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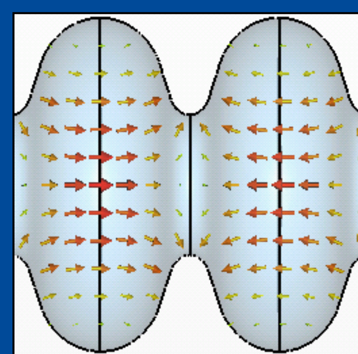
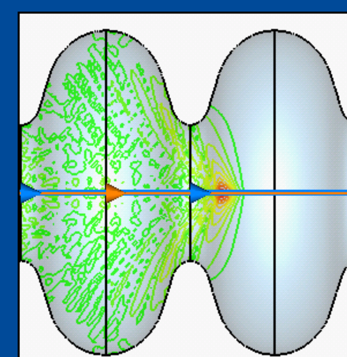
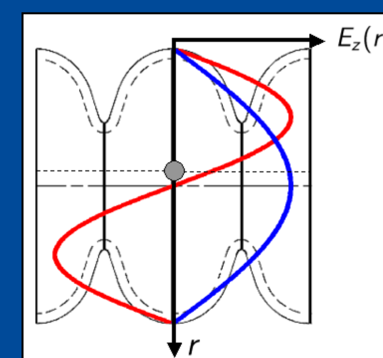






Lumped Equivalent Models of Complex RF Structures

Thomas Flisgen, Johann Heller and Ursula van Rienen

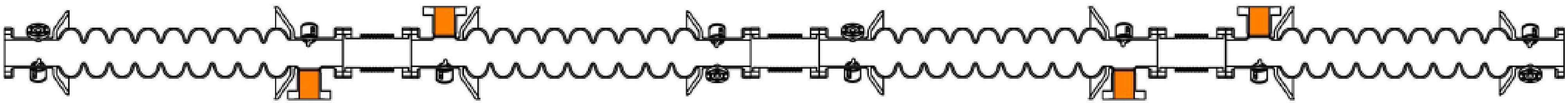
11th International Computational Accelerator Physics Conference (ICAP)
University of Rostock, Warnemünde, Germany, 23rd of August 2012

Outline

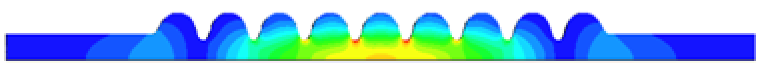
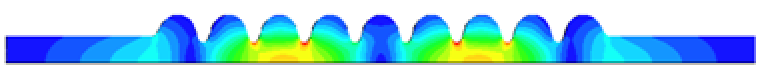
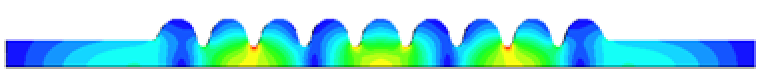
- Challenges in modelling of complex RF structures
- Approach to determine complex structure‘s S-Parameter
- A closer inspection on this approach
- State space coupling for creation of lumped models
- Validation example
- Validation results
- Conclusions

Challenges in Modelling of Complex Structures

String of Cavities in ACC39 @ FLASH Beamline



String of cavities in ACC39 mounted in FLASH*

Electric field pattern of dipole modes (individual cavity)**	$\omega/2\pi$ (GHz)
	4.2953
	4.3580
	4.4460

Cut off Frequencies of beam pipes:

1. TE11 Pol. 1 fco = 4.3920 GHz
2. TE11 Pol. 2 fco = 4.3920 GHz
3. TM01 fco = 5.7371 GHz
4. TE21 Pol. 1 fco = 7.2858 GHz
5. TE21 Pol. 2 fco = 7.2858 GHz
6. TE01 fco = 9.1412 GHz
7. TM11 Pol. 1 fco = 9.1412 GHz
8. TM11 Pol. 2 fco = 9.1412 GHz
9. TE31 Pol. 1 fco = 10.022 GHz
10. TE31 Pol. 2 fco = 10.022 GHz

* Picture courtesy E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008.
** I. R. R. Shinton, N. Juntong, R. M. Jones "Modal Dictionary of Cavity Modes for the Third Harmonic XFEL/FLASH Cavities", DESY note: DESY 12-053.

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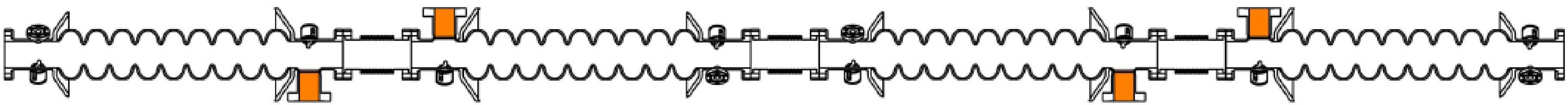
String of Cavities in ACC39 @ FLASH Beamline



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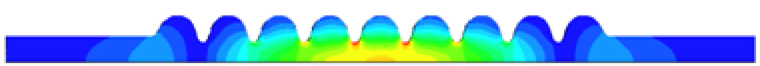
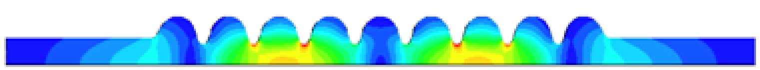
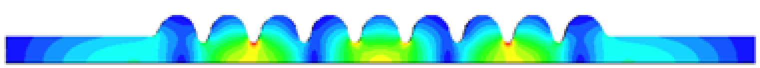
	4.2953
	4.3580
	4.4460

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6. TE01 fco = 9.1412 GHz
7. TM11 Pol. 1 fco = 9.1412 GHz
8. TM11 Pol. 2 fco = 9.1412 GHz
9. TE31 Pol. 1 fco = 10.022 GHz
10. TE31 Pol. 2 fco = 10.022 GHz

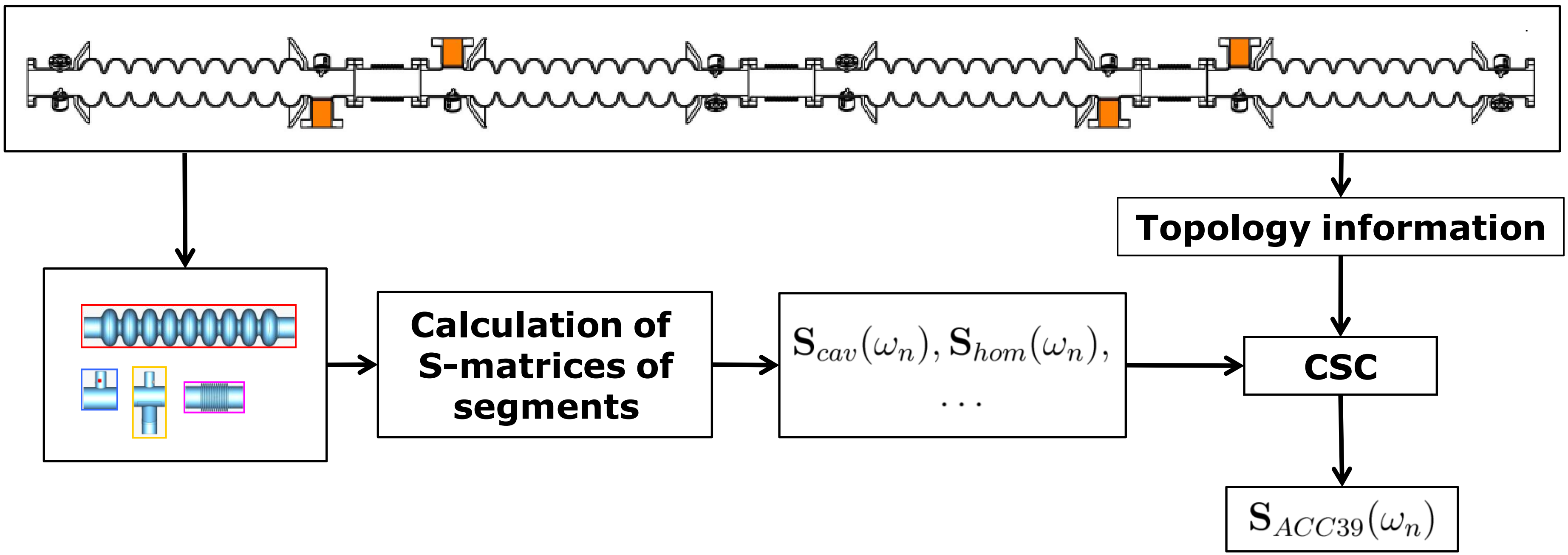
- RF properties are determined by entire string.
- Treatment of string is numerically expensive.

* Picture courtesy E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008.
** I. R. R. Shinton, N. Juntong, R. M. Jones "Modal Dictionary of Cavity Modes for the Third Harmonic XFEL/FLASH Cavities", DESY note: DESY 12-053.

