Eigenmode Computation For Ferrite-Loaded Cavity Resonators

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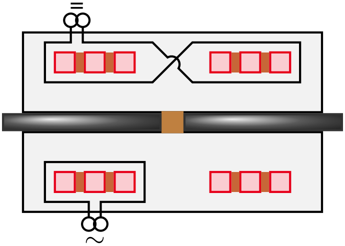

THACC2

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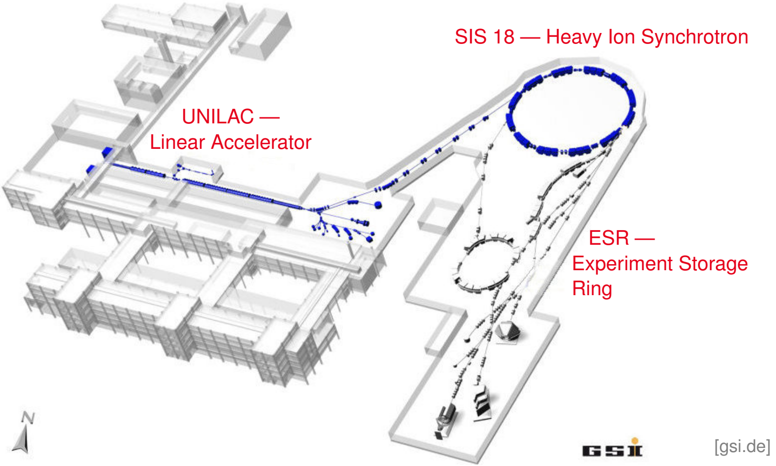
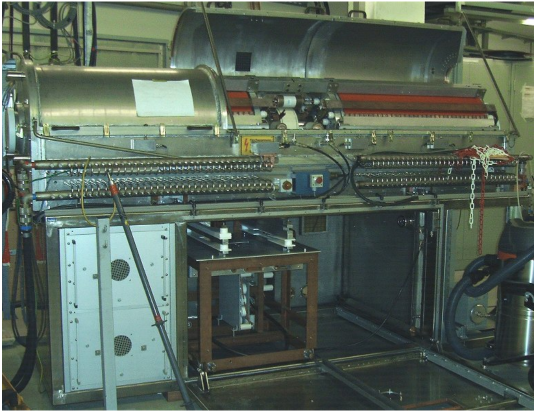
GSI Helmholtzzentrum für Schwerionenforschung



Motivation

GSI SIS18 Ferrite Cavity



Main benefits of ferrite cavities:

- reduction of wavelength
  $\Rightarrow$ more compact cavity
- modification of resonance frequency in a wide range
  SIS 18 cavity:
  $\sim 0.6\,\text{MHz} - 5\,\text{MHz}$

Motivation

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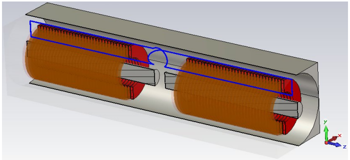
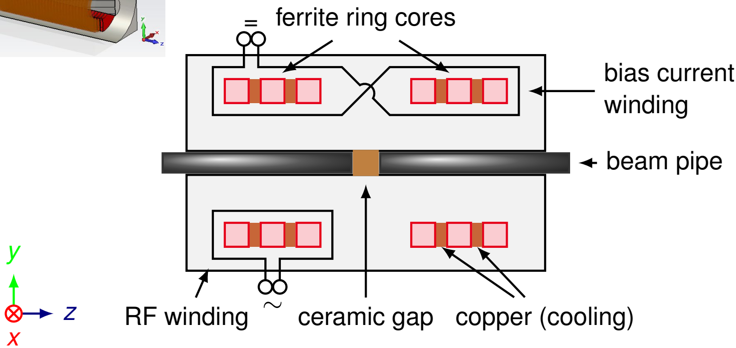
GSI SIS18 Ferrite Cavity: Main components





