

# THE DARK CURRENT AND MULTIPACTING CAPABILITIES IN OPAL: MODEL, BENCHMARKS AND APPLICATIONS

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23-08-2012



## 1 Background and Goal

- Dark current and Multipacting Phenomenas
- Existing Tools: Theory and Codes
- Motivation and Goal of Our Work

## 2 Schemes, Models and Implementations in OPAL

- Geometry Handling
- Surface Physics Models
- Implementation in OPAL

## 3 Benchmark Results

- Code to Code Benchmark of Furman-Pivi's model
- Benchmark Against Non-stationary Theory
- Benchmark Against a Nano-second Time Resolved Experiment

## 4 Preliminary results

- Dark Current Simulation on CTF3 Gun
- Multipacting simulation on Cyclotron Cavity

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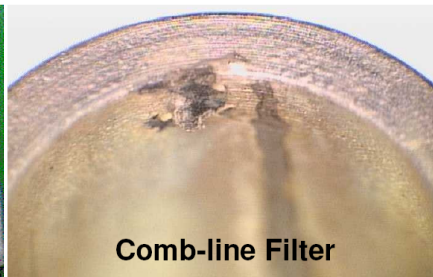
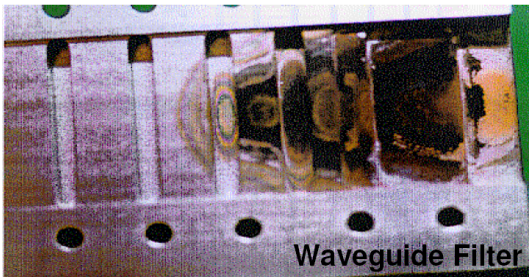
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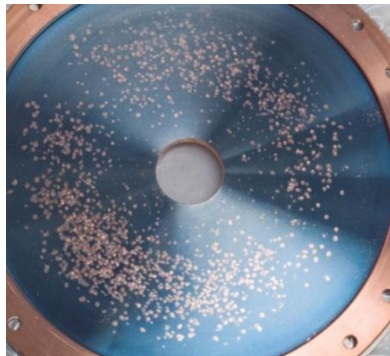
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# Experimental Observations



- Multipacting observed in RF/microwave components in the aerospace community.

# Experimental Observations



- Dark current observed in a RF cavity and a Be window in the Linac community.

# Experimental Observations

- Multipacting is also a very disturbing phenomenon appearing in high-Q RF cavities in the cyclotron community.
- Electrons are pulled out-off the walls of resonators by the RF field. If these electrons then hitting other metallic surfaces, more new secondary electrons are produced.
- This kind of electron multiplication will limit the power level until the surfaces will be cleaned through a conditioning process.
- Conditioning can be a very time-consuming process.

1

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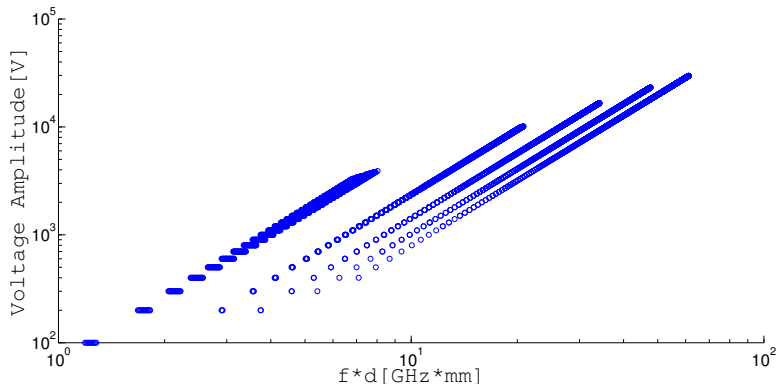
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# Classic Multipacting Theory I

- Simple geometries (parallel plate, rectangular waveguide or coaxial line)
- Deterministic: the emission energy is a constant fraction of the impact energy
- Multipacting is contributed only by the electrons whose transit times through the gap are equal to an odd number of half-periods of the high-frequency field

