

On Accelerator Driven Subcritical Reactor Power Gain

Anna G. Golovkina, Dmitri A. Ovsyannikov¹, Igor V. Kudinovich²

¹Saint-Petersburg State University, St. Petersburg, Russia

²Krylov Shipbuilding Research Institute, St. Petersburg, Russia

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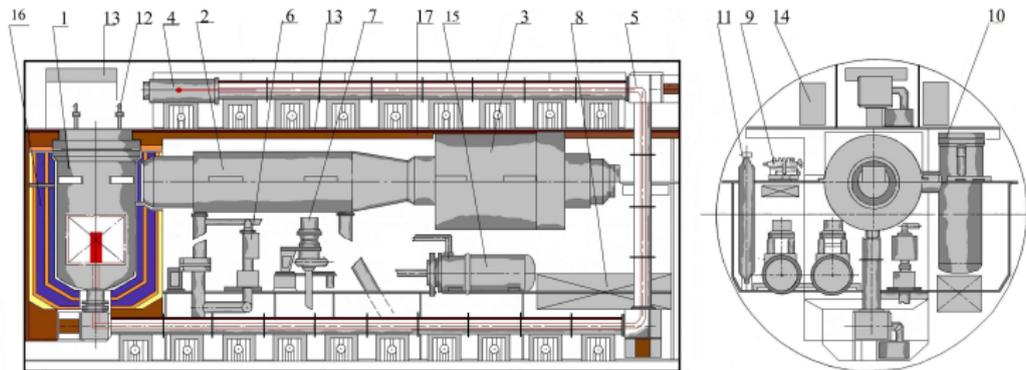


Figure: Configuration of ADS in protecting container 1 – reactor, 2 – turbocompressor with heat exchanger, 3 – main electricity-generating, 4 – accelerator, 5 – bending magnets, 6,7 – pumps of cooling systems, 8 – water storage tank of cooling system, 9 – vacuum aggregate, 10 – aggregate of emergency shut-down cooling, 11 – helium balloons, 12 – monitors, 13 – electrical equipment, 14 – blocks of RF source, 15 – heat-exchanger, 16 – bio and heat shielding.

In 1952 Prof. Rubbia further proposed the concept of an energy amplifier, a novel and safe way of producing nuclear energy exploiting present-day accelerator technologies. Now such systems are called Accelerator Driven Systems (ADS). The (ADS) is a new type of reactor which produces power even though it remains sub-critical throughout its life.

ADS applications:

- Transmuting actinides and fission products;
- Power generation;
- Producing fissile materials (Pu239, U233).

One of the characteristic optimization problems in ADSR is obtaining the maximal reactor power when the effective neutron multiplication factor and the external neutron source intensity are fixed. Some ways to increase reactor power rate:

- External neutron source localization in the reactor core optimization.
- Spallation target sizes optimization and optimized choice of the target material.
- Cascade reactor cores

External neutron source amplification

In the general case the stationary spatial energy neutron flux distribution in subcritical reactor with an external neutron source is described by a linear heterogenous equation

$$\begin{aligned} \mathbf{M} \Phi(r, E) &= -\mathbf{M}_f \Phi(r, E) - q(r, E), \\ r \in V, E_T \leq E \leq E_f, \end{aligned} \quad (1)$$

with boundary condition

$$\Phi(r_S, E) = 0, r_S \in S,$$

The multiplication constant in the reactor core does not depend on the external neutron source intensity and for k_{eff} calculation the equation for quasi-critical reactor is used:

$$\begin{aligned} \mathbf{M} \Phi_0(r, E) &= -\frac{1}{k_{\text{eff}}} \mathbf{M}_f \Phi_0(r, E), \\ \Phi_0(r_S, E) &= 0, r_S \in S. \end{aligned} \quad (2)$$

$$\mathbf{M} \Phi_0(r, E) = -\mathbf{M}_f \Phi_0(r, E) - \frac{1 - k_{\text{eff}}}{k_{\text{eff}}} \mathbf{M}_f \Phi_0(r, E). \quad (3)$$

Equations (2) and (3) are the same if the source spacial energy distribution is

$$q_{\text{pen}}(r, E) = \frac{1 - k_{\text{eff}}}{k_{\text{eff}}} \mathbf{M}_f \Phi_0(r, E).$$

where $\Phi_0(r, E)$ is the solution of Eq. (3).