Computational Model: Fundamental Relations

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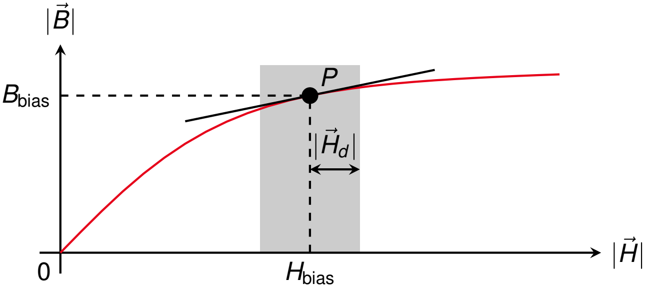
Assumptions: $|\vec{H}_d| \ll |\vec{H}_{\text{bias}}|$, effect of hysteresis negligible

$$\vec{B}(t) = \mu_0 \mu_{\text{bias}} \vec{H}_{\text{bias}} + \mu_0 \overset{\leftrightarrow}{\mu}_d \vec{H}_d(t) = \mu_0 \mu_{\text{bias}} \vec{H}_{\text{bias}} + \mu_0 \overset{\leftrightarrow}{\mu}_d \, \text{Re}\left(\vec{H}_d \cdot e^{-\mathrm{i}\omega t}\right)$$

- Linearization at working point



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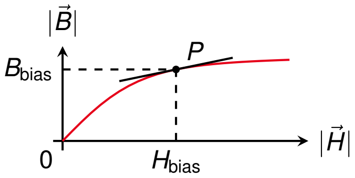

- **Modification of bias current**
  $\Rightarrow$ Modification of differential permeability
  $\Rightarrow$ Adjustment of eigenfrequency

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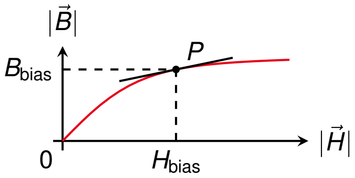

- Eigensolutions determined by:

$$\epsilon^{-1} \nabla \times \left(\mu_0^{-1} \overleftrightarrow{\mu}_d^{-1} \nabla \times \vec{E}\right) = \omega^2 \vec{E}$$

Boundary condition: $\vec{n} \times \vec{E} = 0$ on cavity boundary

Computational Model: Fundamental Relations

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[D. Polder, Phil. Mag., 40 (1949)]

Properties of the differential permeability tensor $\overleftrightarrow{\mu}_d$:

- Fully occupied (3×3)-tensor, for $\vec{H} = H_{\text{bias}} \cdot \vec{e}_z$ reduces to the Polder tensor

$$\overleftrightarrow{\mu}_d = \begin{pmatrix} \mu_1 & \mathrm{i}\mu_2 & 0 \\ -\mathrm{i}\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{with} \qquad \mu_{1,2} = \mu_{1,2}\left(\vec{H}_{\text{bias}}, \omega\right)$$

- If magnetic losses are included:
  $\mathrm{Im}(\mu_{1,2}) \neq 0 \Rightarrow$ Non-Hermitian

Computational Model: Implementation

$$\epsilon^{-1}\nabla \times \left(\mu_0^{-1}\overleftrightarrow{\mu}_d^{-1}\nabla \times \vec{E}\right) = \omega^2\vec{E}$$

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Discretization by Finite Integration Technique (FIT):

$$M_\epsilon^{-1}\widetilde{\mathbf{C}}M_{d,\mu}^{-1}\mathbf{C}\widehat{\mathbf{e}} = \omega^2\widehat{\mathbf{e}}$$

Computational Model: Implementation

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- permeability tensor $M_{d,\mu}$:
  - non-diagonal
  - dependend on $\vec{H}_{\text{bias}}$ and ω

Computational Model: Implementation

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- permeability tensor $M_{d,\mu}$:
  - non-diagonal
  - dependend on $\vec{H}_{\text{bias}}$ and ω
  - if magnetic losses included:
- requirements on eigensolver:

  $\Rightarrow$ nonlinear

  $\Rightarrow$ non-Hermitian

Computational Model: Implementation

Concept

Magnetostatic solver —— $\mathbf{M}_{d,\mu}^{-1}$ ——> Eigensolver

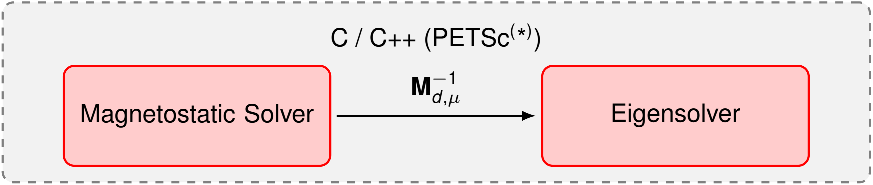
- calculation of bias magnetic field
- determination of permeability matrix $\mathbf{M}_{d,\mu}^{-1}$

- calculation of eigenmodes

General requirements:

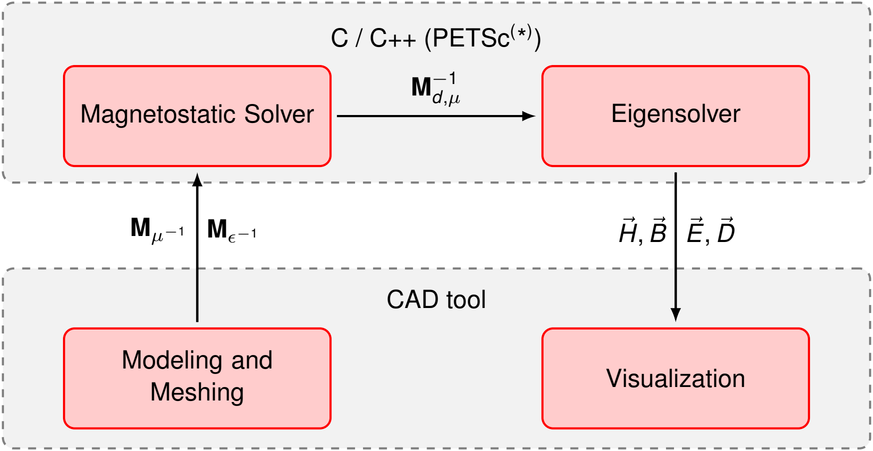
- support of nonlinear material
- support of lossy material
- parallel computation with distributed memory (scalability)

Computational Model: Implementation



(*) Portable, Extensible Toolkit for Scientific Computation

Computational Model: Implementation



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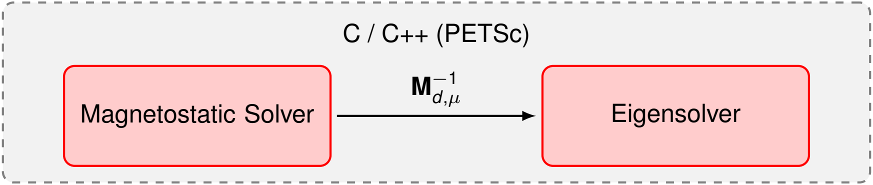
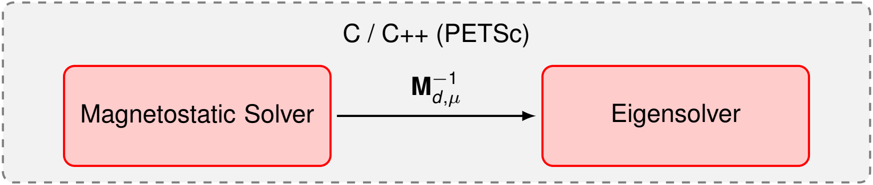


- *H_i***-algorithm**
  Helmholtz decomposition
  $\vec{H} = \vec{H}_i + \vec{H}_h$ with
  $\nabla \times \vec{H}_i = \vec{J}$ and $\vec{H}_h = -\nabla\varphi$
- Solution of nonlinear equation: successive substitution or Newton method

Computational Model: Implementation



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- **Jacobi-Davidson algorithm**
  harmonic Ritz-values for computation of interior eigenvalues
- Solution of nonlinear problem: successive substitution

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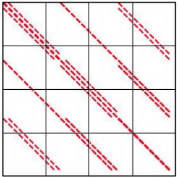
Aim: Efficient distributed computing

Structure of system matrix

standard arrangement



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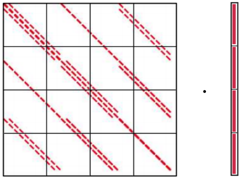