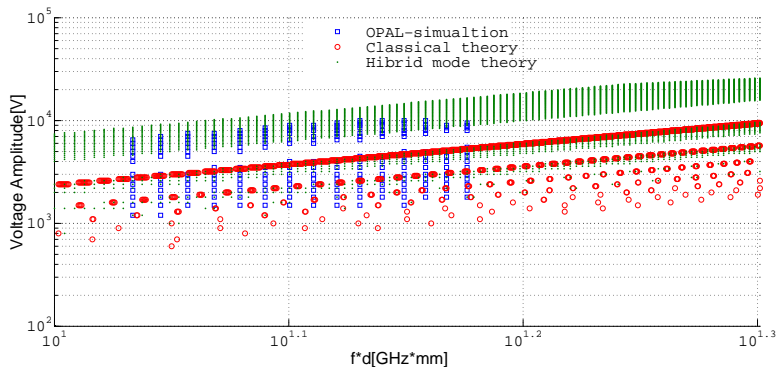


# Classic Multipacting Theory II



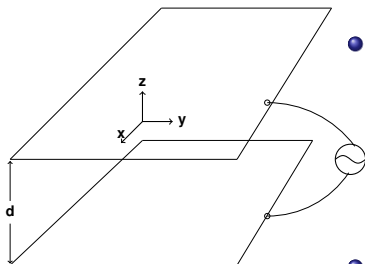
- Example resonant zone in phase space

# Classic Multipacting Theory III



- Fail to predict multipacting zone due to missing the single side impact which also plays an important role in multiplication

# Non-stationary Multipacting Theory I



- First published in S.Anza's paper.
- Parallel Plate:

$$\begin{aligned}\frac{d^2 z}{dt^2} &= -\frac{e}{m} E_0 \sin \omega t \\ &= -\frac{e}{m} \frac{V_0}{d} \sin \omega t \quad (1)\end{aligned}$$

- Random nature of emission energy( velocity): random impact phase, energy ...

*S. Anza et al., Phys. Plasmas 17, 062110  
2010*

# Non-stationary Multipacting Theory(2)

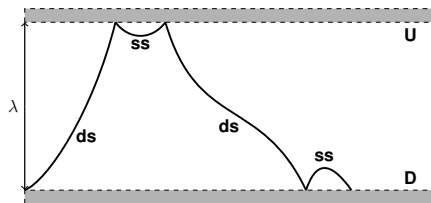
- Integrating equation (1) w.r.t variable  $t$ , and using initial conditions  $\frac{dz}{dt}|_{t=t_0} = v_0$ ,  $z|_{t=t_0} = 0$ , normalized variables:  $v_\omega = eV_0/m\omega d$ ,  $\lambda = \omega d/v_\omega$ ,  $u = v_0/v_\omega$ ,  $\omega t_0 = \varphi_0$ ,  $\omega t = \varphi$

$$z = -\frac{d}{\lambda} \sin \omega t + \frac{d}{\lambda} (u + \cos \varphi_0) \omega t + \frac{d}{\lambda} \sin \varphi_0 - \frac{d}{\lambda} (u + \cos \varphi_0) \varphi_0. \quad (2)$$

- if we define  $\xi = \omega z/v_\omega$  and  $\tau = \varphi - \varphi_0$  in consequence (2) can be rewritten by using this new variable as:

$$\xi(\varphi, \varphi_0, u) = (u + \cos \varphi_0) \tau + \sin \varphi_0 - \sin(\varphi_0 + \tau). \quad (3)$$

# Non-stationary Multipacting Theory(3)



- Now double-side(ds) and single-side(ss) impacting exist:  
 $\xi(\varphi, \varphi_0, u) = \lambda$  and  $\xi(\varphi, \varphi_0, u) = 0$  in equation (3)
- More (most!) complete description of multipacting in PP

# Non-stationary Multipacting Theory(4)

- The initial velocity  $u$  of emitted particles is a random variable, the solution of equation (3) w.r.t time  $\tau$  that particles hit the plates is also a random variable.
- As long as we know the probability density function (PDF) of the initial velocity  $u$ , which usually is a thermal distribution, then the PDF of time  $\tau$  can be derived according to the rule of change of variable in probability theory.

# Non-stationary Multipacting Theory(5)

- The electron emission rates and impact rates in each plate can be described by the PDF of  $\tau$ , at which particles hitting the plates, and the secondary emission yield coefficient w.r.t  $\tau$  and  $u$ .
- The particle number can be obtained by integrating the emission rates and impact rates w.r.t time (details are in the appendix of this talk and also in S. Anza's paper).