External neutron source amplification

Full intensity of the external neutron source is defined by:

$$Q_1 = \int_V \int_{E_T}^{E_f} q(r, E) dE dV$$

Fission neutron generation intense in the sub-critical system:

$$Q_f = \int_V \int_{E_T}^{E_f} \mathbf{M}_1 \Phi(r, E) dE dV.$$

For the reference source Q_f and Q_1 are connected by

$$\left(\frac{Q_f}{Q_1} \right)_{\text{ref}} = \frac{k_{\text{eff}}}{1 - k_{\text{eff}}} \quad (4)$$

Introduce a criteria of amplification

$$k_{\text{ampl}} = \left(\frac{Q_f}{Q_1} \right) / \left(\frac{Q_f}{Q_1} \right)_{\text{ref}}$$

according to (4)

$$k_{\text{ampl}} = \frac{1 - k_{\text{eff}}}{k_{\text{eff}}} \cdot \left(\frac{Q_f}{Q_1} \right). \quad (5)$$

Neutron producing target

Neutron yield from the target depends on parameters of the charged particles beam, target composition and dimensions.

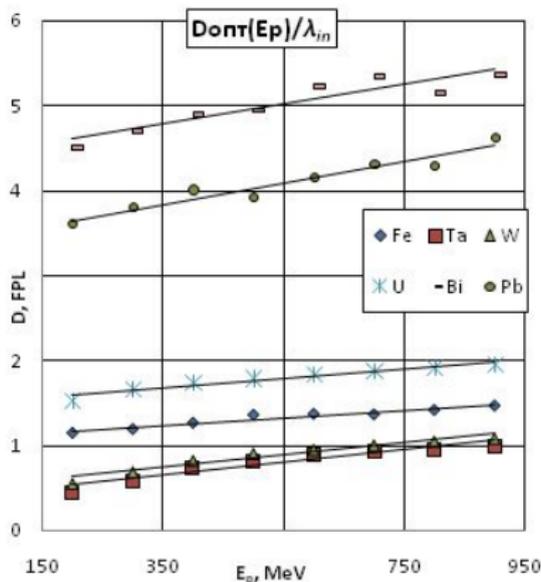


Figure: Optimized target diameter in dimensionless form (in λ_{in} lengths) in dependence of proton beam energy E_p

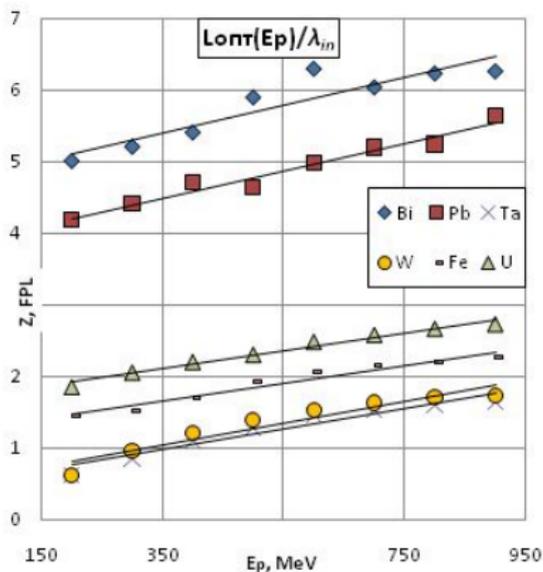


Figure: Optimized target length in dimensionless form (in λ_{in} lengths) in dependence of proton beam energy E_p

