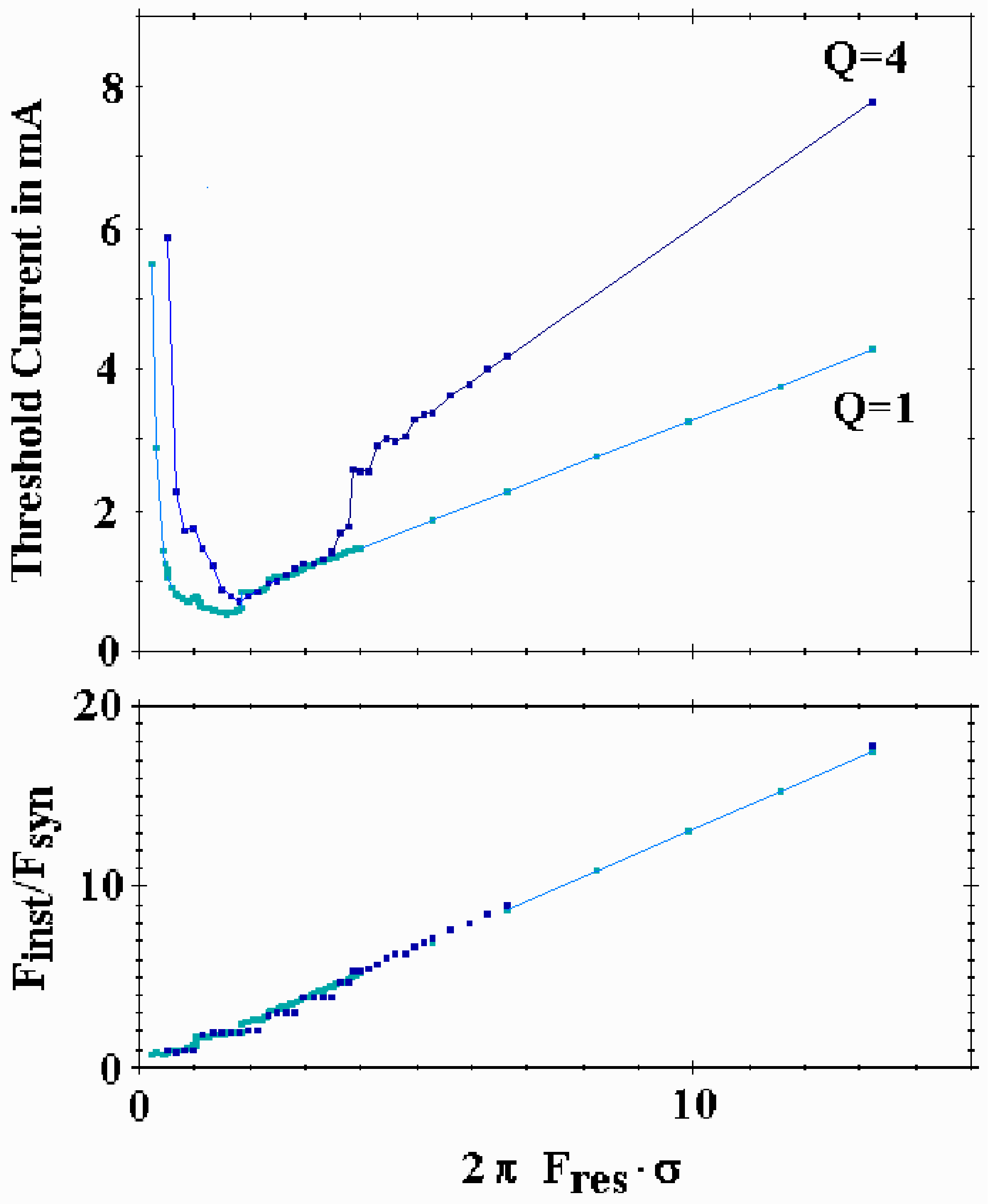
IV. Test with Broad Band Resonator



K. Oide, K. Yokoya, „Longitudinal Single-Bunch Instability in Electron Storage Rings“, KEK Preprint 90-10, April 1990

K.L.F. Bane, et al., „Comparison of Simulation Codes for Microwave Instability in Bunched Beams“, IPAC'10, Kyoto, Japan and references there in

IV. New Features for BBR-Interaction



broad band resonator with:

R_s=10 kΩ, Q=1 and F_{res} variable

solution of VFP-equation: $f(q,p,\tau)$

line density:

$$\rho(q,\tau)=\int_{-\infty}^{\infty} f(q,p,\tau)dp$$

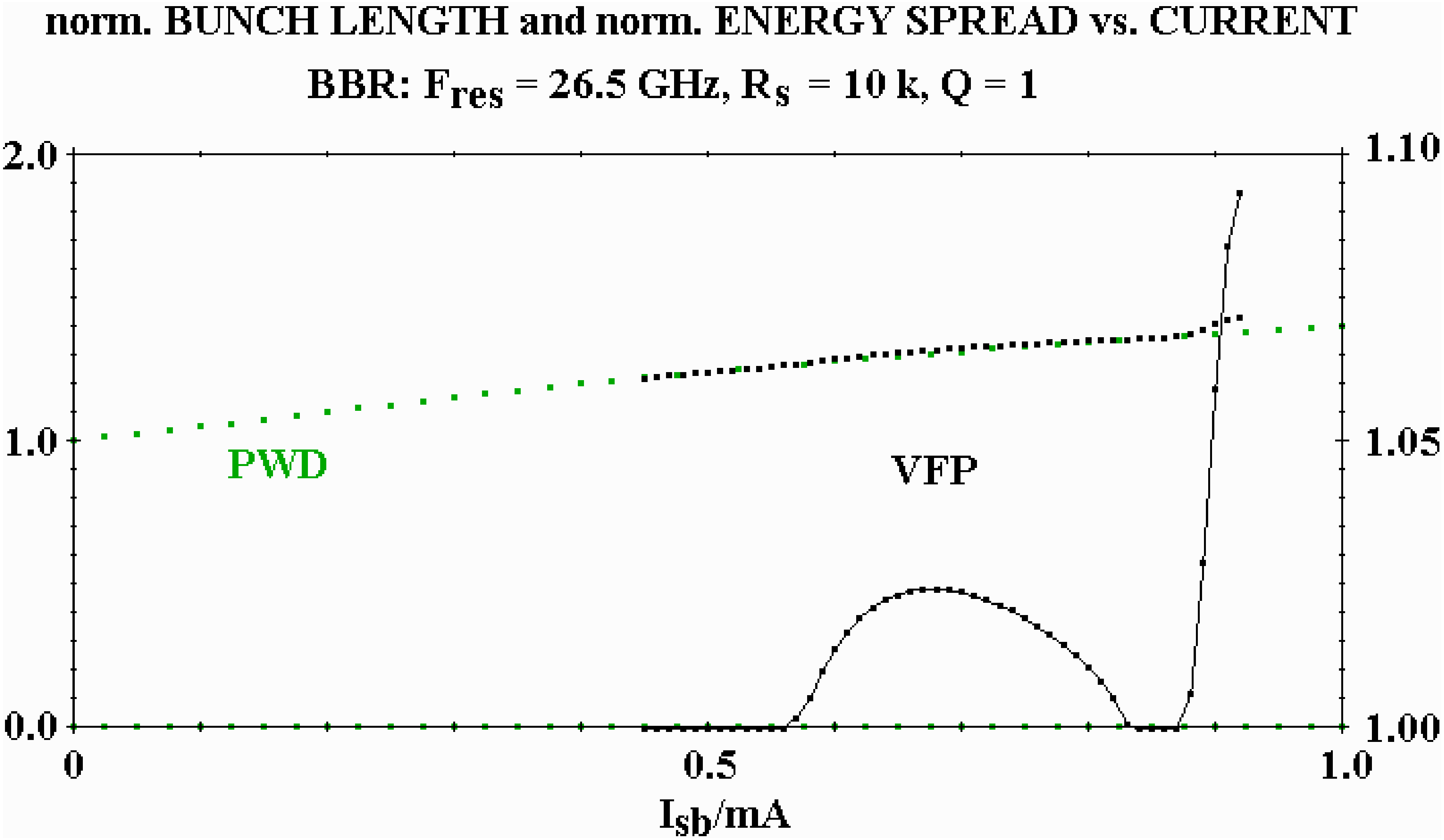
instantaneous CSR:

$$P_{coh}(\omega,\tau)\sim\left|\int_{-\infty}^{\infty}\rho(q,\tau)\cdot e^{-i\omega q}dq\right|^2$$

time dependent CSR-power:

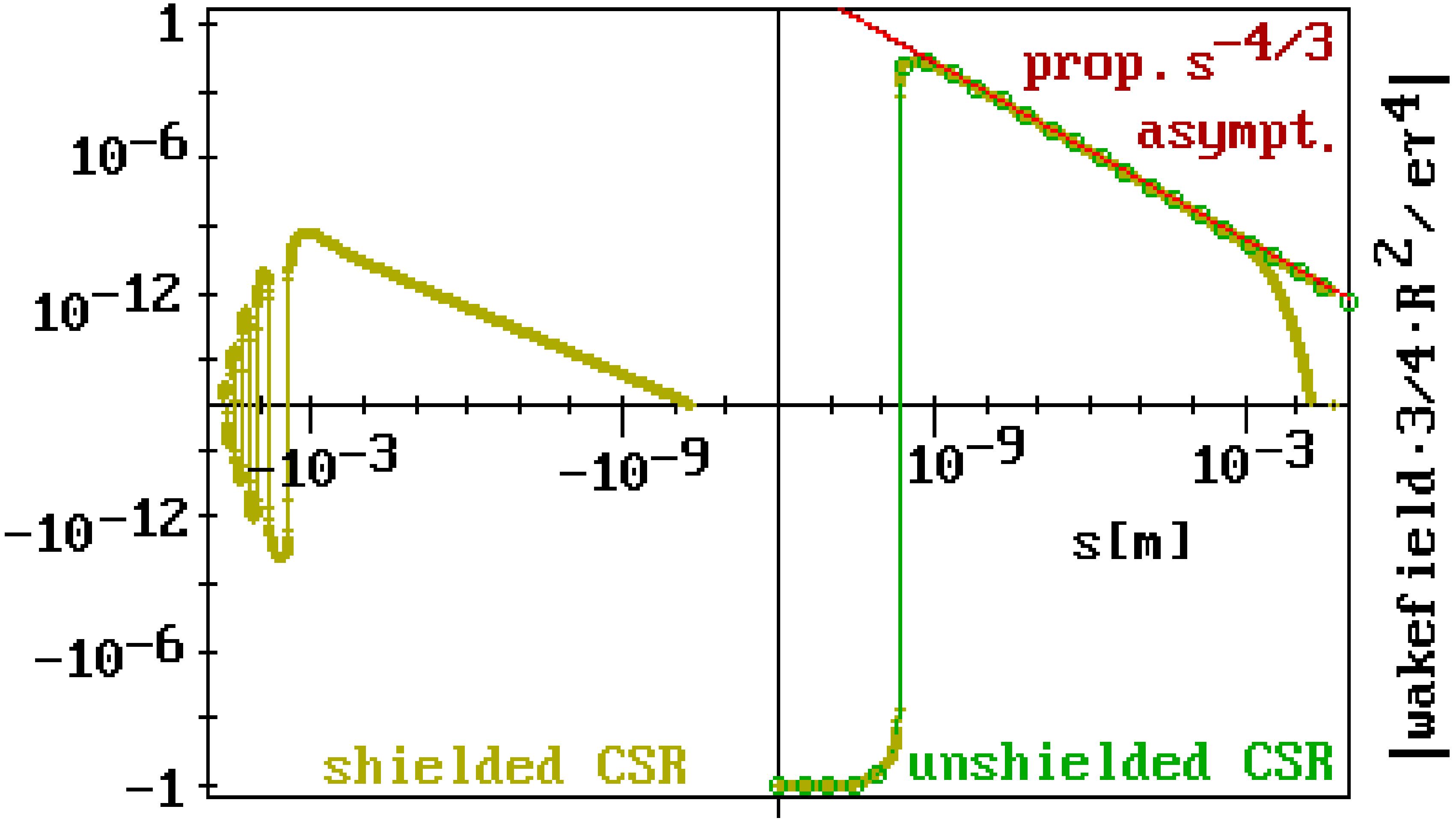
$$P_{coh}^{tot}(\tau)=\int_{cutoff}^{\infty}P_{coh}(\omega,\tau)d\omega$$

IV. New Features – Islands of Stability



V. Parallel Plate Shielded CSR-Wake

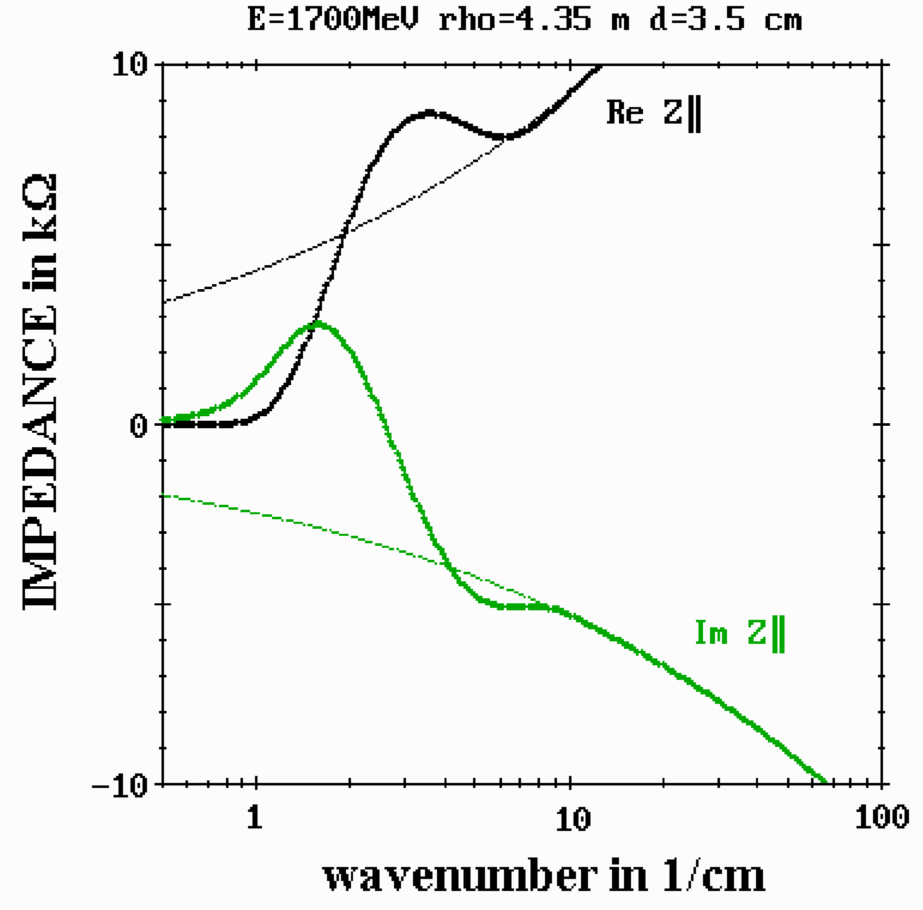
J. B. Murphy, et al. Part. Acc. 1997, Vol. 57, pp 9-64