Approach to determine S-Parameters of large/long Structures: Coupled S-Parameter Calculations*

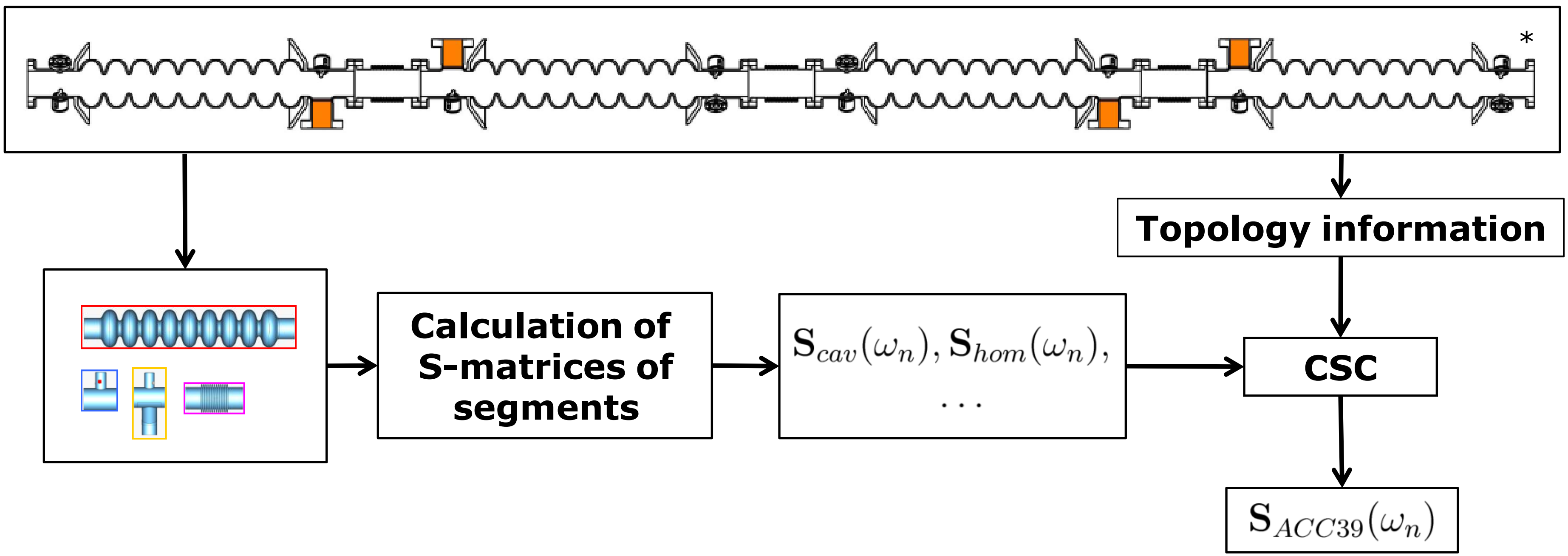
* H.-W. Glock, K. Rothemund, U. van Rienen: "CSC - A System for Coupled S-Parameter Calculations", TESLA-Report 2001-25

CSC Workflow



* Picture courtesy E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008.

CSC Workflow



Some further advantages:

- properties of equal segments need to be computed only once
- symmetry of segments can be employed to reduce computation costs
- suitable to perform parameter studies

* Picture courtesy E. Vogel et al.: “Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008”, Proc. LINAC 2008.

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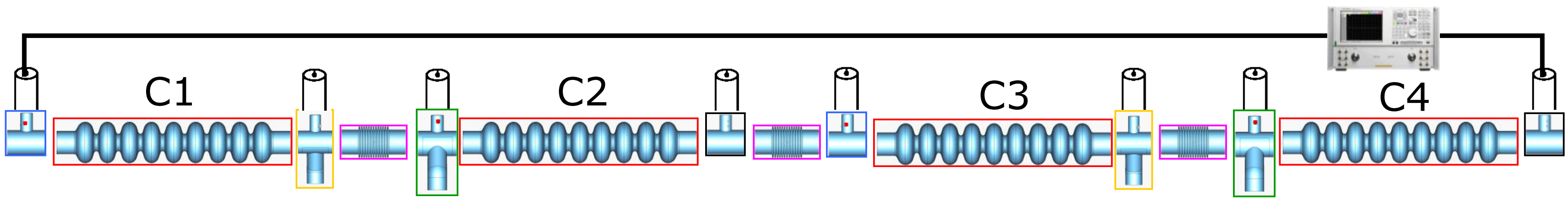
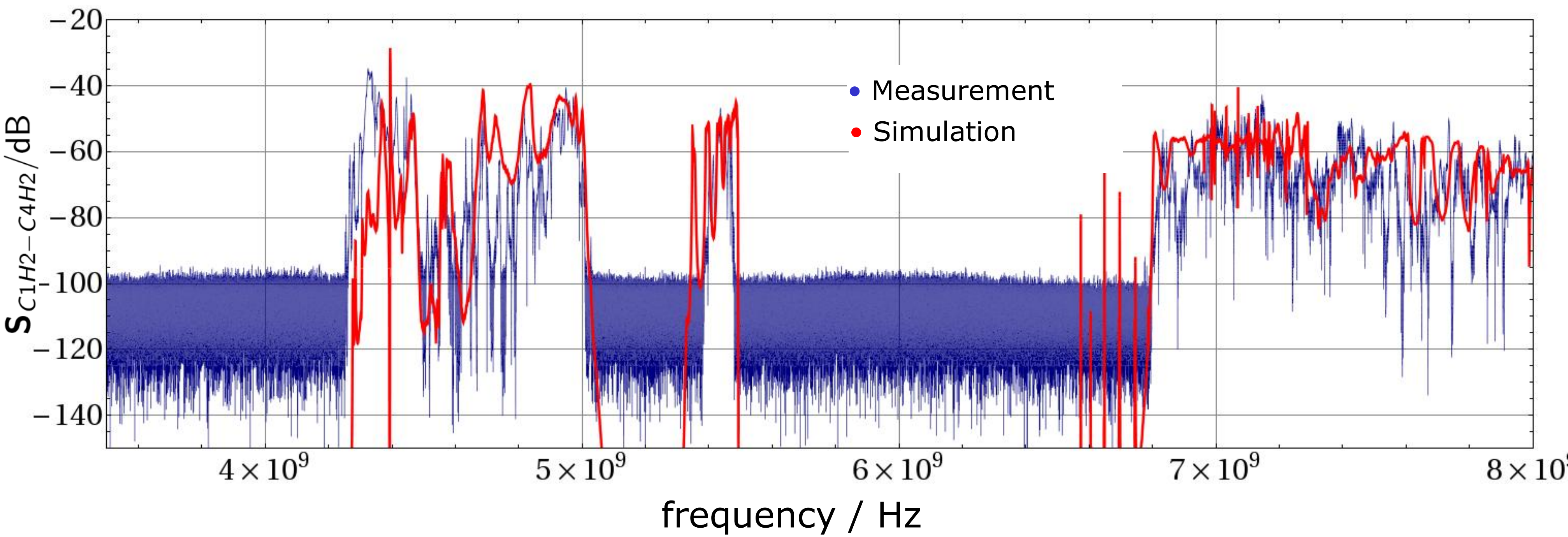
Measured* vs. Simulated Transmission via String





* done by N. Baboi (DESY), T. Flisgen and H.-W. Glock (UROS), I. Shinton (UMAN / CI), P. Zhang (UMAN /DESY)

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A Closer Inspection on Input Data Needed for the CSC scheme