# Code Review

	Code Name	EM Field Solver	Tracking Algorithm	Emission Effects <sup>&amp;</sup>	Geometry	Scanning Parameters <sup>§</sup>	Multipacting Decision <sup>+</sup>
<b>Helsinki</b>	MultiPac	Included	Runge-Kutta	SE, $E_{kin} = \text{user}$	2D	$s, \phi, \alpha, E_a$	CF/ECF/DF
<b>Saclay</b>	MUPAC	Superfish <sup>*</sup>	Runge-Kutta	$\alpha=0$ , SE $E_{kin} = \text{user}$	2D	$s, \phi, E_a$	ECF/DF
<b>Genoa</b>	TRAJEC TTWTR AJ	OSCAR2D <sup>*</sup>	Standard Newton	SE, scattering, $E_{kin} = \text{user}$	2D	$s, \phi, \alpha, E_a$	Spatial/time focusing
<b>Cornell I</b>	MULTIP	SUPERLAN S (Superfish <sup>*</sup> )	Runge-Kutta	SE, FE, $E_{kin} = \text{user}$	2D	$s, \phi, \alpha, E_a$	Time focusing
<b>Cornell II</b>	XING	MAFIA, analytic	Leapfrog Runge-Kutta	$\alpha=0$ , SE $E_{kin} = 2\text{eV}$	3D	$s, \phi, E_a$	CF/ECF/DF
<b>Albuquerque</b>	TRAK- 3D	Included	Runge-Kutta	SE, FE, $E_{kin} = \text{user}$	3D	$s, \phi, \alpha, E_a$	Spatial Focusing
<b>Moscow</b>	MULTP	Superfish, MAFIA <sup>*</sup>	Adams-2D Leapfrog-3D	SE, $E_{kin} = \text{user}$	2D, 3D	$s, \phi, \alpha, E_a$	Phase Focusing

- F.L.Krawczyk's review paper in the 10th Workshop on RF Superconductivity, 2001, Tsukuba, Japan.
- CST, Vorpahl and Track3p.



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- Dark current problem in SwissFEL project at PSI
- Multipactor prediction for the RF cavities of the CYCIAE-100 cyclotron at CIAE

- Extend the 3D parallel particle tracking code OPAL with complex geometry handling capabilities
- Add dark current and multipacting simulation capabilities in OPAL to handle complex RF structures with arbitrary geometries
- Post processing and visualization

1

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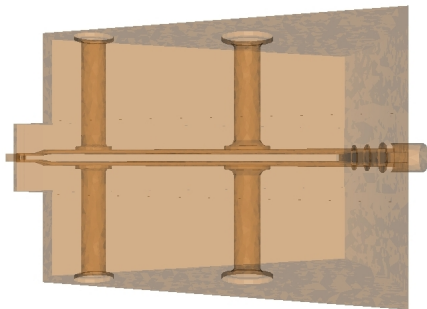
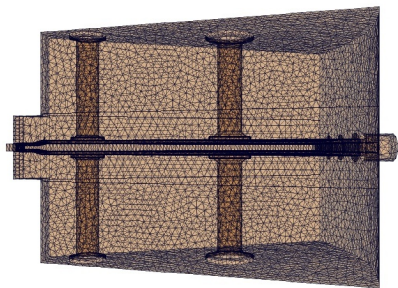
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# 3D Geometry Handling Capability for OPAL

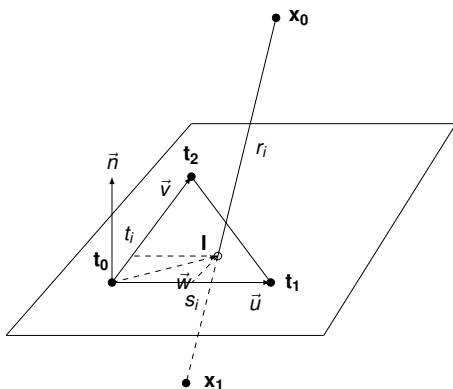
- Read in surface mesh generated by GMSH (step-file) in H5hut format
- Triangle-line segment intersection test and boundary box strategy based collision test
- Handle arbitrary structure as long as it is closed, or more generally speaking: arbitrary structure with pre-defined inward normals