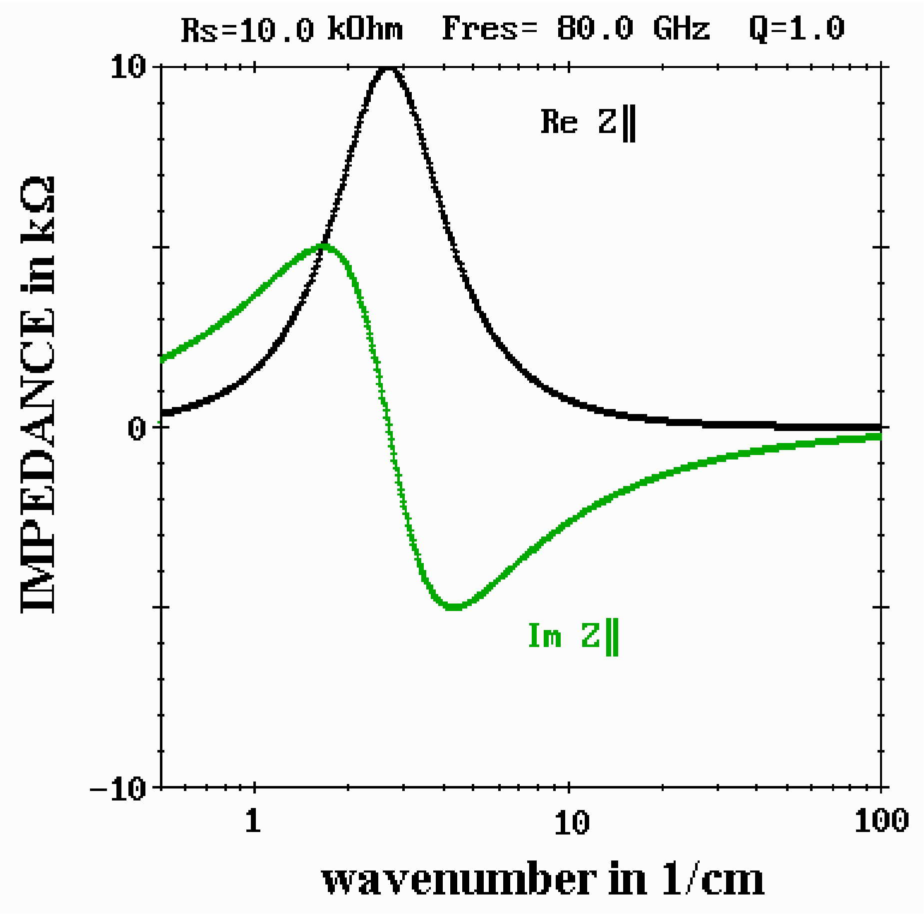


tail

head

V. Comparison of CSR- and BBR-Impedance

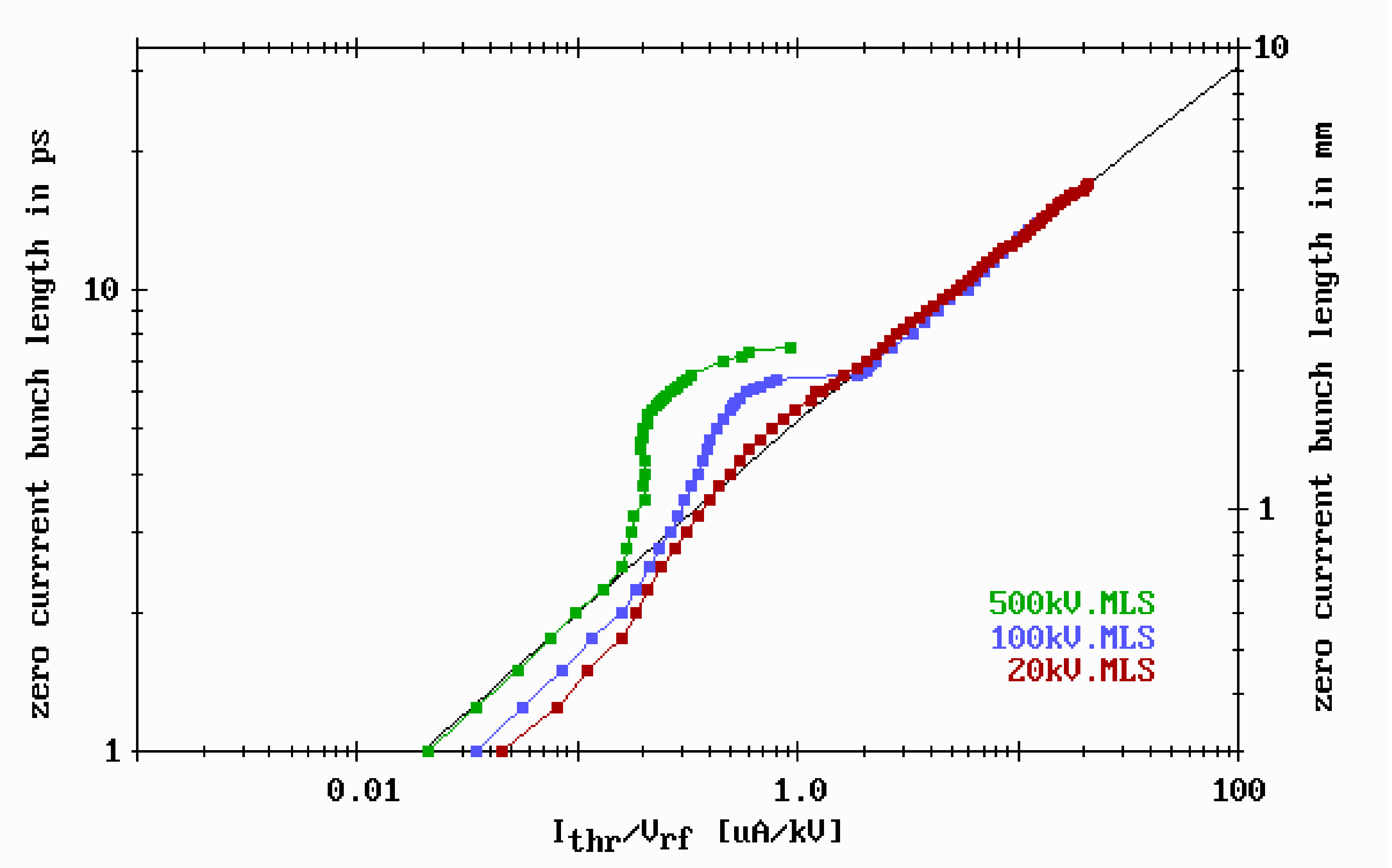




$F_{res}/c=(\pi\rho/24h^3)^{1/2}$ BESSY II: F_{res} ~ 100 GHz

R.L. Warnock, PAC'91, PAC1991_1824, http://www.JACoW.org

V.1 CSR-Threshold Currents for the MLS



Solid black line: K.L. Bane, et al., Phys. Rev. ST-AB **13**, 104402 (2010)