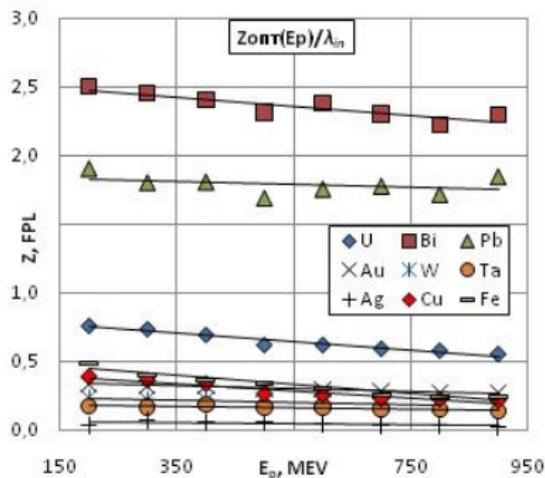


Figure: Optimized deepening of the beam injection point in dimensionless form (in λ_{in} lengths) in dependence of proton beam energy E_p

Neutron producing target

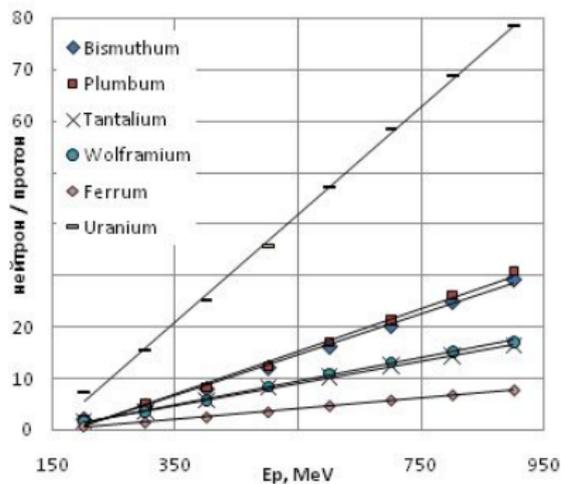
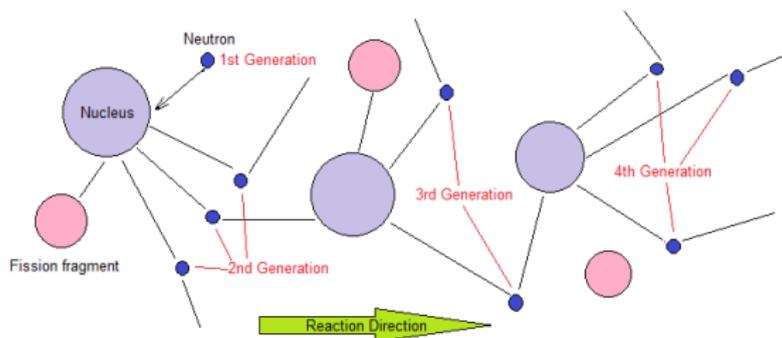


Figure: Neutron yields from target (with optimized dimensions) surface in dependance of proton beam energy E_p

Intensity of the electronuclear neutron source

$$Q_1 = \frac{I_p m_0}{e} \quad (6)$$

It is possible to reduce intensity of external neutron source locating one in the center of the core and hence decrease neutron leakage.



$$Q = \sum_{k=1}^{\infty} Q_k, \quad (7)$$

$$Q = Q_f + Q_1 \quad (8)$$
$$Q_f = \sum_{k=2}^{\infty} Q_k$$

Eq. (1) in diffusion Fermi age approximation

$$\Delta j^k(\vec{r}, \tau) - \frac{\partial j^k(\vec{r}, \tau)}{\partial \tau} - \frac{j^k(\vec{r}, \tau)}{L^2(\tau)} = 0, \quad (9)$$

$$D_T \Delta \Phi_T^k(\vec{r}) - \Sigma_{aT} \Phi_T^k(\vec{r}) + j^k(\vec{r}, \tau_T) = 0. \quad (10)$$

For $k = 1$

$$j^k(\vec{r}, \tau) = 0, \quad \text{при } \tau > \tau_0 \quad j^k(\vec{r}, \tau_0) = q(\vec{r}) \quad (11)$$