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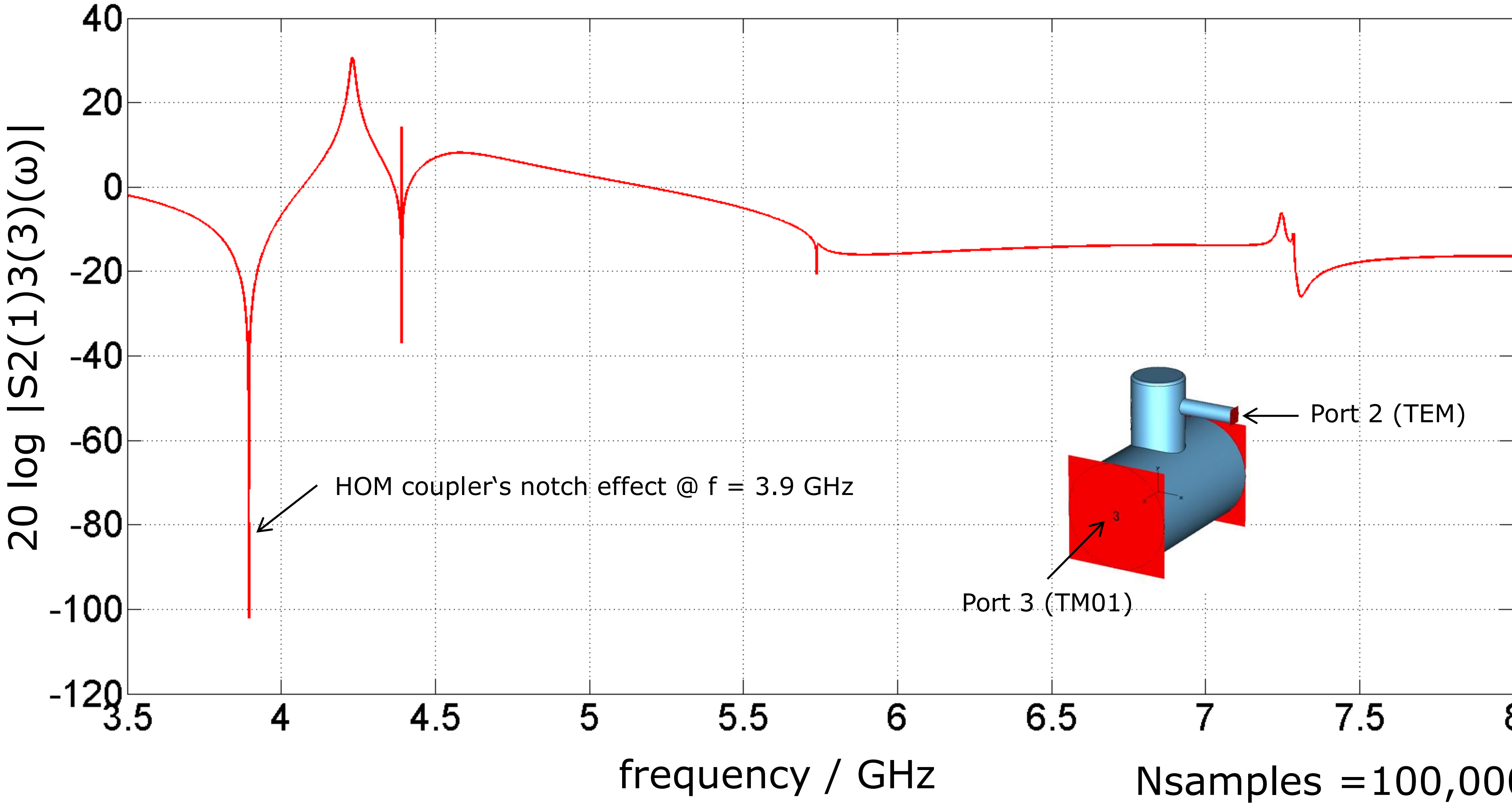
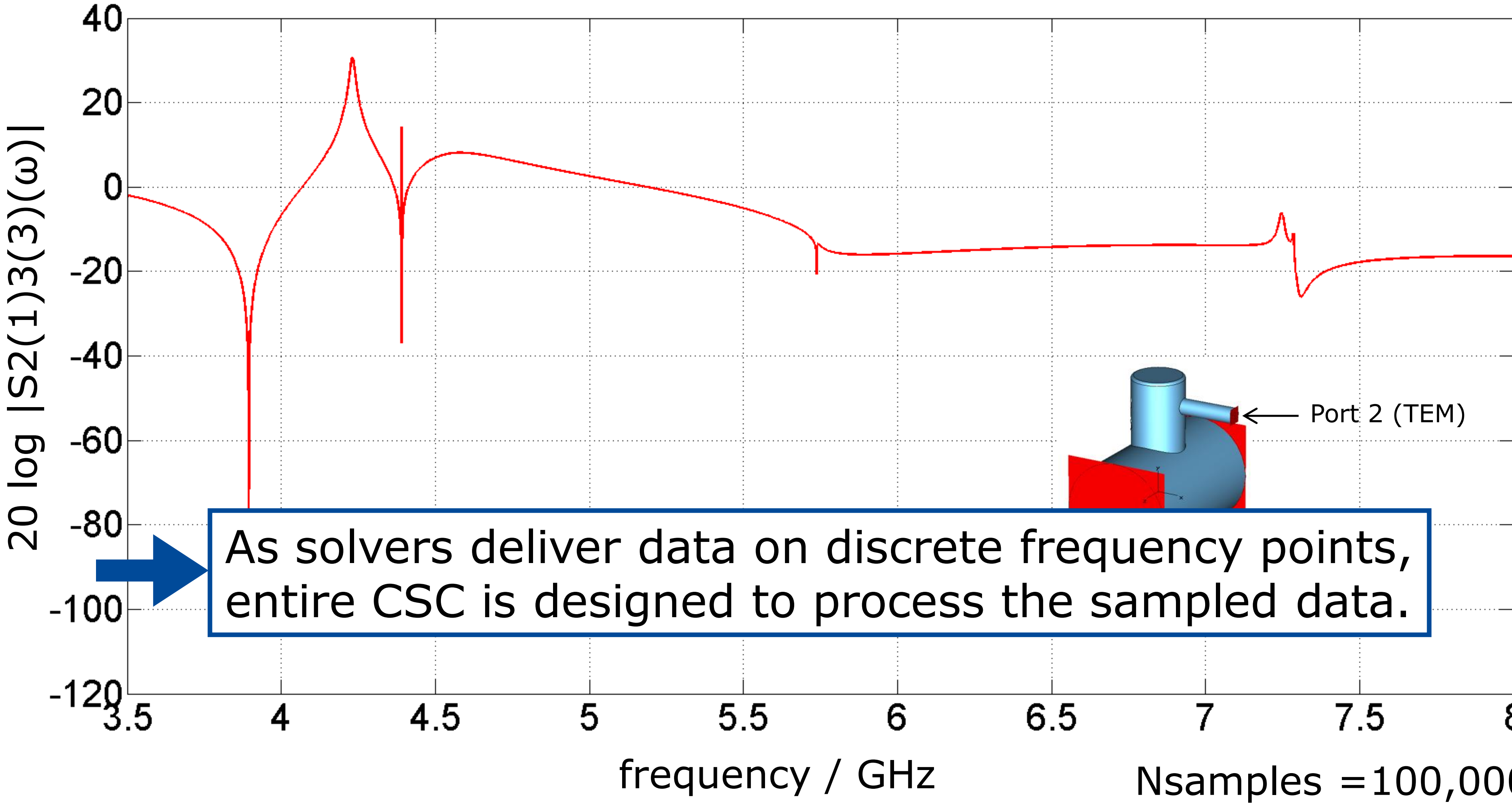
Example: Simulated Transmission HOM Coupler



Nsamples =100,000

Example: Simulated Transmission HOM Coupler



As solvers deliver data on discrete frequency points, entire CSC is designed to process the sampled data.

Nsamples =100,000

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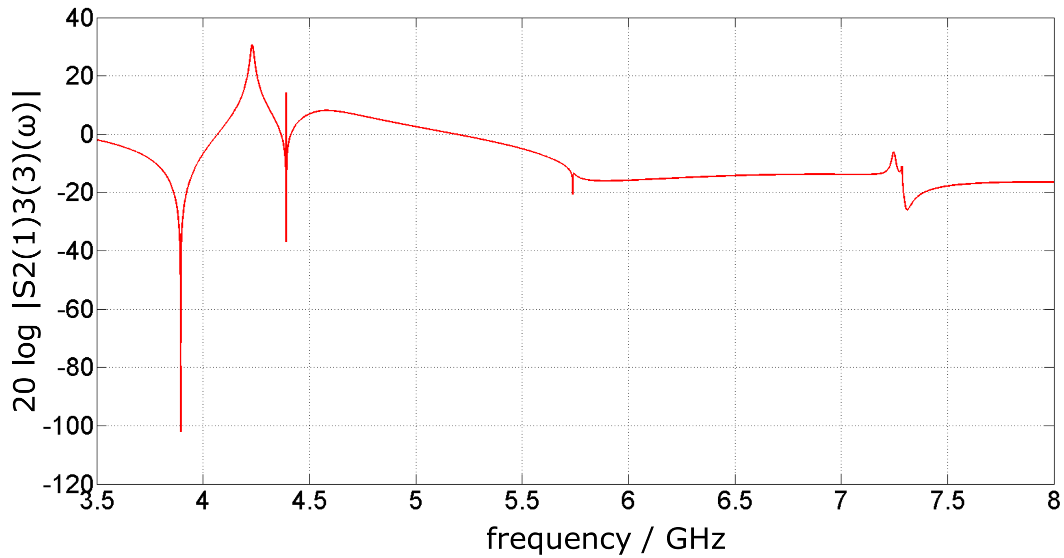
Pole-Zero based Description

- From theory it is known that

$$s(\omega) = \sum_{k=1}^{\infty} \frac{a_k}{j\omega - p_k}$$

as parameters are integral quantities derived from a field problem governed by set of PDEs.