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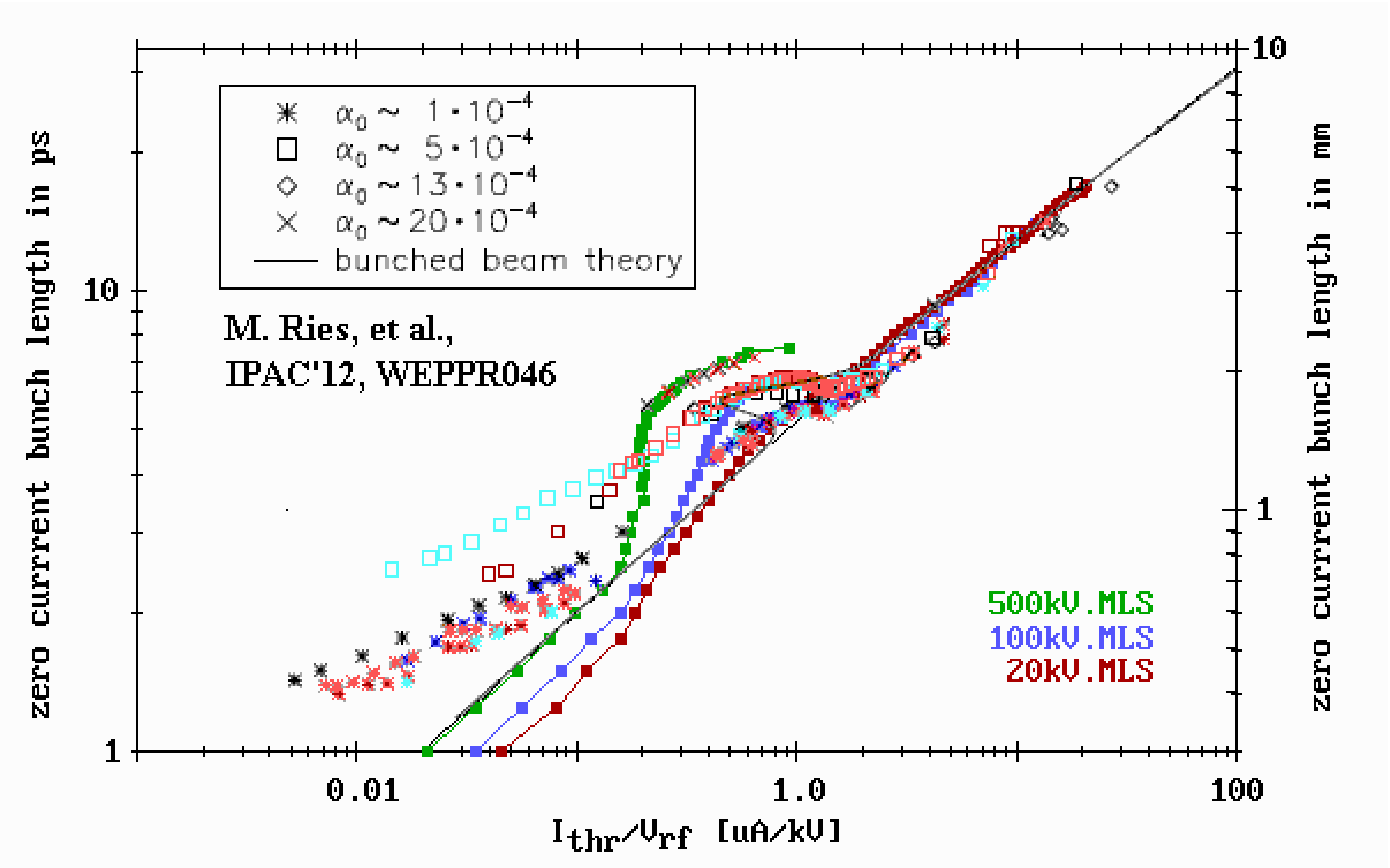
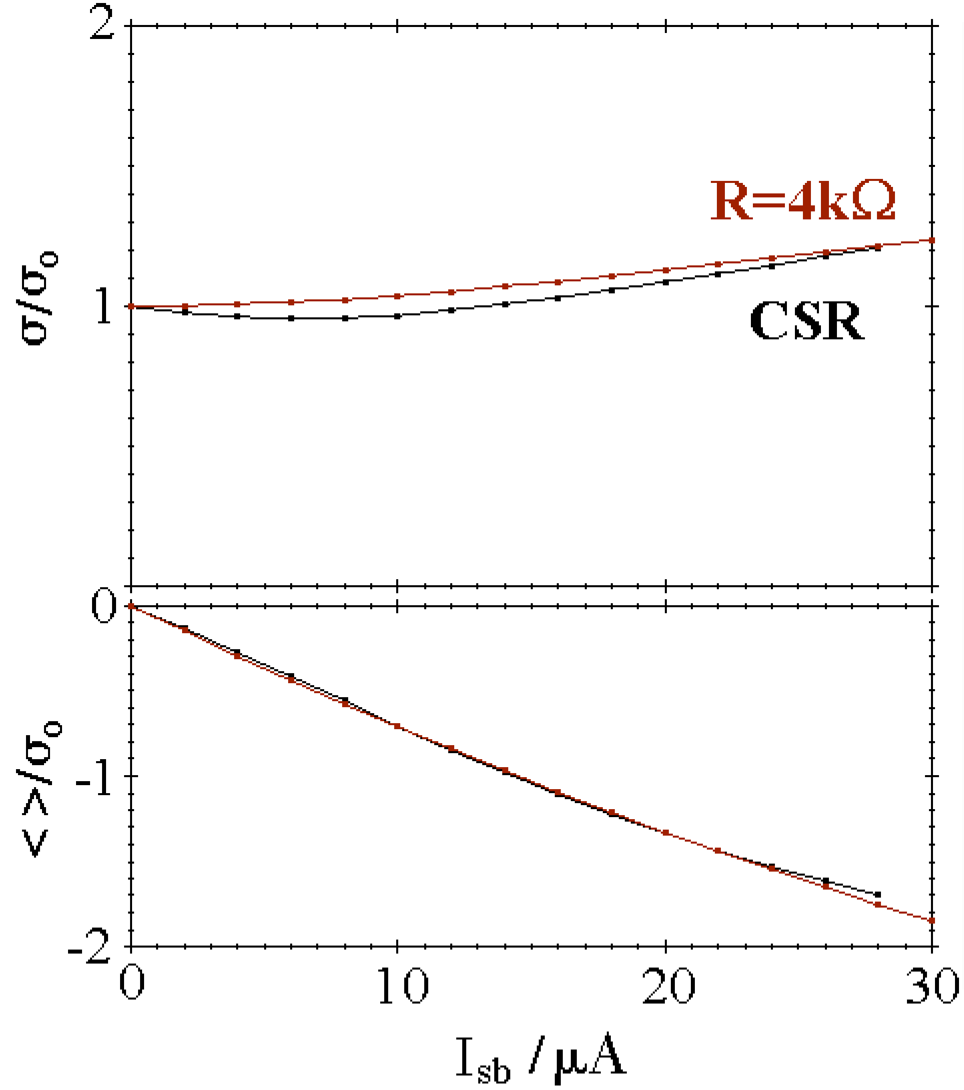
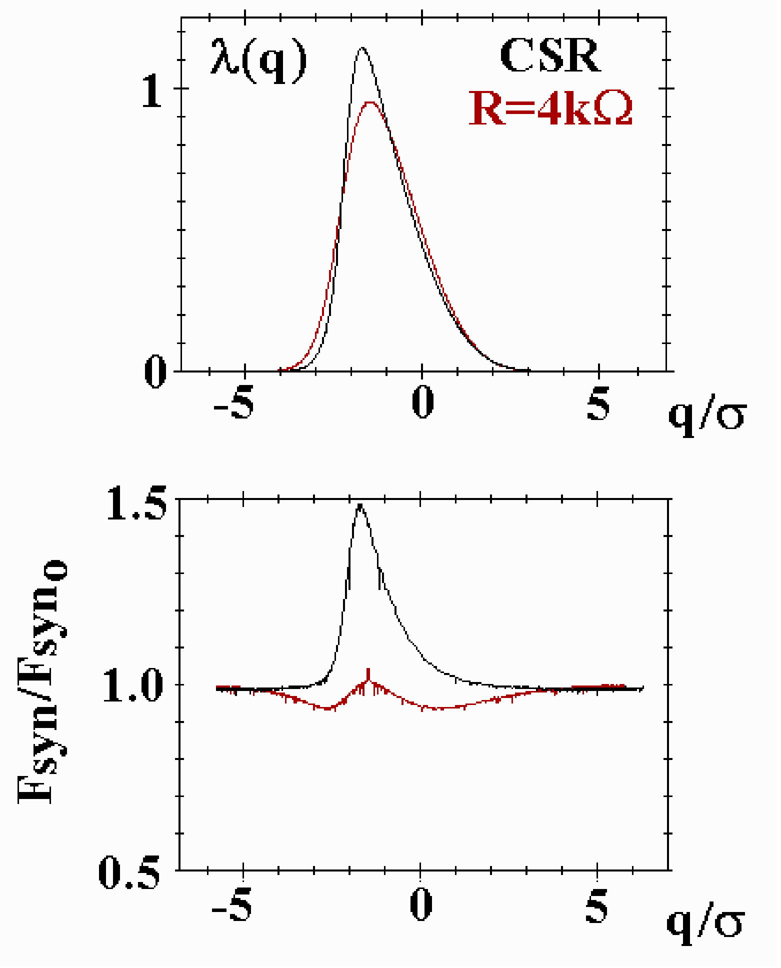