# 3D Geometry model in OPAL I

- Geometry represented by triangulated surface

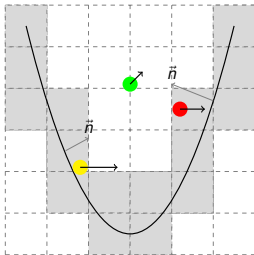


# 3D Geometry model in OPAL II



- Line segment-Triangle intersection test

# 3D Geometry model in OPAL III



- Boundary bounding box to speedup the collision tests



1

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- Fowler-Nordheim formula introduced by R. H. Fowler, L. Nordheim:  $J(\mathbf{r}, t) = \frac{A(\beta E)^2}{\varphi t(y)^2} \exp\left(\frac{-B\varphi(y)\varphi^{3/2}}{\beta E}\right)$
- Child-Langmuir law: space charge limited current at the surface

$$\begin{aligned} J(\mathbf{r}, t) &= \frac{4\epsilon_0}{9} \sqrt{2\frac{e}{m}} \left( \frac{V^{3/2}}{d^2} \right) \\ &= \frac{4\epsilon_0}{9} \sqrt{2\frac{e}{m}} \left( \frac{E^{3/2}}{d^{1/2}} \right) \end{aligned}$$

# Secondary Emission Model by Furman & Pivi

- Mathematically self-consistent
- Phenomenological- don't involve secondary physics but fit the data
- A number of parameters to fit the measured SEY data
- Built-in SEY data for copper and stainless steel
- Monte Carlo technique has been used
- Detailed description on algorithms can be found in M. A. Furman and M. Pivi's paper.

*M. A. Furman and M. Pivi, Phys. Rev. ST Accel. Beams 5, 124404 (2002)*

# Secondary Emission Model: Vaughan's Formula I

- For material other than copper and stainless steel or material with different SEY curve from the built-in SEY curve in Furman-Pivi's model, Vaughan's model has less parameters than Furman-Pivi's model thus relatively easier to be adjusted to fit the new SEY curve
- Vaughan's formula:

$$\delta(E, \theta) = \delta_{max}(\theta) \cdot (ve^{1-v})^k, \text{ for } v \leq 3.6 \quad (4a)$$

$$\delta(E, \theta) = \delta_{max}(\theta) \cdot 1.125/v^{0.35}, \text{ for } v > 3.6 \quad (4b)$$

where

$$v = \frac{E - E_0}{E_{max}(\theta) - E_0},$$

$$k = 0.56, \text{ for } v < 1,$$

# Secondary Emission Model: Vaughan's Formula II

$$k = 0.25, \text{ for } 1 < \nu \leq 3.6,$$

$$\delta_{max}(\theta) = \delta_{max}(0) \cdot (1 + k_{\theta}\theta^2/2\pi),$$

$$E_{max}(\theta) = E_{max}(0) \cdot (1 + k_E\theta^2/2\pi).$$

- User adjustable parameters:  $E_{max}(0)$ ,  $E_0$ ,  $\delta_{max}(0)$ ,  $\delta_0$ ,  $k_{\theta}$  and  $k_E$

1

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- OPAL is the short for “Object-Oriented Parallel Accelerator Library”.
- Based on the CLASSIC library and the  $IP^2L$  framework.
- CLASSIC: building portable accelerator models and algorithms, MAD input language to specify complicated accelerator systems in general.
- $IP^2L$ : providing an integrated, layered system of parallel objects relayed to large scale 3D particle and field calculations.

# Dark current and multipacting module in OPAL

- Implemented in one of the flavors of OPAL, the object-oriented parallel ESPIC code OPAL-t.
- Geometry, particle position, momentum and particle type (primary bunch, field emitted electrons or secondaries), are stored in the H5hut file format.
- A re-normalization of simulation particle number approach is used to prevent the exponentially growth of particles in the computational domain.
- Post processing tools are provided in the H5hut library and by the use of Paraview or Visit.