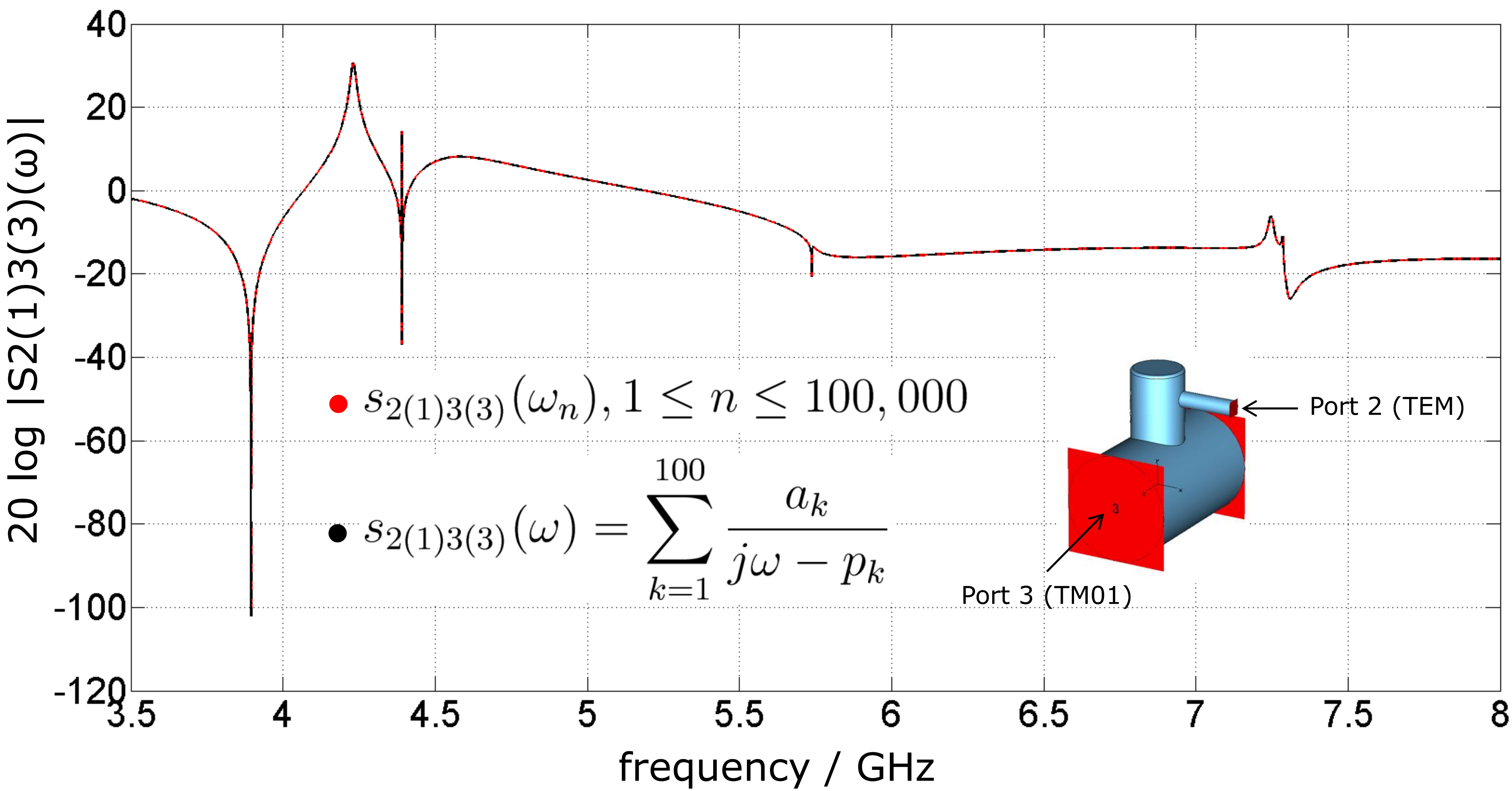


- Restriction on finite frequency interval:

$$s(\omega) = \sum_{k=1}^{N_{poles}} \frac{a_k}{j\omega - p_k} = c_0 \frac{\prod_{k=1}^{N_{poles}-1} (j\omega - \tilde{z}_k)}{\prod_{k=1}^{N_{poles}} (j\omega - z_k)}$$

→ **Sampling leads to redundant information!**

Redundancy in Description of HOM Coupler



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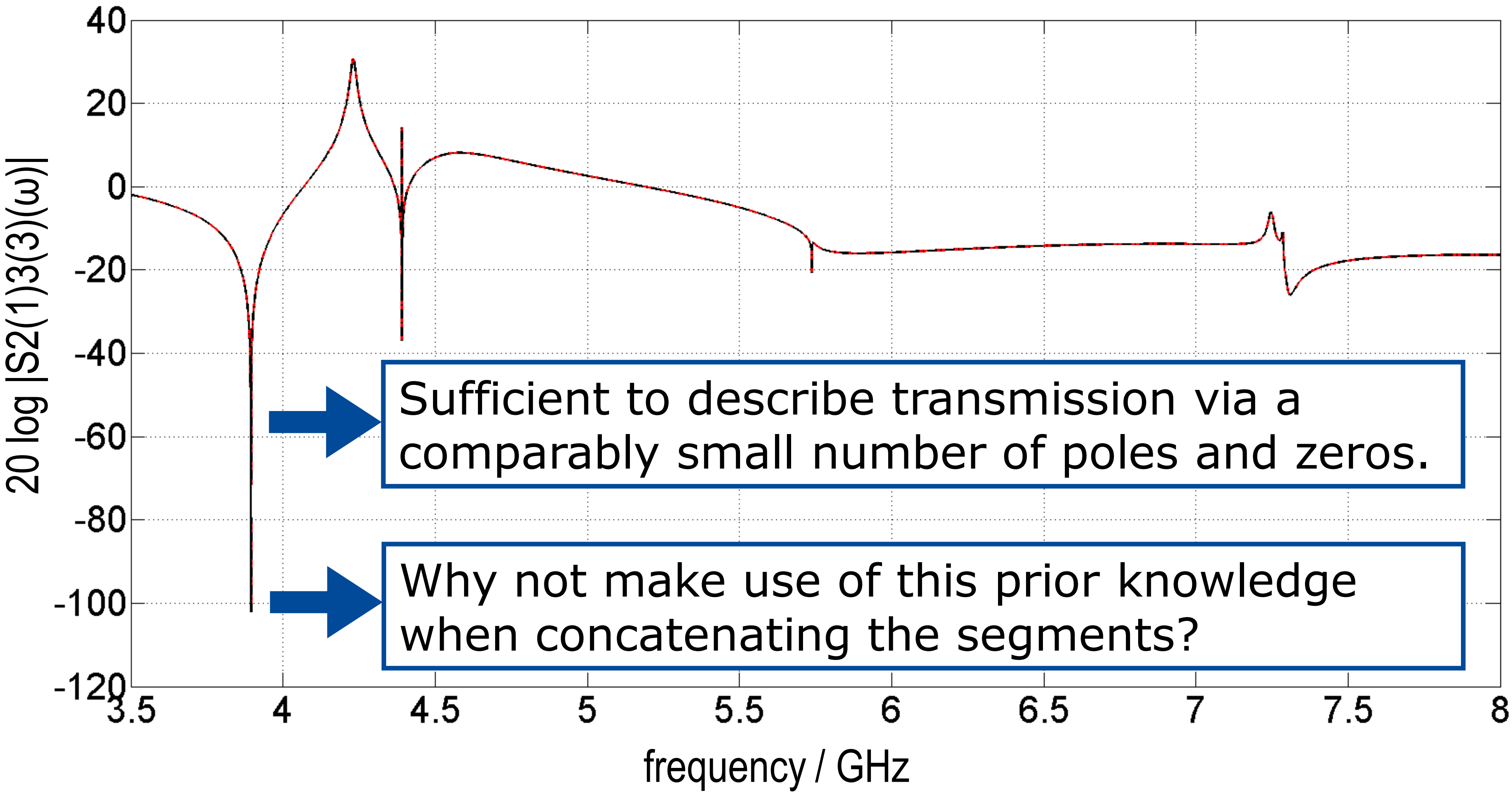
Redundancy in Description of HOM Coupler



State Space Coupling* for Creation of Lumped Model of Complete Structure

*Inspired by M. Dohlus, R. Schuhmann, T. Weiland: "Calculation of frequency domain parameters using 3D eigensolutions", Int. J. Numer. Model. 12, 41-68 (1999) and H.-W. Glock, K. Rothemund, U. van Rienen: "CSC - A System for Coupled S-Parameter Calculations", TESLA-Report 2001-25

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Description of Segments via Lumped Models

- Redundant-free description of segment´s in an impedance formulation by:

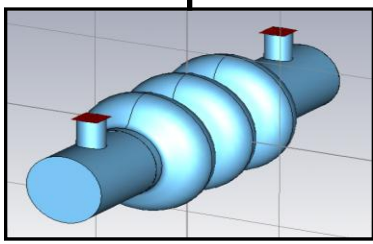
$$\frac{\partial}{\partial t}\underbrace{\mathbf{x}_r(t)}_{\text{state}} = \underbrace{\mathbf{A}_r}_{\text{state matrix}}\mathbf{x}_r(t) + \overbrace{\mathbf{B}_r}^{\text{input matrix}}\underbrace{\mathbf{i}_r(t)}_{\text{port currents}} \qquad \overbrace{\mathbf{v}_r(t)}^{\text{port voltages}} = \underbrace{\mathbf{C}_r}_{\text{output matrix}}\mathbf{x}_r(t)$$

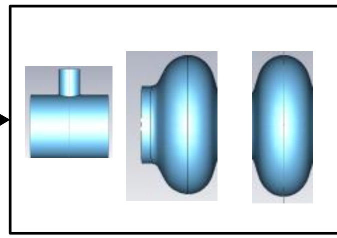
- Respective matrices computed solving real eigenproblems for each segment*
- Upper equations are referred to as lumped equivalent model as they do not have spatial expanses or spatial derivatives.
- Segment‘s transfer function in freq. domain:

$$\mathbf{Z}_r(j\omega) = \mathbf{C}_r\left(j\omega\,\mathbf{I} - \mathbf{A}_r\right)^{-1}\mathbf{B}_r = \sum_{k=1}^{N_{poles}} \frac{\mathbf{M}_k}{j\omega - p_k}$$

*M. Dohlus, R. Schuhmann, T. Weiland: "Calculation of frequency domain parameters using 3D eigensolutions", Int. J. Numer. Model. 12, 41-68 (1999)

Demonstration Example for State Space Coupling





Solve real (!) eigenproblem for each segment + correction term

$$\frac{\partial}{\partial t}\mathbf{x}_r(t) = \mathbf{A}_r\,\mathbf{x}_r(t) + \mathbf{B}_r\,\mathbf{i}_r(t)$$
$$\mathbf{v}_r(t) = \mathbf{C}_r\,\mathbf{x}_r(t)$$

Topology information

State Space Coupling

$$\frac{\partial}{\partial t}\mathbf{x}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{i}(t)$$
$$\mathbf{v}(t) = \mathbf{C}\,\mathbf{x}(t)$$

Impedance formulation of full structure in time domain

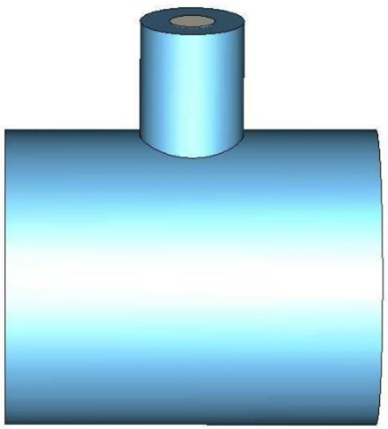
$$\frac{\partial}{\partial t}\mathbf{x}_{sc}(t) = \mathbf{A}_{sc}\,\mathbf{x}_{sc}(t) + \mathbf{B}_{sc}\,\mathbf{a}(t)$$
$$\mathbf{b}(t) = \mathbf{C}_{sc}\,\mathbf{x}_{sc}(t) - \mathbf{I}\,\mathbf{a}(t)$$

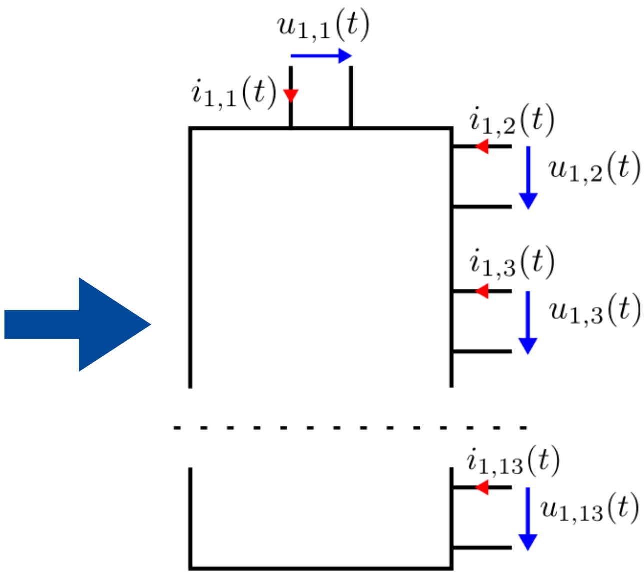
Scattering formulation of full structure in time domain*

*transient system response available using Ordinary Differential Equations (ODE) Solver

Creation of Lumped Model for Coupler

- 50 eigenmodes on hexahedral grid with 34,200 mesh cells used for the 3D modal expansion computed with CST MWS® eigenmode solver (T = 33 min).
- PMC boundary condition used for port planes and PEC boundary condition for the remaining boundaries.
- 13 2D port modes and 10 equidistant sampled impedance matrices for correction term* determined by CST MWS® frequency domain solver.





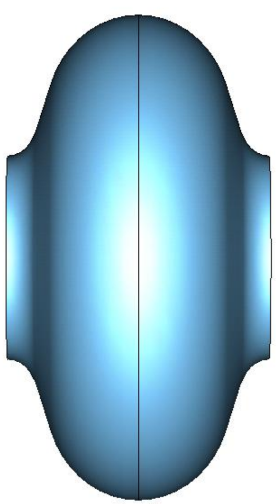
$$\frac{\partial}{\partial t}\mathbf{x}_{HOM}(t) = \mathbf{A}_{HOM}\,\mathbf{x}_{HOM}(t) + \mathbf{B}_{HOM}\,\mathbf{i}_{HOM}(t)$$

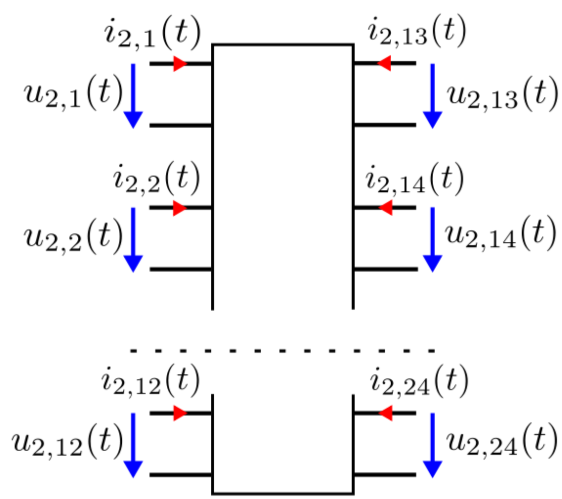
$$\mathbf{v}_{HOM}(t) = \mathbf{B}^{\mathrm{T}}_{HOM}\,\mathbf{x}_{HOM}(t)$$

*M. Dohlus, R. Schuhmann, T. Weiland: "Calculation of frequency domain parameters using 3D eigensolutions", Int. J. Numer. Model. 12, 41-68 (1999)

Creation of Lumped Model for Middle Cell*

- 50 eigenmodes on hexahedral grid with 39,600 mesh cells used for the 3D modal expansion computed with CST MWS® eigenmode solver (T = 34 min).
- PMC boundary condition used for port planes and PEC boundary condition for the remaining boundaries.
- 24 2D port modes and 10 equidistant sampled impedance matrices for correction term** determined by CST MWS® frequency domain solver.





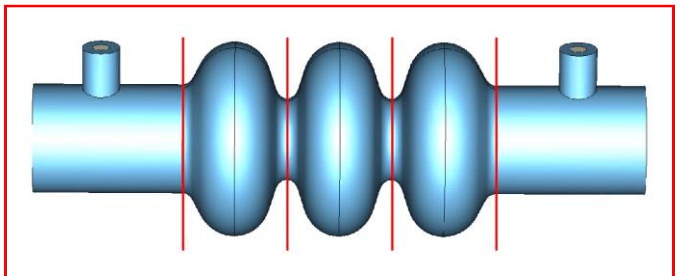
$$\frac{\partial}{\partial t}\mathbf{x}_{cell}(t) = \mathbf{A}_{cell}\,\mathbf{x}_{cell}(t) + \mathbf{B}_{cell}\,\mathbf{i}_{cell}(t)$$

$$\mathbf{v}_{cell}(t) = \mathbf{B}_{cell}^{\mathrm{T}}\,\mathbf{x}_{cell}(t)$$

* Treatment of end cell is analoguous.
**M. Dohlus, R. Schuhmann, T. Weiland: "Calculation of frequency domain parameters using 3D eigensolutions", Int. J. Numer. Model. 12, 41-68 (1999)