For $k > 1$

$$j^k(\vec{r}, 0) = \nu_T \Sigma_{fT} \Phi_T^{k-1}(\vec{r}) + \int_0^{\tau_T} \frac{\nu(\tau') \Sigma_f(\tau')}{L^2(\tau') \Sigma_a(\tau')} j^{k-1}(\vec{r}, \tau') d\tau' \quad (12)$$

$$\begin{aligned} j^k(\vec{r}_S, \tau) &= 0 \\ q(\vec{r}_S) &= 0 \end{aligned} \quad (13)$$

$$\begin{aligned} \Phi_T(\vec{r}_S) &= 0, \\ \vec{r} \in V; \quad \vec{r}_S \in S, \end{aligned} \quad (14)$$

Eq. (9) could be rewritten as:

$$\Delta j_0^k(\vec{r}, \tau) - \frac{\partial j_0^k(\vec{r}, \tau)}{\partial \tau} = 0 \quad (15)$$

with initial and boundary conditions:

$$j_0^k(\vec{r}, 0) = j^k(\vec{r}, 0) \quad (16)$$

$$j^k(\vec{r}_S, \tau) = 0 \quad (17)$$

where

$$j^k(\vec{r}, \tau) = j_0^k(\vec{r}, \tau) \cdot \varphi^k(\tau) \quad (18)$$

$$\text{where } \begin{cases} \varphi^1(\tau) = \exp\left(-\int_{\tau_0}^{\tau} \frac{d\tau'}{L^2(\tau')}\right), & k = 1 \\ \varphi^k(\tau) = \exp\left(-\int_0^{\tau} \frac{d\tau'}{L^2(\tau')}\right), & k > 1 \end{cases} \quad \text{– probability to avoid}$$

absorption during neutron slowing-down from energy level E_0 to E ($\varphi^1(0) = 1$, $\varphi^k(\tau_0) = 1$).

Eq. (15) solution, corresponding to non-stationary heat conduction equation with homogeneous boundary conditions of first kind, is known:

$$j_0^1(\vec{r}, \tau) = \sum_{n=1}^{\infty} b_n^1 \cdot \exp(-B_n^2(\tau - \tau_0)) \cdot \psi_n(\vec{r}), \quad (19)$$

$$j_0^k(\vec{r}, \tau) = \sum_{n=1}^{\infty} b_n^k \cdot \exp(-B_n^2\tau) \cdot \psi_n(\vec{r}), \quad (20)$$

где $\psi_n(\vec{r})$ – Laplas operator eigenfunctions, satisfying wave equation:

$$\begin{aligned} \Delta\psi_n(\vec{r}) &= -B_n^2\psi_n(\vec{r}) \\ \psi_n(\vec{r}_S) &= 0 \end{aligned}$$

$B_n^2(r_s)$ – eigenvalues, corresponding to eigenfunctions ψ_n and satisfying:

$$0 < B_1^2 \leq B_2^2 \leq \dots \leq B_n^2 \leq \dots$$

$$\Phi_T^k(\vec{r}) = \sum_{n=1}^{\infty} b_n^{\Phi(k)} \cdot \psi_n(\vec{r}) \quad (21)$$

$$b_n^{\Phi(1)} = \frac{C_n \exp(-B_n^2(\tau_T - \tau_0)) \varphi(\tau_T)}{D_T + \Sigma_{aT}} \quad (22)$$

$$b_n^{\Phi(k)} = \frac{b_n^{(k)} \exp(-B_n^2 \tau_T) \varphi(\tau_T)}{(\Sigma_{aT} + D_T B_n^2)} \quad (23)$$

Thus, fission neutron generation intensity in the reactor Q_f is defined by the following eq.:

$$Q_f^{\tau_0} = \sum_{k=2}^{\infty} \sum_{n=1}^{\infty} \left(\nu(\tau) \Sigma_{fT} b_n^{\Phi(k)} + b_n^{(k)} \int_0^{\tau} \frac{\nu(\tau') \Sigma_f(\tau')}{\xi \Sigma_s(\tau')} d\tau' \int_{V(r_s)} \psi_n(\vec{r}) dr \right) \quad (24)$$

Spatial localization of external neutron source

Spatial localization of external neutron source (radius r_{source}) in the center of the core (radius R_{core}) can be characterized by source localization factor \bar{a} .