1

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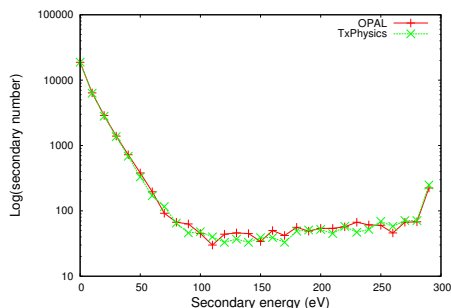
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# Benchmark Against the TxPhysics Library



- Validate the implementation of Furman-Pivi's model
- Logarithm of total secondary emission number (backscattered + re-diffused + true secondaries) vs. energy of emitted particles

1

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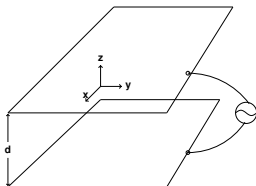
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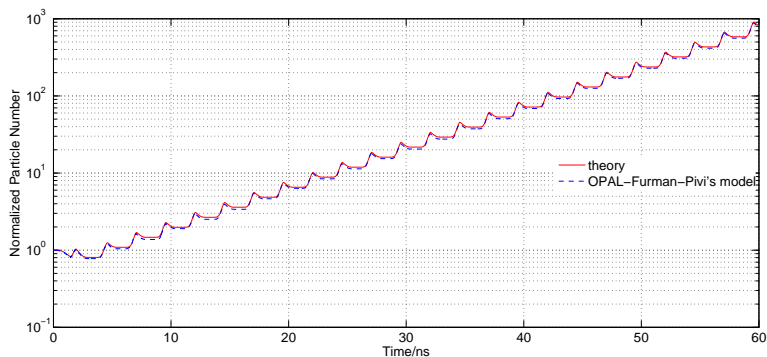
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# Why the Non-stationary Theory?

- Only benchmarking the implementation of secondary emission model is not sufficient, the tracking process and the non-trivial geometry handling algorithms need also to be benchmarked
- Simple geometry

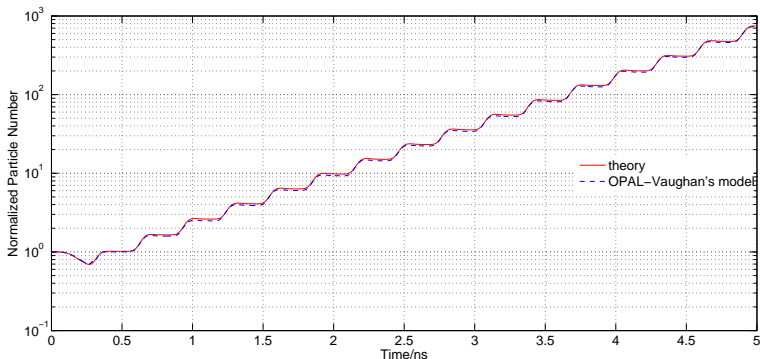


# Real Number of Simulation Particles



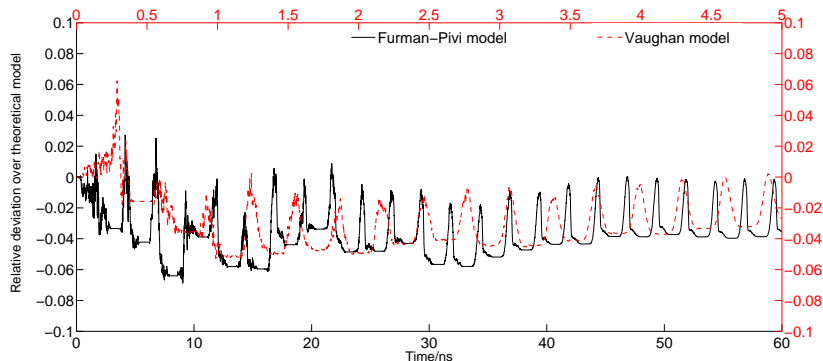
- $f = 200\text{MHz}$ ,  $V_0 = 120\text{V}$ ,  $d = 5\text{mm}$ , Furman and Pivi's model and copper's SEY data