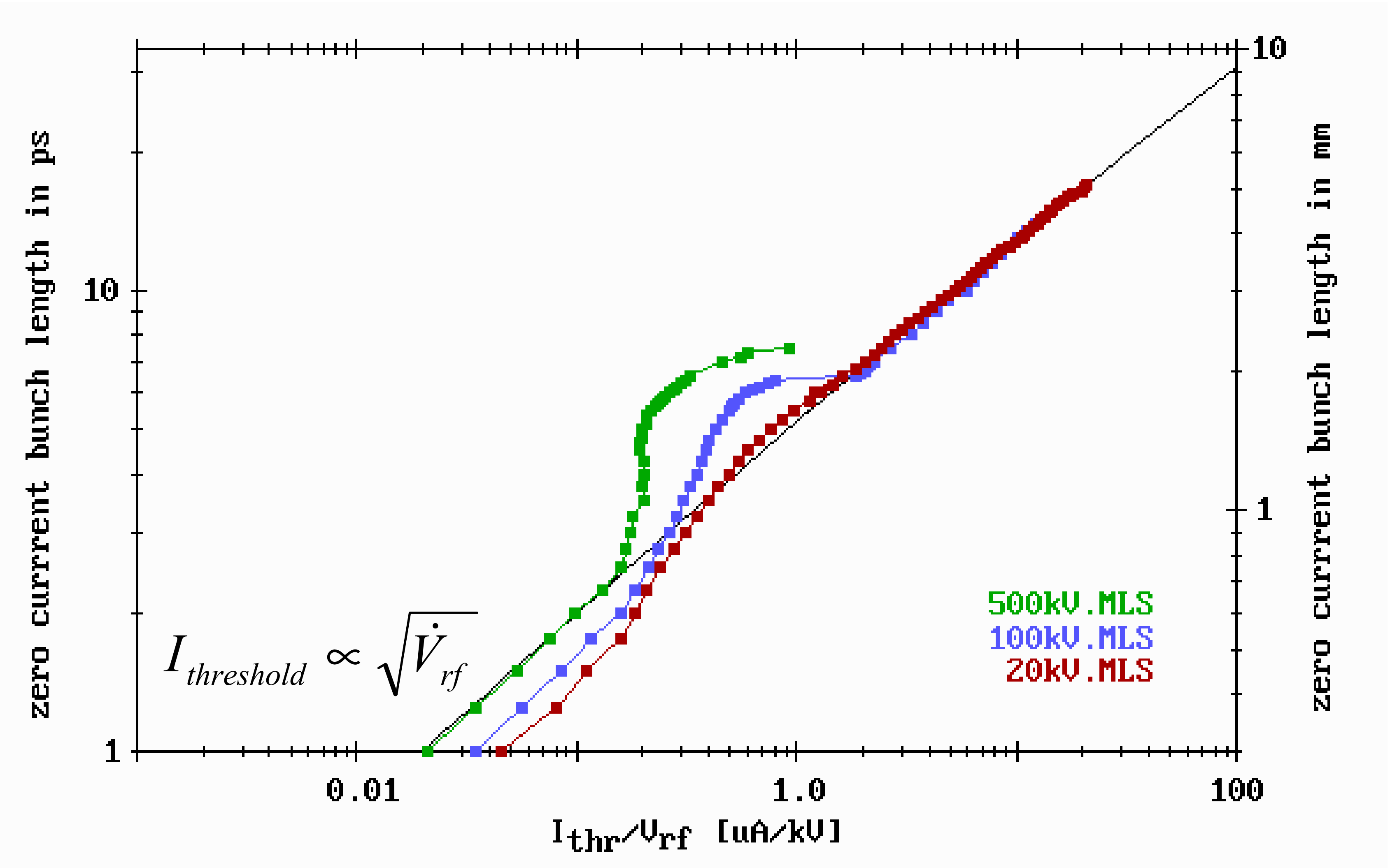
V.2 Comparison of CSR- and Resistive Wake

MLS: Vrf=330kV, α= 1.3 10^{-4}, σ_0=1.55ps





V.1 CSR-Threshold Currents for the MLS



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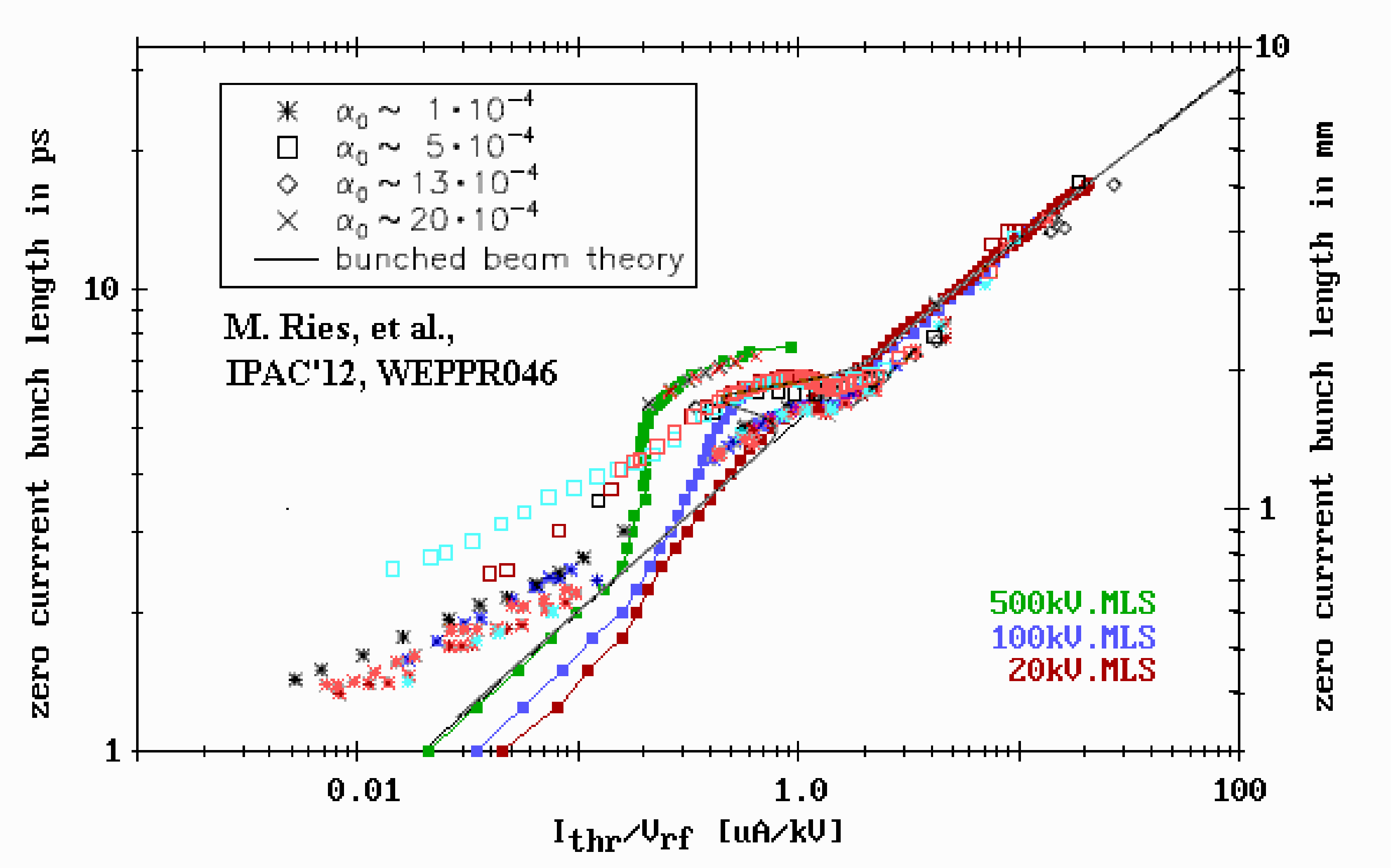
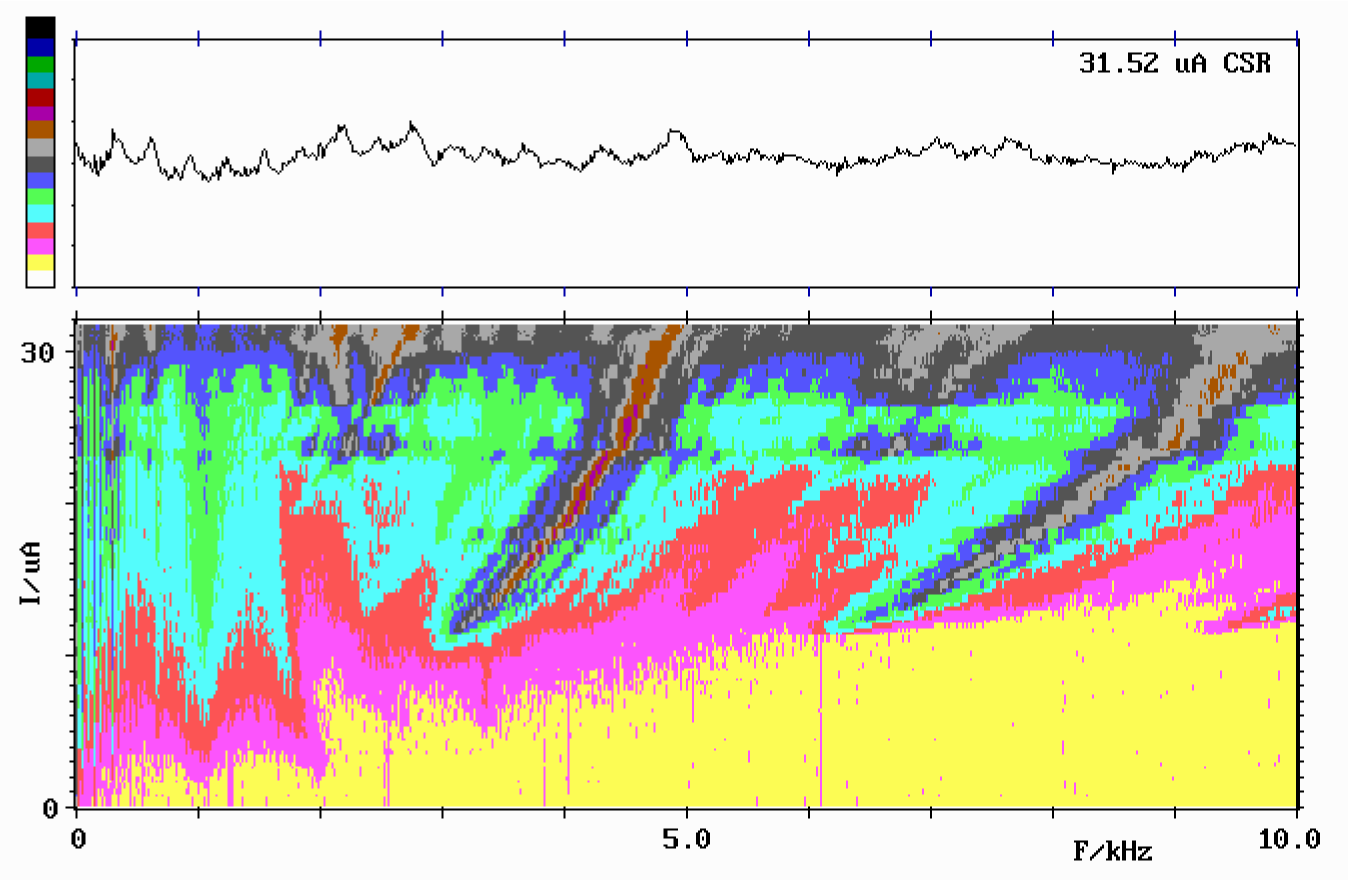