Validation Results

Validation using S-Parameters

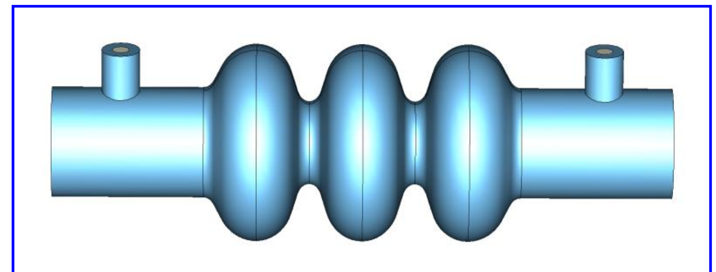


$$\frac{\partial}{\partial t}\mathbf{x}_{sc}(t) = \mathbf{A}_{sc}\,\mathbf{x}_{sc}(t) + \mathbf{B}_{sc}\,\mathbf{a}(t)$$

$$\mathbf{b}(t) = \mathbf{C}_{sc}\,\mathbf{x}_{sc}(t) - \mathbf{I}\,\mathbf{a}(t)$$

$$\mathbf{S}(j\omega) = \mathbf{C}_{sc}\left(j\omega\,\mathbf{I} - \mathbf{A}_{sc}\right)^{-1}\mathbf{B}_{sc} - \mathbf{I}$$

(T = 6 s)



Full structure's S-Parameter Calculation using CST MWS® FR Solver (T = 4 min)

Comparison of S-Parameters

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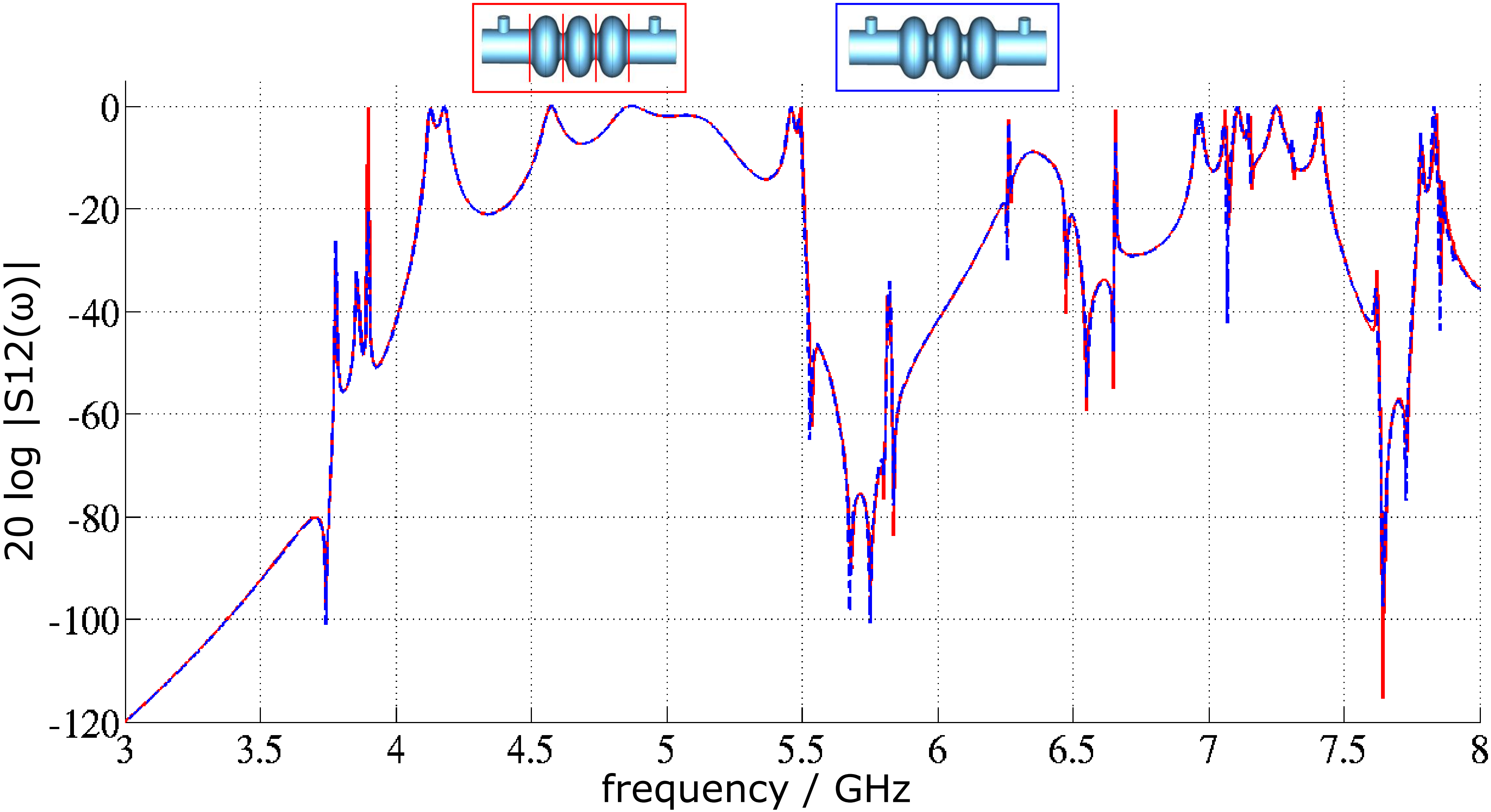
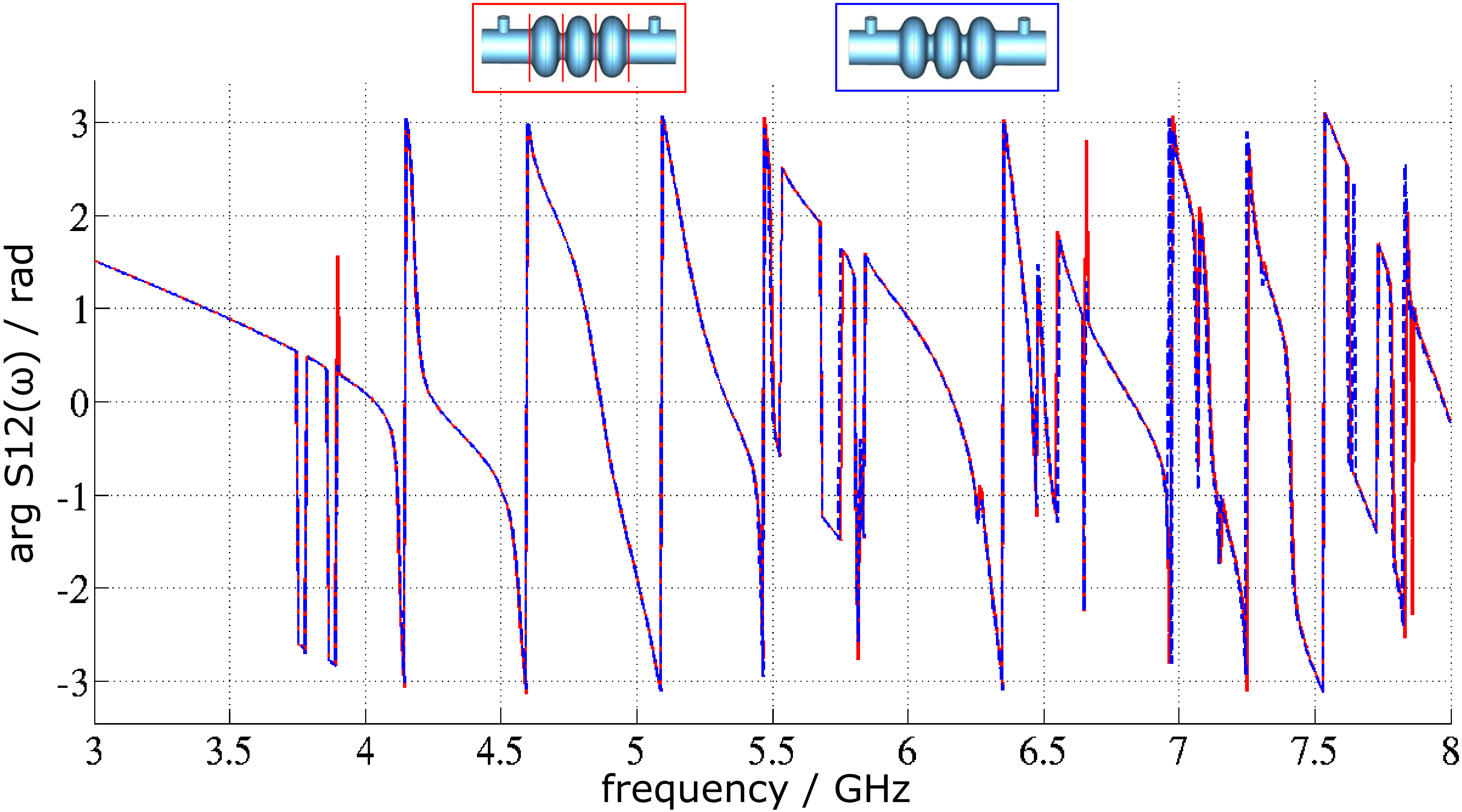
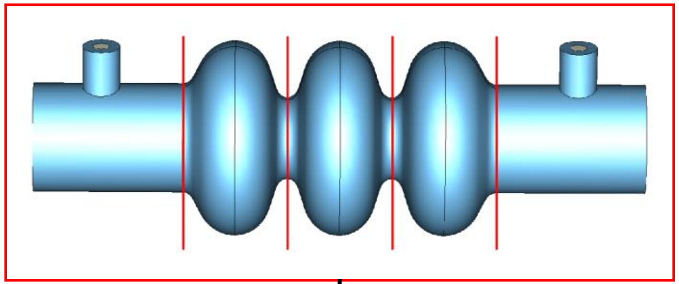
Comparison State Space Coupling vs. Direct