# Real Number of Simulation Particles



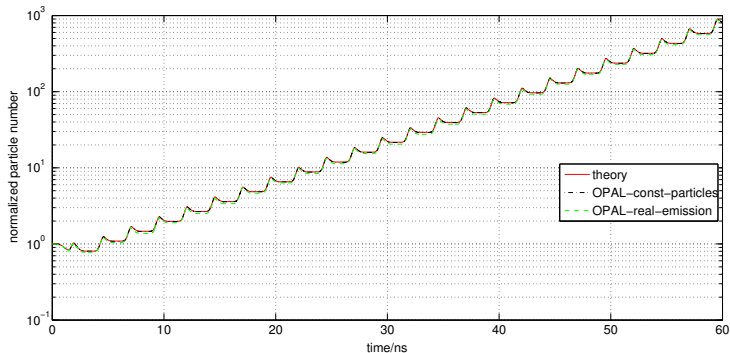
- $f = 1640\text{MHz}$ ,  $V_0 = 120\text{V}$ ,  $d = 1\text{mm}$ , using Vaughan's model and silver's SEY data

# Real Number of Simulation Particles



- Relative deviations from the above simulation results w.r.t the theoretical model predictions

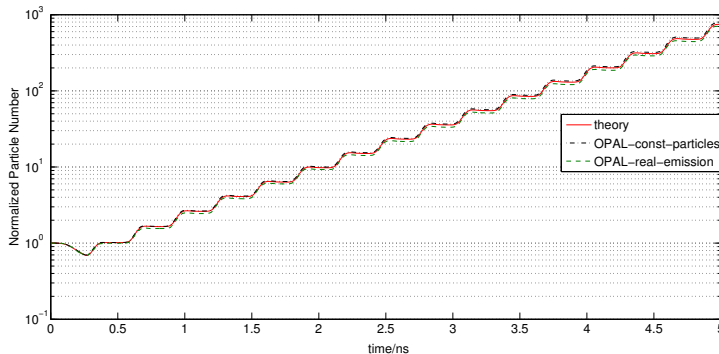
# Renormalized Simulation Particles I



- $f = 200\text{MHz}$ ,  $V_0 = 120\text{V}$ ,  $d = 5\text{mm}$ , Furman-Pivi's model, copper and re-normalize to a constant number of simulation particles

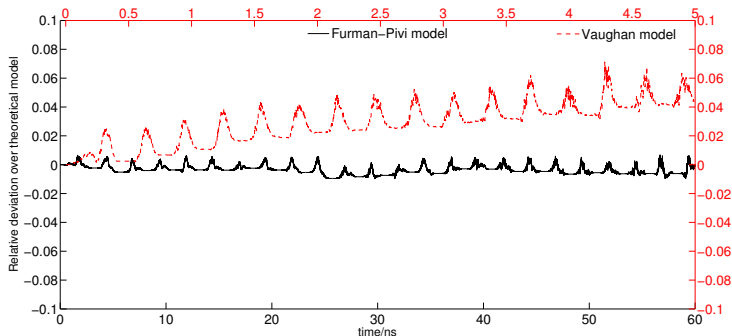


# Renormalized Simulation Particles II



- $f = 1640\text{MHz}$ ,  $V_0 = 120\text{V}$ ,  $d = 1\text{mm}$ , Vaughan's model, silver and re-normalize to a constant number of simulation particles

# Renormalized Simulation Particles III



- Relative deviations of simulation results from the theoretical predicted values (re-normalize to const. simulation particle)

1

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- Why did we do a benchmark experiment after we have already done a perfectly matched code-theory benchmark?

- The ultimate test of a model/code is the comparison with "Nature"!

# Experiment configurations I

- The experiment has been done on a 73 MHz,  $\lambda/4$  transmission line resonator.



Figure: The RF resonator after installation

# Experiment configurations II

- Electron pickup with the vacuum feed-through is mounted through a small hole in the middle of ground plate.

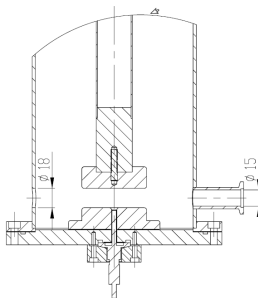
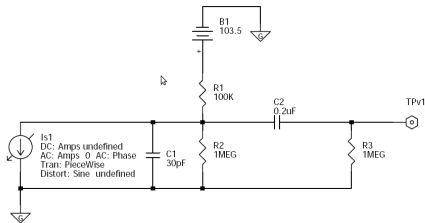


Figure: The configuration of parallel plates and pickup

# Experiment configurations III

- Nano-second time resolved measurement circuit.