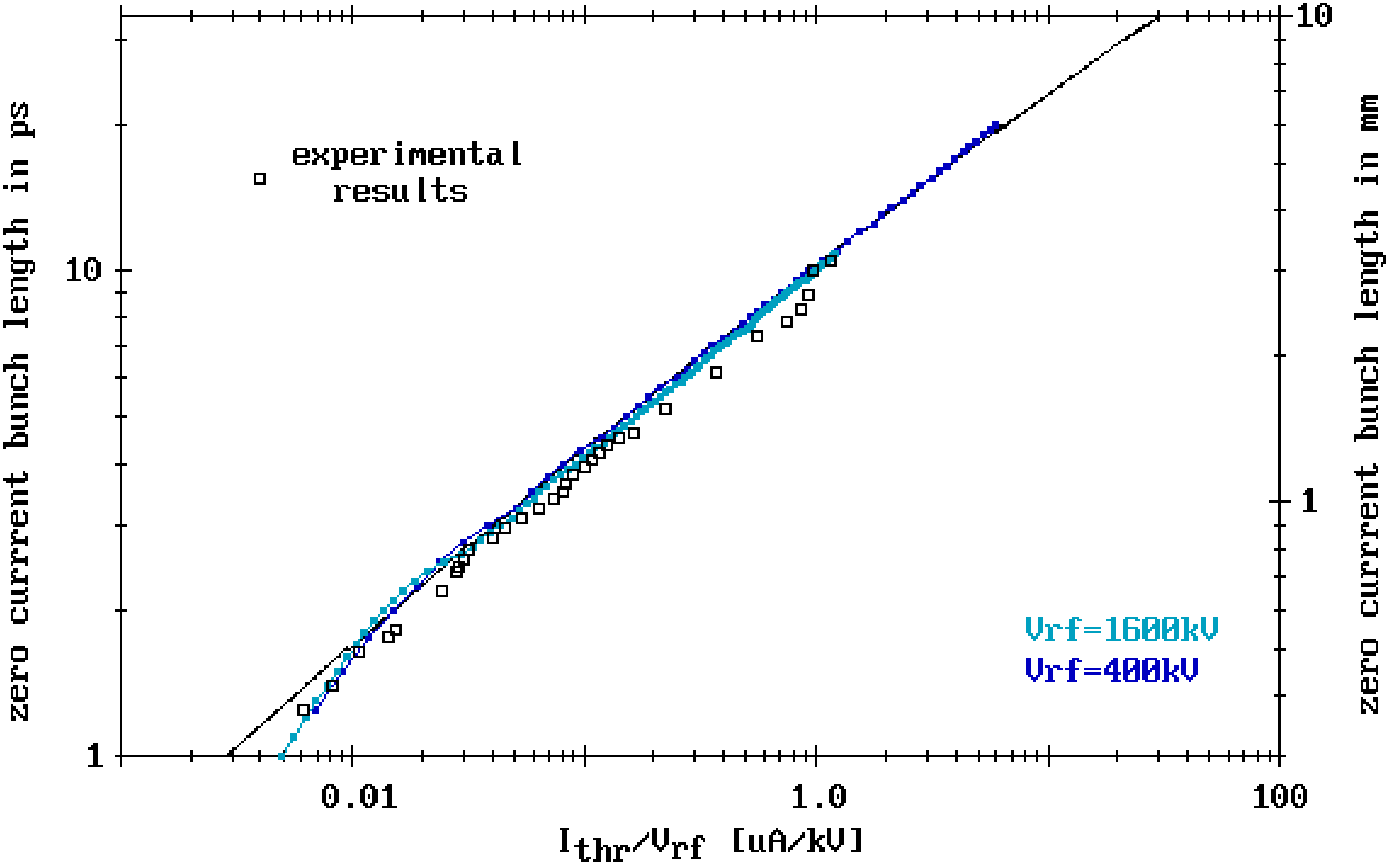
CSR-Threshold Current Measurement

BESSY II, F_{syn_o}=1 kHz, σ_o~1.5 ps



Many modes visible in the Fourier transformed CSR

CSR-Threshold Currents for BESSY II



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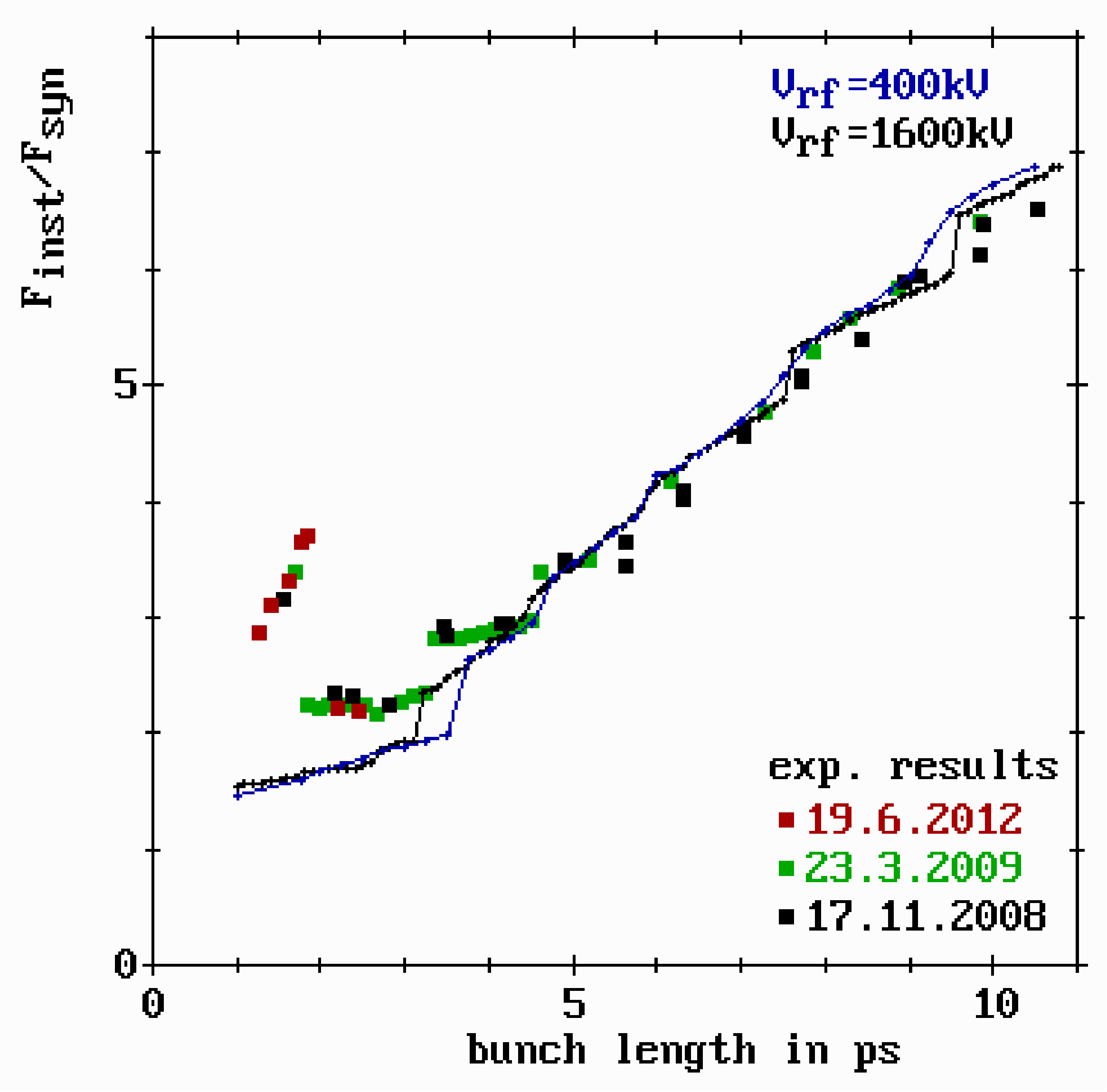
First Unstable Modes BESSY II