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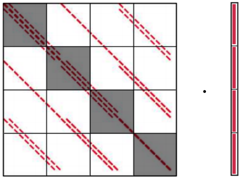

Parallel Computing

Aim: Efficient distributed computing

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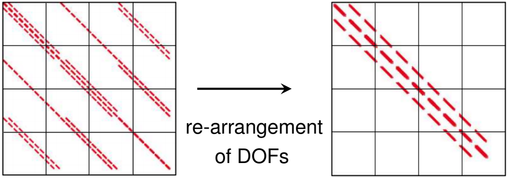

⇒ higher computation to communication ratio

Parallel Computing

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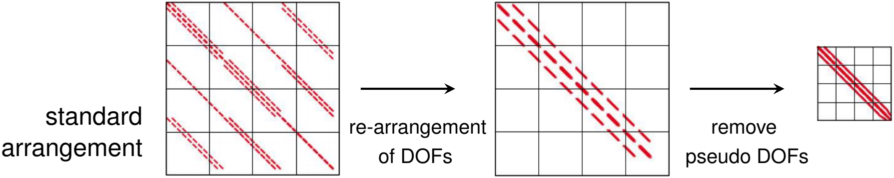

⇒ higher computation to communication ratio

⇒ better load balancing

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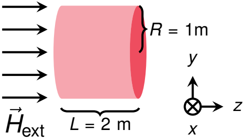
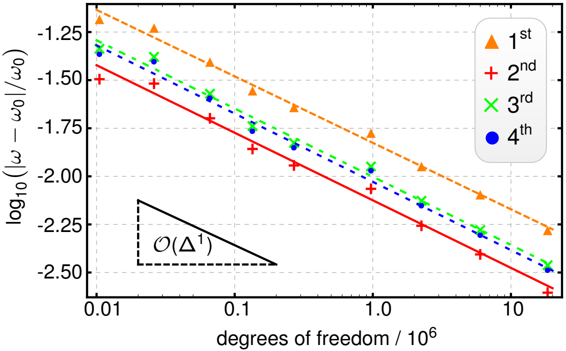
Numerical Examples
Biased Cylinder Resonator

- test model: $(|\vec{H}_{\text{ext}}| = 2750\,\frac{\text{A}}{\text{m}};\ \mu_r = 7)$
  lossless, ferrite-filled cylindrical cavity resonator
  longitudinally biased by homogeneous magnetic field
- semi-analytical solution available [Chinn, Epp and Wilkins]





Numerical Examples
Biased Cylinder Resonator

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  lossless, ferrite-filled cylindrical cavity resonator
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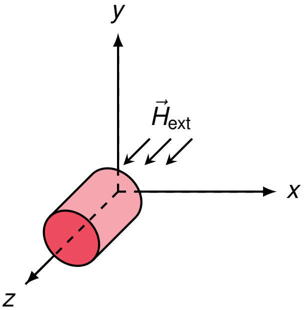

$\vec{H}_{\text{ext}} \parallel \vec{e}_z$:

$$\overleftrightarrow{\mu}_d = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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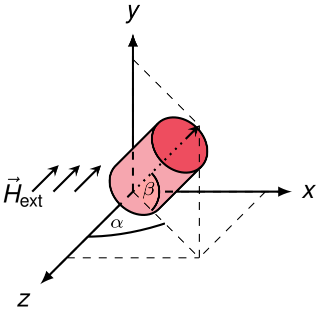
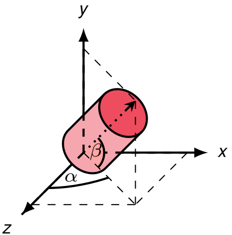