**Figure:** Sketch of measurement circuit



**Figure:** Real measurement circuit in a sealed metal box



# Experiment configurations IV

- The current source of the measurement circuit for SPICE simulation is the electron impact rate which can be predicted either by the none-stationary theory or by OPAL simulation and simulated output can be directly compared with the signal in the oscilloscope.

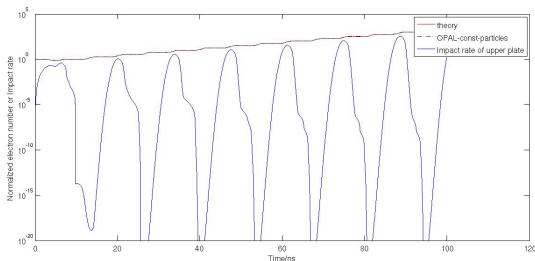


Figure: Simulated impact rate as current source of the measurement circuit

# Results of the experiment I

- Signal directly from the oscilloscope.

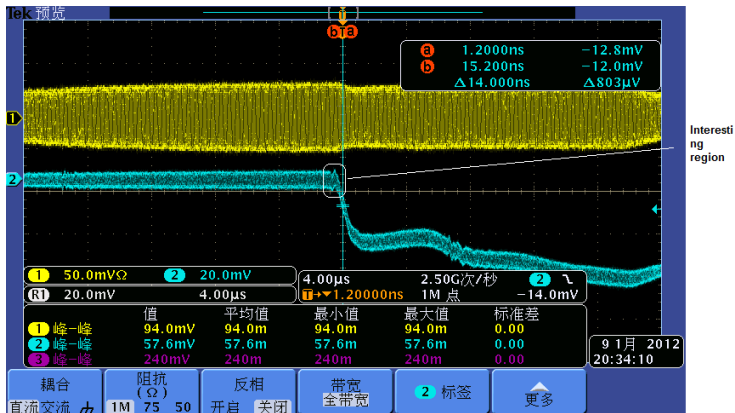
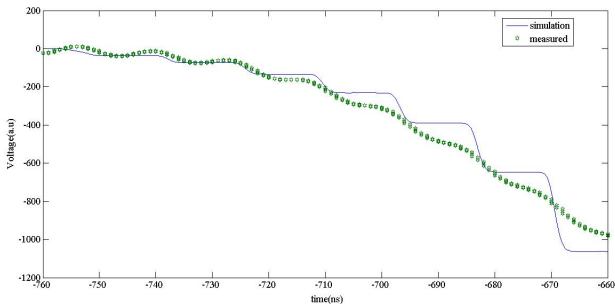


Figure: The measured multipacting signal

# Results of the experiment II

- Comparison between simulation and measurement.



**Figure:** The comparison between measured multipacting signal after digital filtering and the simulated one

1

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# ANIMATION OF DARK CURRENT SIMULATION

- We add a post processing feature which shows the origin positions and phase of dark current particles which are alive beyond user specified positions
- Animation of CTF3 gun

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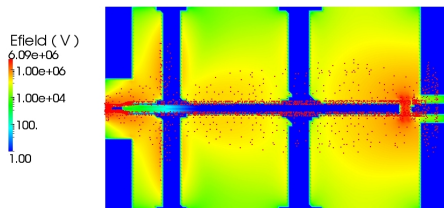
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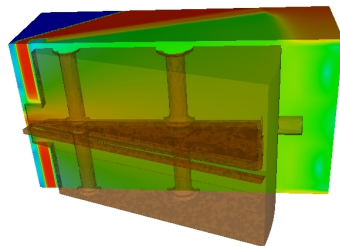
# Multipacting of CYCIAE-100MeV Cyclotron I

- Preliminary results only on full RF power case, further extension needed to evaluate prone multipacting conditions on different power level



**Figure:** The electric field in the cavity and initial distribution of electrons (projection view at the symmetric plane of the cavity along the radius)