Slope agrees with resonance F_{res}~100 GHz

VI Summary

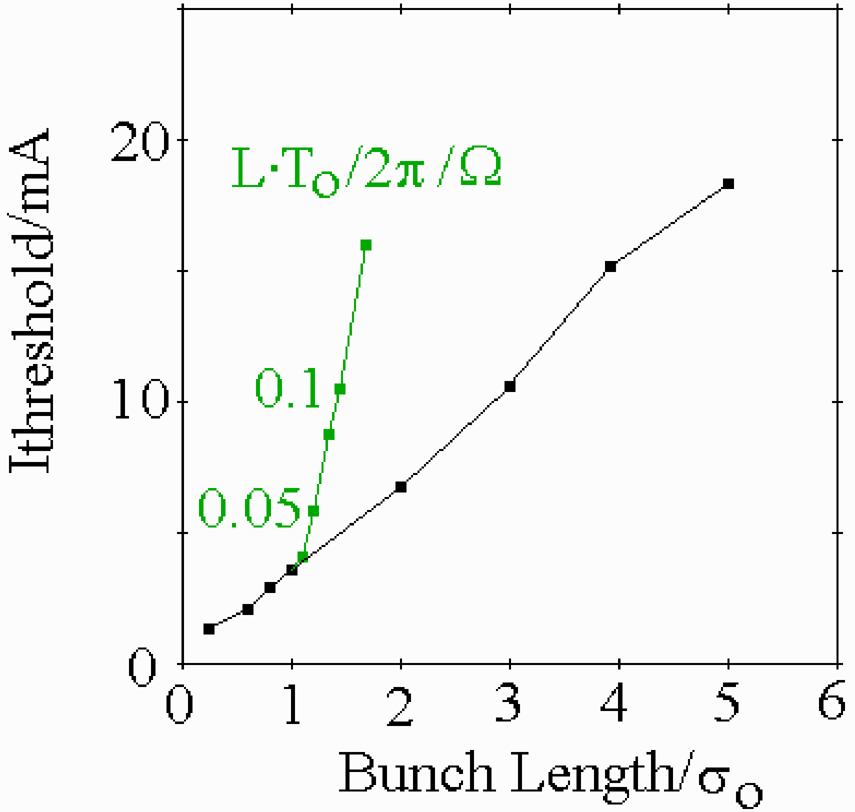
- This VFP solver reproduces earlier results – for the resistive, inductive, BBR, and CSR wakes.
- Simulations for the shielded CSR wake are in surprisingly good agreement with measurements at BESSY II and the MLS. The observed resonance-like features show that the vertical spacing of the vacuum chamber is important.
- Simulations have demonstrated the weak nature of the CSR driven instability, also in the region of short bunches where the shielding is less important.
- The VFP solver is currently used to model the behavior of bunches above the threshold currents.

In the future:

- Looking back – wave approximation or the fourth order interpolation is probably not essential. I plan to make comparisons with a simplified code.
- A more realistic wake with upstream radiation and realistic vacuum chambers should be used.
- A comparison should be made in terms of speed, accuracy, and results of the 3 different solvers for the VFP equation.

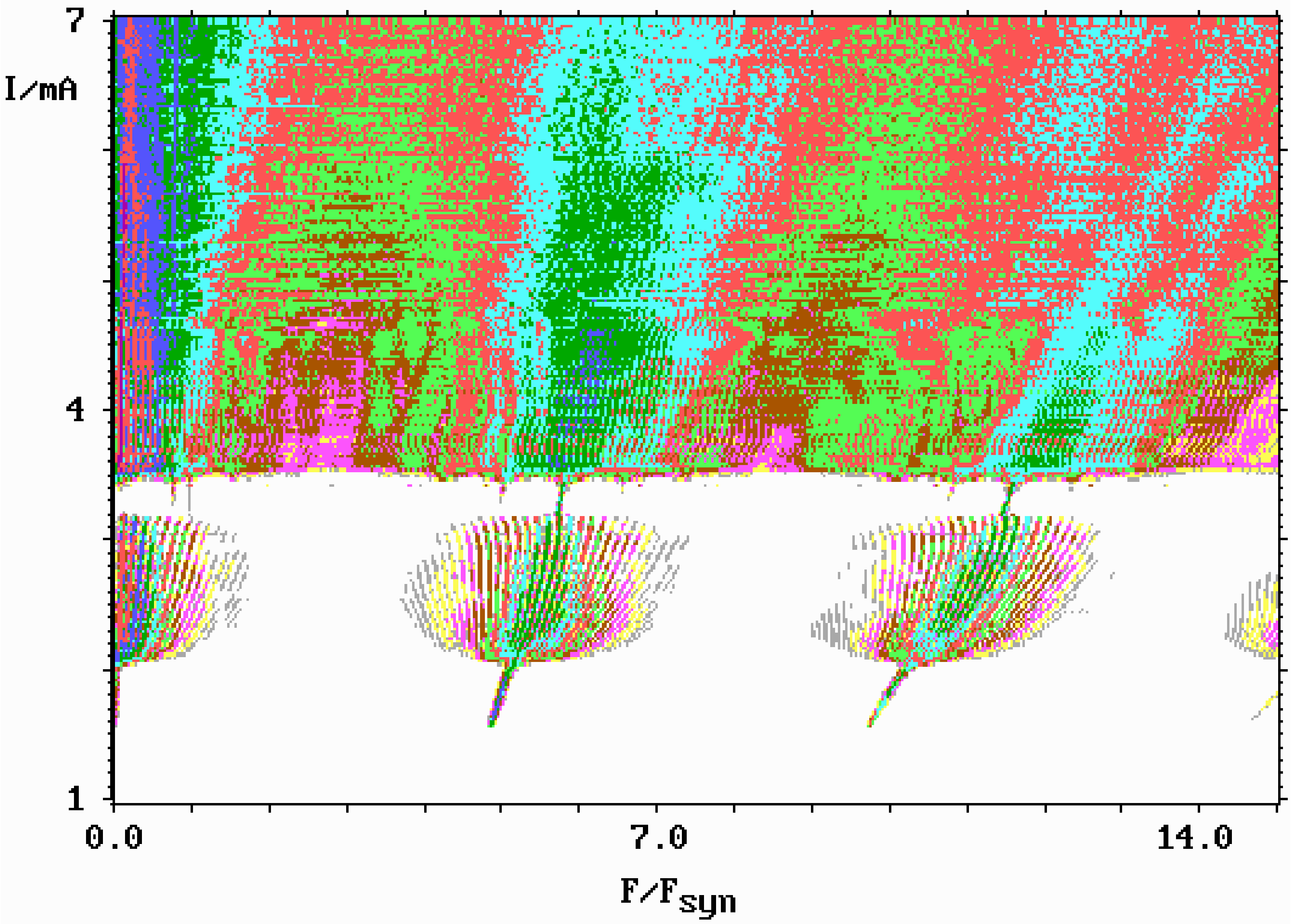
V.4 Combination of Wakes

CSR-Instability Thresholds for the SLS
with 3rd Harmonic Cavity and Inductive Impedance