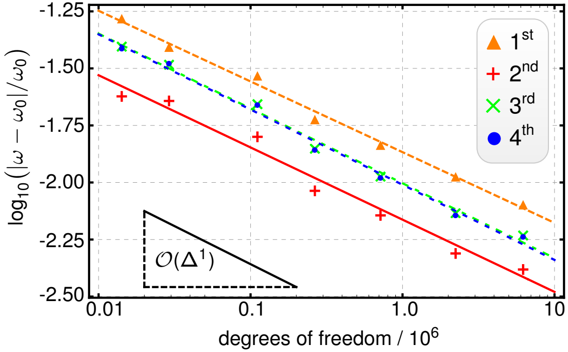
$\vec{H}_{ext} \nparallel \vec{e}_z$:

$$\overleftrightarrow{\mu}_d = \begin{pmatrix} \mu_{x,x} & \mu_{x,y} & \mu_{x,z} \\ \mu_{y,x} & \mu_{y,y} & \mu_{y,z} \\ \mu_{z,x} & \mu_{z,y} & \mu_{z,z} \end{pmatrix}$$

Numerical Examples
Biased Cylinder Resonator

- test of construction of permeability tensor for different orientations of cylinder axis to coordinate axes
- example shown for $\alpha = 45^\circ$ and $\cos\beta = \sqrt{2/3}$





Numerical Examples
Biased Cavity With Ferrite Ring Cores

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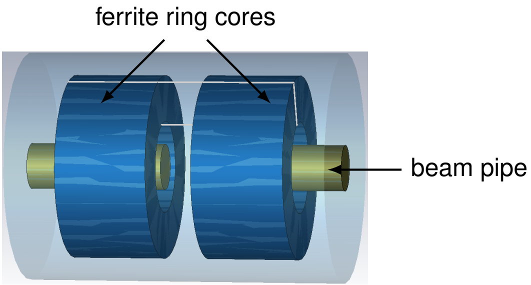
Model description



- Ferrite material is characterized by
$B(H) = \mu_0\, 2.5 \cdot 10^4 \tanh\left(H \cdot 10^{-2}\frac{\mathrm{m}}{\mathrm{A}}\right)\frac{\mathrm{A}}{\mathrm{m}} + \mu_0 H$ and $\epsilon_r = 1$

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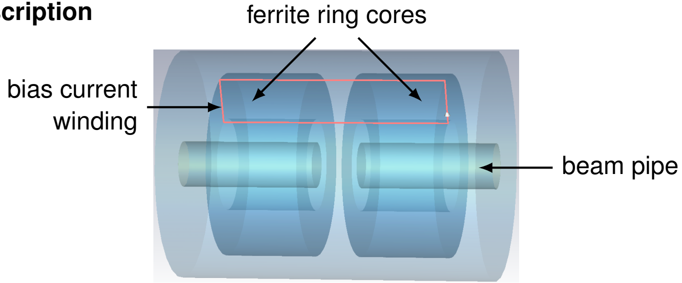

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- bias magnetic field excited by current winding (2 kA)

Numerical Examples
Biased Cavity With Ferrite Ring Cores

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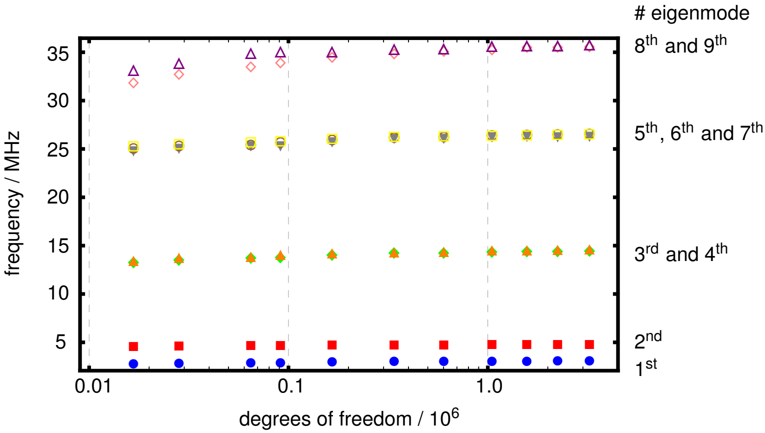
Results of fully nonlinear computation