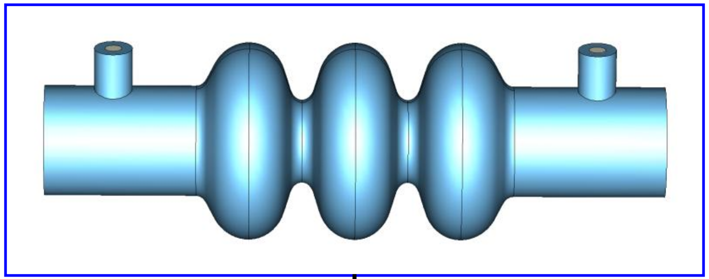


Comparison State Space Coupling vs. Direct



Validation using External Q Factors





$$\frac{\partial}{\partial t}\mathbf{x}_{sc}(t) = \mathbf{A}_{sc}\,\mathbf{x}_{sc}(t) + \mathbf{B}_{sc}\,\mathbf{a}(t)$$

$$\mathbf{b}(t) = \mathbf{C}_{sc}\,\mathbf{x}_{sc}(t) - \mathbf{I}\,\mathbf{a}(t)$$

$$Q_{ext,\nu} = \frac{\omega_\nu W_{stored,\nu}}{P_{ports,\nu}} = \frac{\mathrm{Im}(\lambda_\nu)}{2\mathrm{Re}(\lambda_\nu)}$$

$$\mathrm{eig}(\mathbf{A}_{sc}) = \{\lambda_1, \ldots, \lambda_\nu, \ldots\}$$

(T = 1 s)

Full structure‘s eigenmode calculation using CST MWS® EM Solver (T ≈ 3 h)

Comparison of External Q‘s

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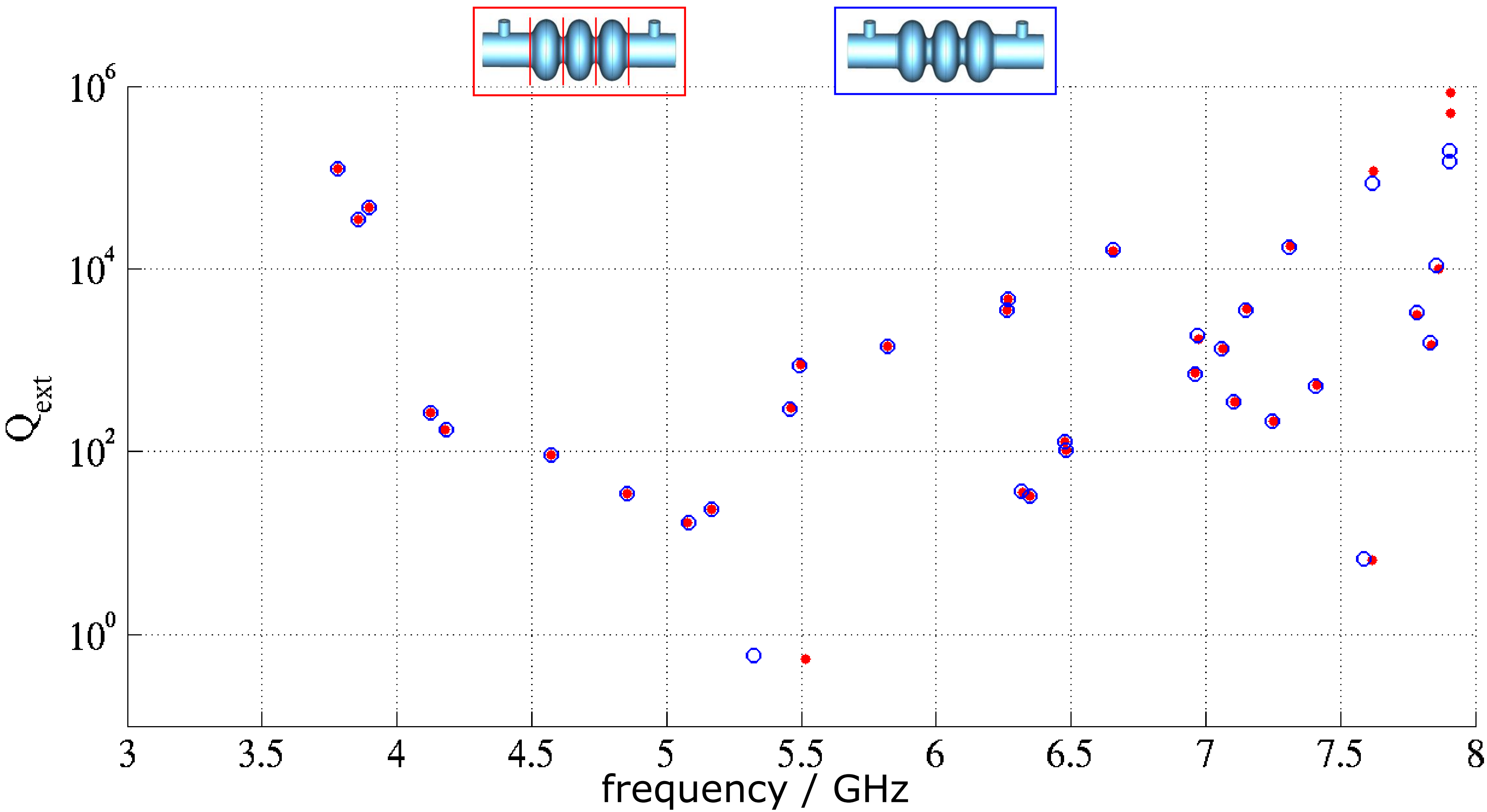
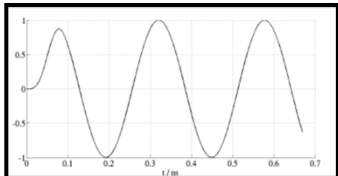
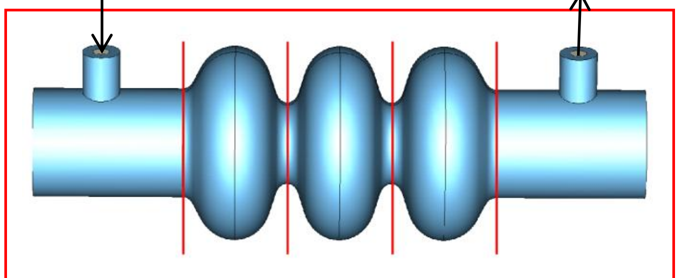
Comparison State Space Coupling vs. Direct

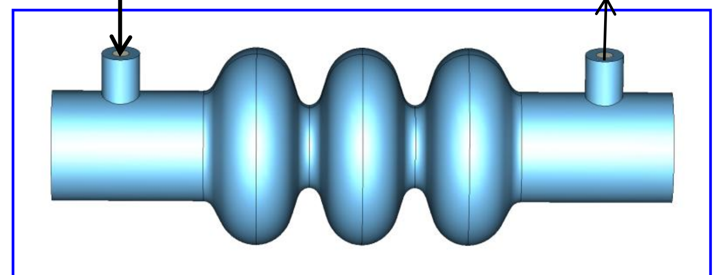


Validation using Transient System Responses





?