$$\bar{a} = \frac{r_{\text{source}}}{R_{\text{core}}}$$

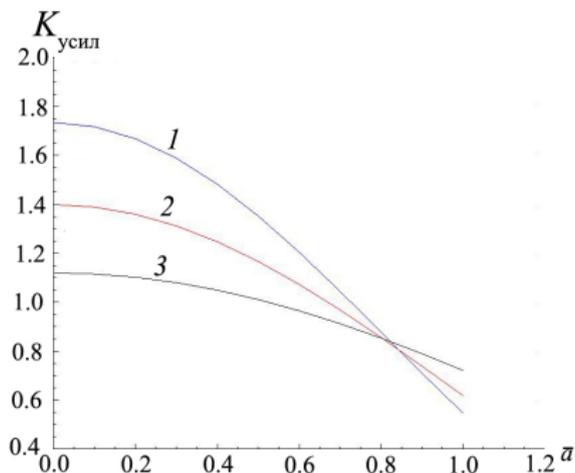


Figure: k_{ampl} depends on source localization factor, 1 – spherical geometry, 2 – cylindrical, 3 – slab

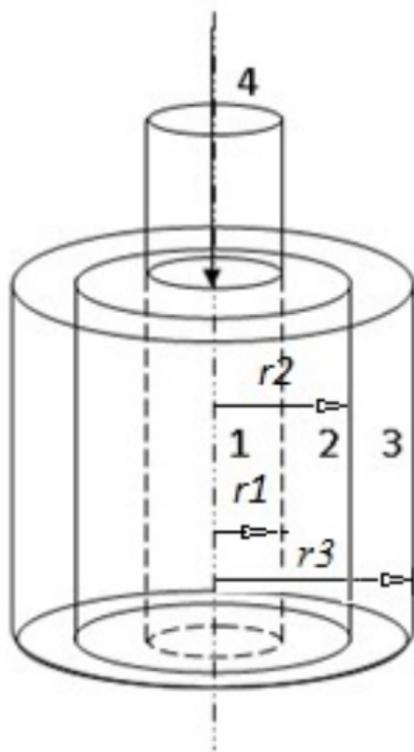


Figure: Cascade reactor core scheme. 1 – inner section, 2 – neutron gate, 3 – outer section, 4 – charged particles beam

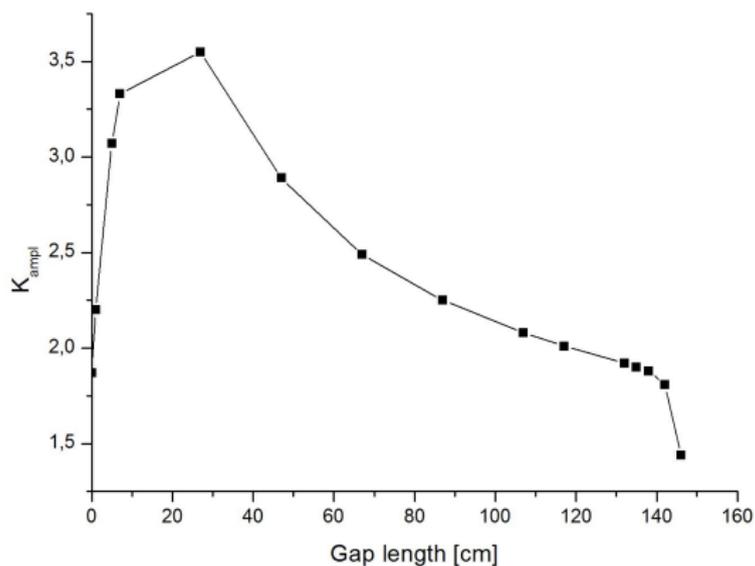


Figure: Dependence of k_{ampl} on gap length for fast cascade core with $r_3 = 150$ cm (cylindrical geometry)

Thus, ADS power rate could be increase by several ways:

- localization of spatial distribution of external neutron source in the reactor core;
- optimization of spallation target size;
- using of coupling fast-neutron reactors (cascade core with “geometrical” gate).