V.4

Theoretical Results – Calculation of Spectra with BBR + R + L



BBR: Rs=10 kΩ, Fres=40 GHz, Q=1; R= 850 Ω and L=0.2 Ω

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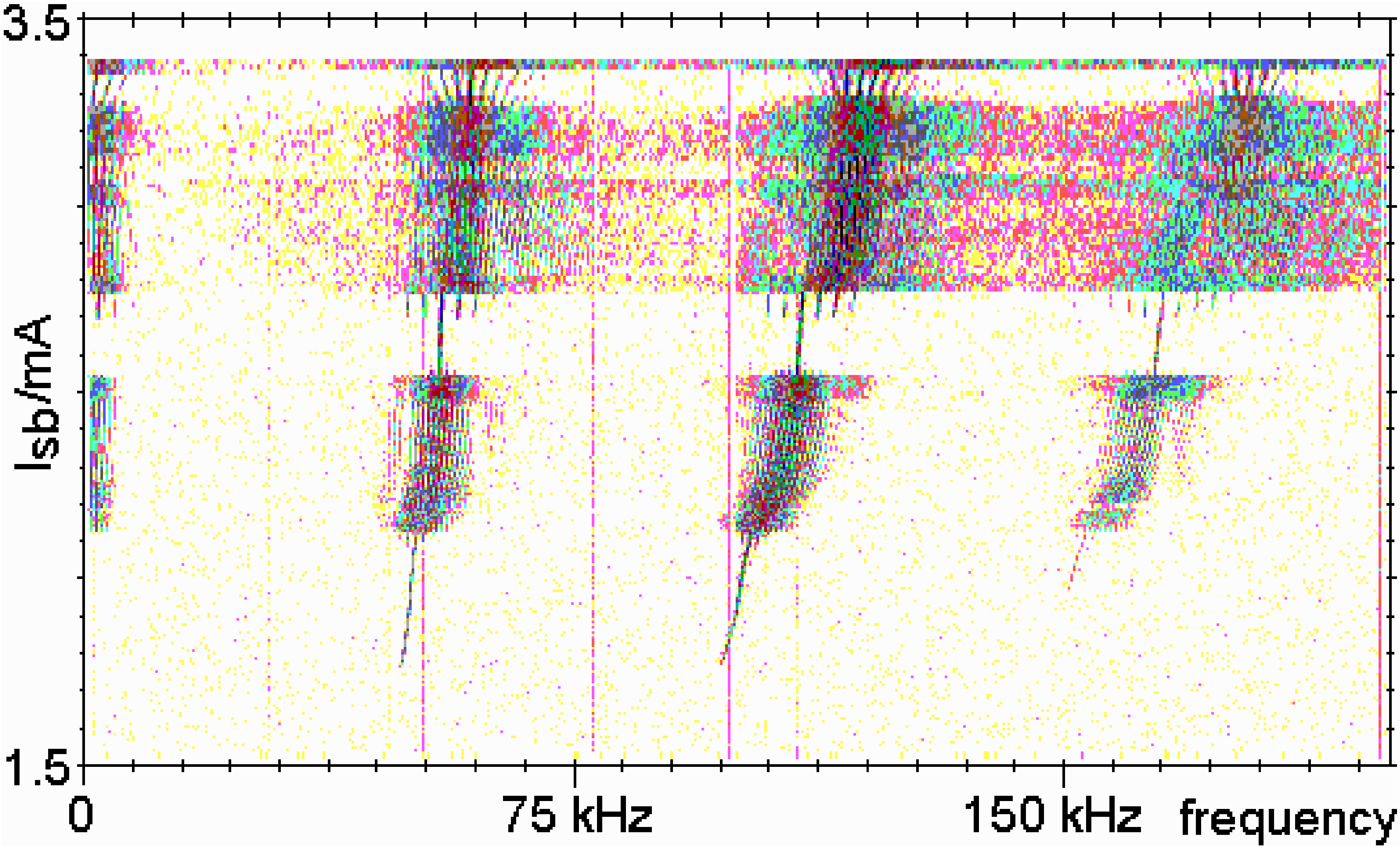
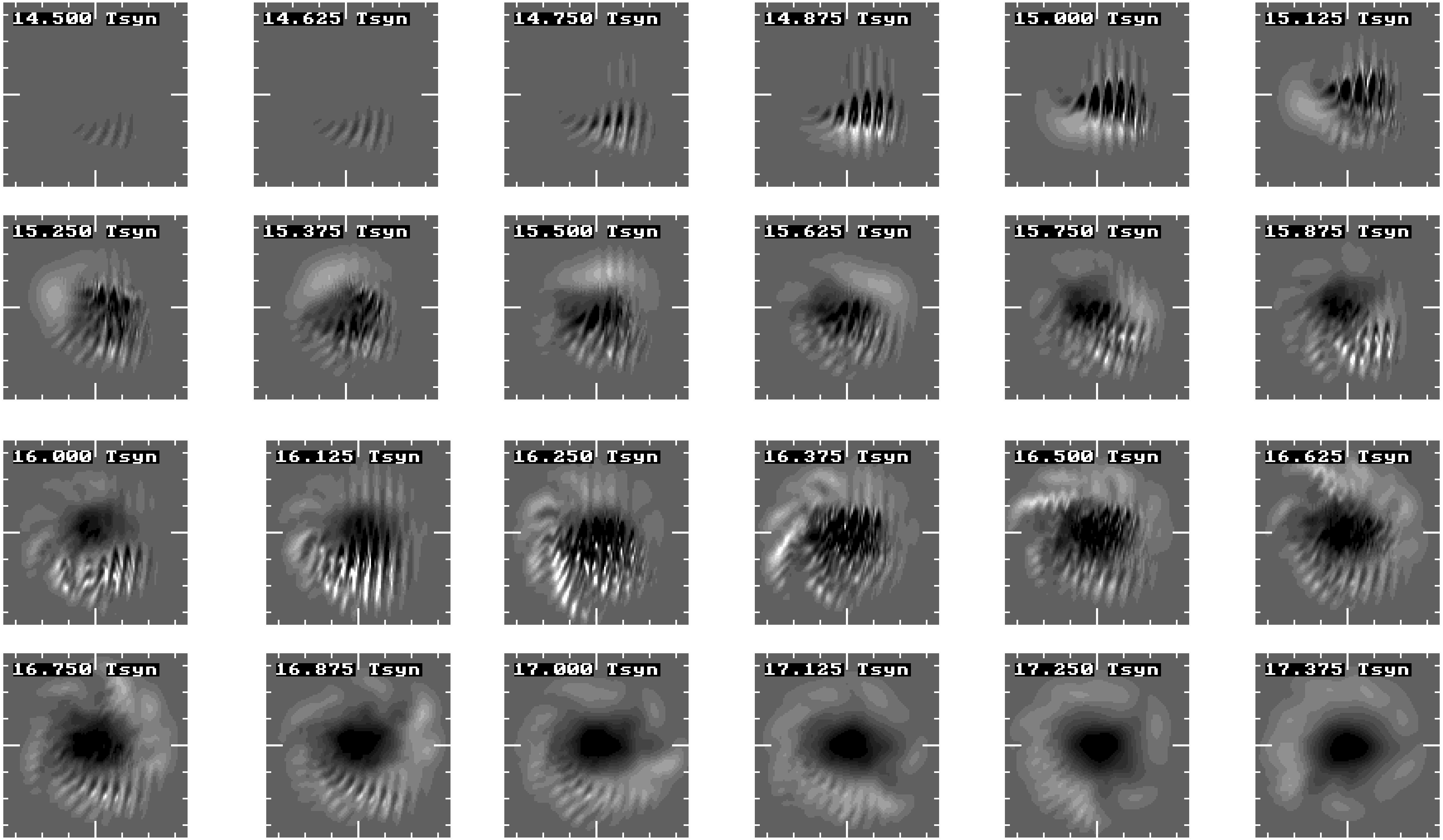


V.4 Theoretical Results - Burst

BBR: Rs=10kΩ, Fres=200GHz, Q=4 and Isb=7.9mA



Perturbation of f(q,p,τ), q horizontal axis, p vertical axis; in units of σ_0.

Theoretical Results – Lowest Unstable Mode
BBR: R_s=10kΩ, F_{res}=100GHz, Q=1 and I_{sb}=2.275mA