?

$$\frac{\partial}{\partial t}\mathbf{x}_{sc}(t) = \mathbf{A}_{sc}\,\mathbf{x}_{sc}(t) + \mathbf{B}_{sc}\,\mathbf{a}(t)$$

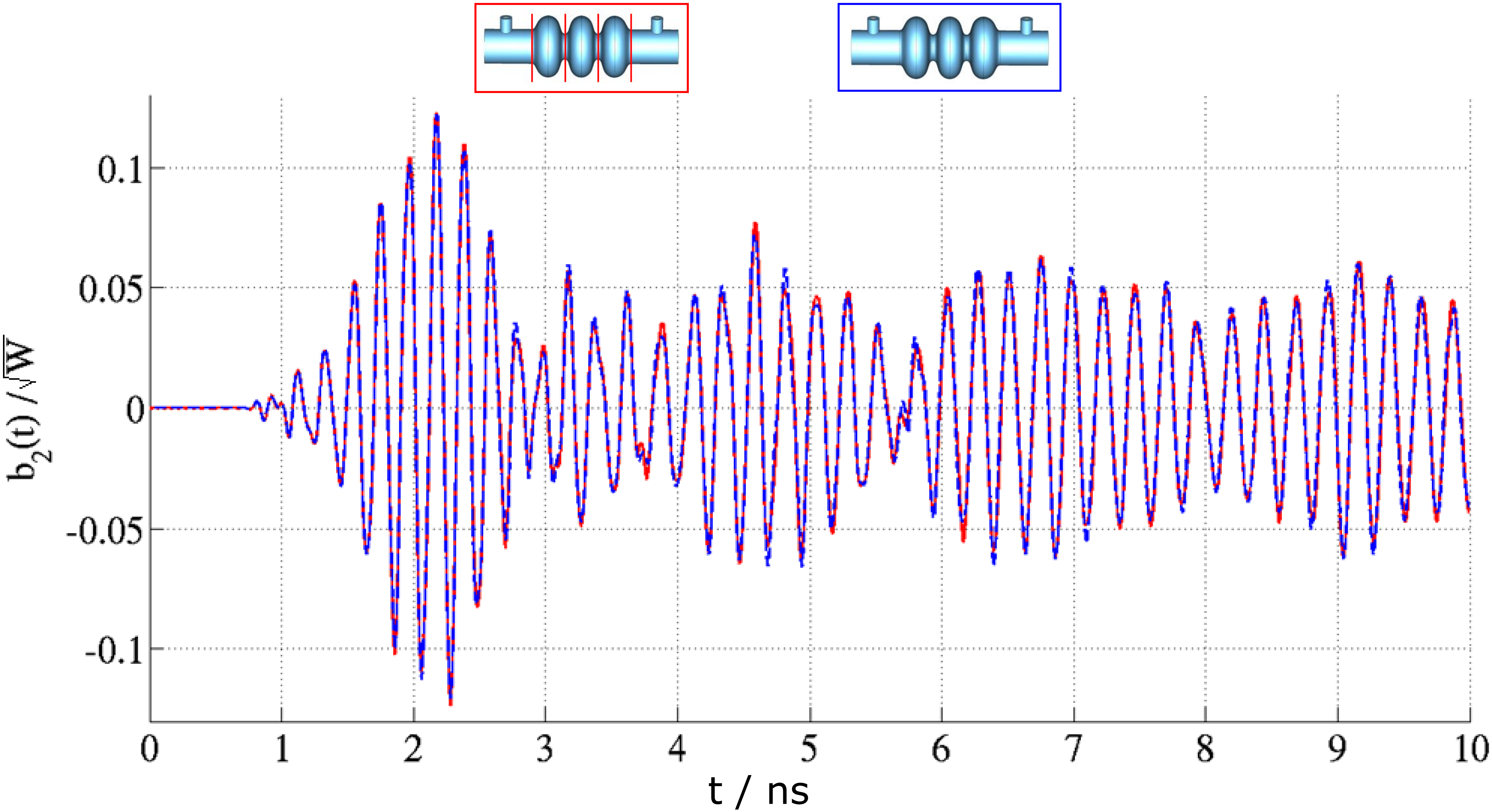
$$\mathbf{b}(t) = \mathbf{C}_{sc}\,\mathbf{x}_{sc}(t) - \mathbf{I}\,\mathbf{a}(t)$$

Computation of system response by means of ODE solver (T = 1s)

Full structure‘s transient simulation using CST MWS® Transient Solver (T ≈ 30 s)

Comparison of Responses

Comparison Direct vs. State Space Coupling



Conclusions

Conclusions

- State space coupling enables the creation of lumped equivalent models of complex structures.
- In comparision to CSC no redundant sampling of S-matrices.
- The lumped model directly allows for computation of S-Parameters, transient system responses, external quality factors and other secondary quantities.
- The validation example shows a good agreement between results obtained by direct and piecewise computations over a wide frequency range.

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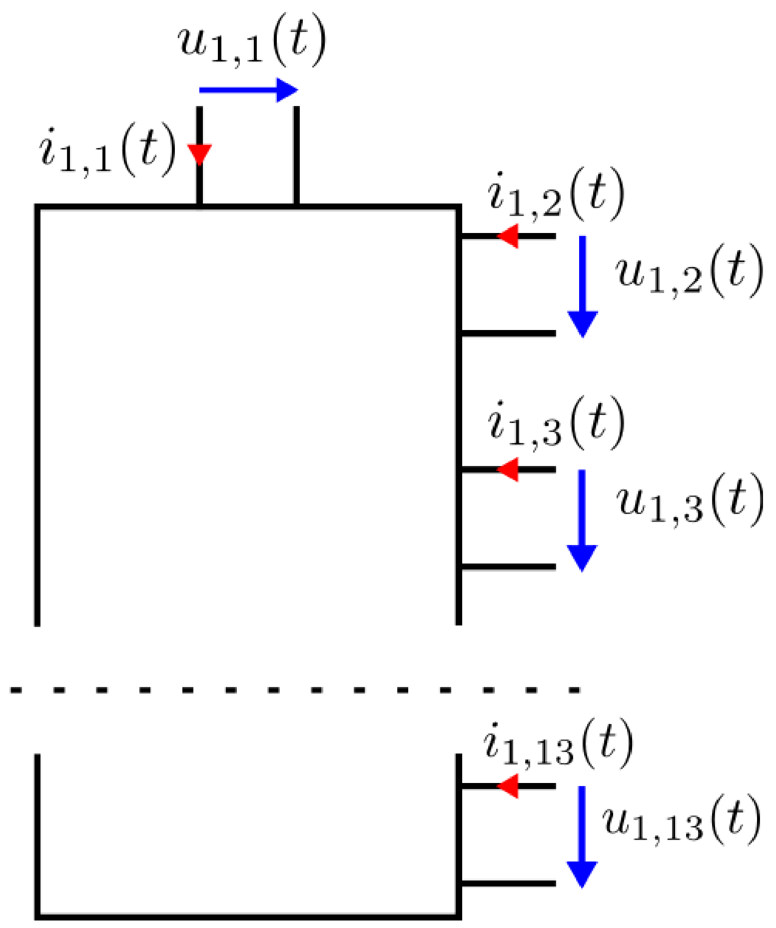
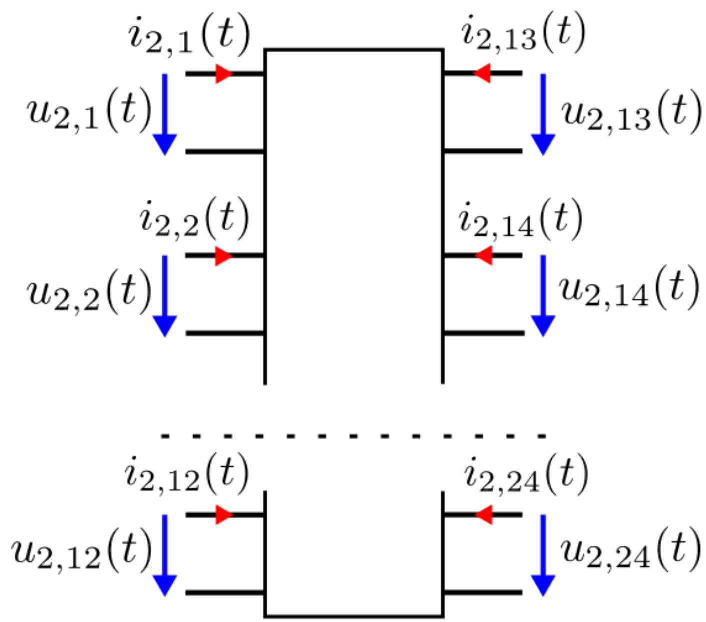
Cutoff Frequencies of Coupler



Index	Type	f_co / GHz
1	TEM	0
2	TE11	4.3918
3	TM01	5.7364
4	TE21	7.2837
5	TM11	9.1354
6	TE31	10.0119
7	TM21	12.2313
8	TE41	12.6647
9	TE12	13.1486
10	TM02	13.1486
11	TM31	15.1878
12	TE51	15.2665
13	TE22	15.9647

Cutoff Frequencies of End Cell

Index	Type	f_co / GHz
1	TE11	4.3918
2	TM01	5.7364
3	TE21	7.2837
4	TM11	9.1354
5	TE31	10.0119
6	TM21	12.2313
7	TE41	12.6647
8	TE12	13.1486
9	TM02	13.1486
10	TM31	15.1878
11	TE51	15.2665
12	TE22	15.9647



Index	Type	f_co / GHz
13	TE11	5.85489
14	TM01	7.64644
15	TE21	9.70892
16	TM11	12.1716
17	TE31	13.338
18	TM21	16.2819
19	TE41	16.8615
20	TE12	16.9082
21	TM02	17.5047
22	TM31	20.2098
23	TE51	20.3156
24	TE22	21.2472