Spectrum of the lowest nine eigenmodes



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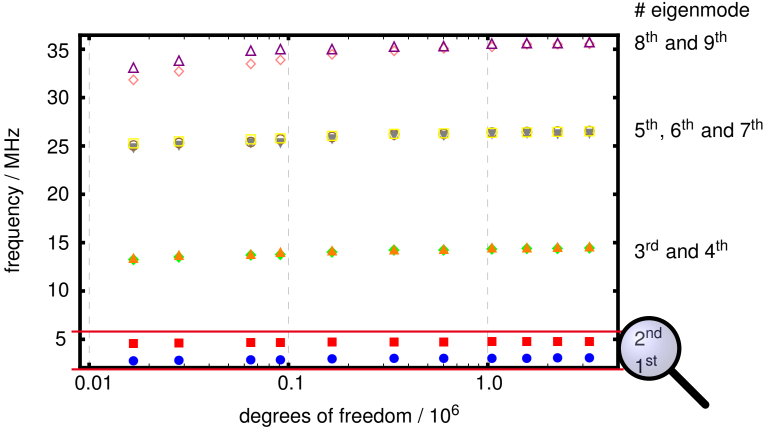

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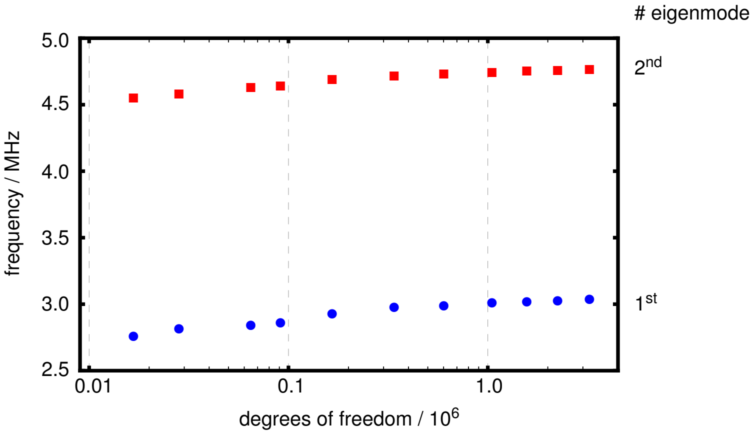

Numerical Examples
Biased Cavity With Ferrite Ring Cores

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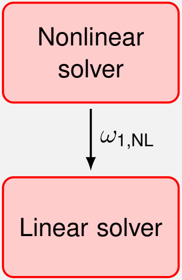
Comparison nonlinear — linear computation

Linear approach

1. Compute lowest eigenfrequency $\omega_{1,\mathrm{NL}}$

2. Set μ_r homogeneously such that $\omega_{1,\mathrm{Lin}} \backsimeq \omega_{1,\mathrm{NL}}$



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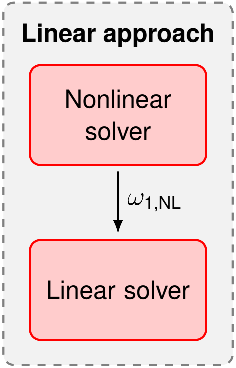

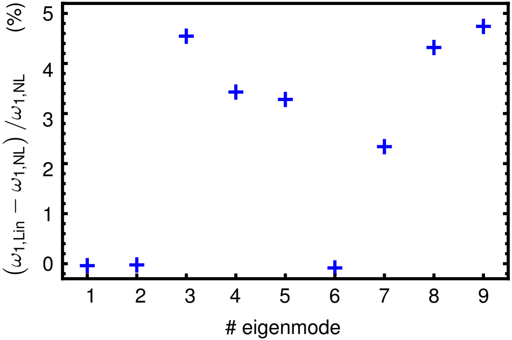

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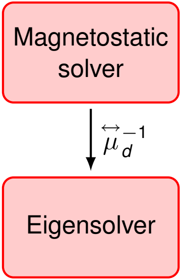
Summary

- **Goal:**
  calculation of eigenvectors for biased ferrite cavities

  1. Magnetostatic solver (nonlinear material):

     $\Rightarrow$ permeability tensor $\overleftrightarrow{\mu}_d$

  2. Eigensolver:
     $\Rightarrow$ nonlinear complex eigenvalue problem



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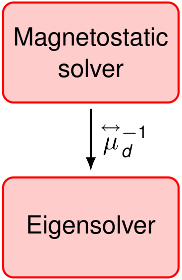

- **Status:**
  Functionality of fully nonlinear solver for Hermitian problems demonstrated