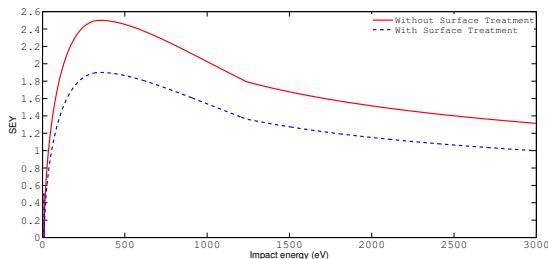
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**Figure:** The RF cavity of CYCIAE-100 cyclotron under the magnetic stray field

# Multipacting of CYCIAE-100MeV Cyclotron II

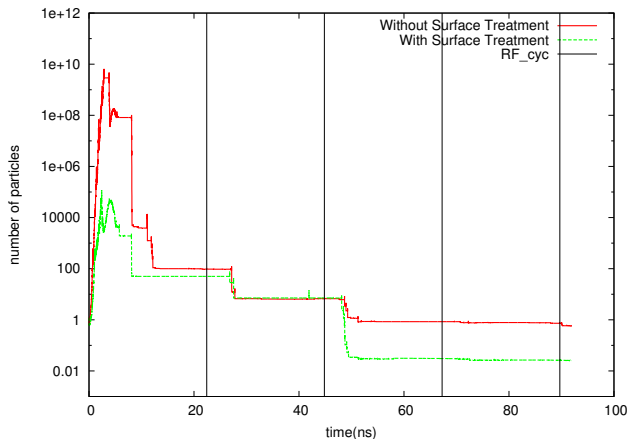
- According to experiments done by LHC project in CERN, the secondary emission curve for copper varies for different surface treatments:



- Multiplication within 1 RF cycle has been observed:



# Multipacting of CYCIAE-100MeV Cyclotron III



- Hot spot: Animation of hot spot where particle hit the surface
- Dumped for each 100 time steps (0.4ns)

- We have successfully modeled, implemented and benchmarked dark current and multipacting modeling capabilities in OPAL
- Thanks to the parallel nature of OPAL, large scale structures can be analyzed
- A full set of pre- and post-processing tools are available in order to enable complex studies [arXiv:1205.3098v2]

- Further multipacting study on different RF power level is useful to predict and understand the behavior of the cavities of CYCIAE-100
- Obtain hot spots in different RF power level by simulations to determine the position where a special surface treatment is needed to suppress multipacting
- Add GUI to pre-define different surface materials on a geometry, in order to make multiple surface material simulation possible.

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# Appendix: Formulas for Non-Stationary Theory I

- The solution of (3) when electron hit the plates, i.e., when  $\xi(\varphi, \varphi_0, u) = \lambda$  or 0, is a probabilistic number, as the emission velocity  $u$  is a random number
- The probability density of the least root(on variable  $\tau$ ) of equation (3) can be expressed by the known distribution

$$f_u = \frac{uv_{\omega}^2}{v_t^2} \exp\left(-\frac{u^2 v_{\omega}^2}{2v_t^2}\right) \text{ of velocity } u:$$

$$G(\tau|\varphi_0; \lambda) = \left| \frac{dg(\tau|\varphi_0; \lambda)}{d\tau} \right| f_u[g(\tau|\varphi_0; \lambda)]$$

$$G(\tau|\varphi_0; 0) = \left| \frac{dg(\tau|\varphi_0; 0)}{d\tau} \right| f_u[g(\tau|\varphi_0; 0)]$$

where,  $u = g(\tau|\varphi_0; \lambda)$  and  $u = g(\tau|\varphi_0; 0)$  respectively (monotonic function)

# Appendix: Formulas for Non-Stationary Theory II

- Emission rate (electrons/radian) in plate  $U/D$  at phase  $\varphi$ :

$$C_U(\varphi) = \int_0^\varphi C_D(\varphi') G_{ds,D}(\varphi - \varphi'|\varphi') \delta_{ds,D}(\varphi - \varphi'|\varphi') d\varphi' \\ + \int_0^\varphi C_U(\varphi') G_{ss,U}(\varphi - \varphi'|\varphi') \delta_{ss,U}(\varphi - \varphi'|\varphi') d\varphi' + \Psi_U(\varphi)$$

$$C_D(\varphi) = \int_0^\varphi C_D(\varphi') G_{ss,D}(\varphi - \varphi'|\varphi') \delta_{ss,D}(\varphi - \varphi'|\varphi') d\varphi' \\ + \int_0^\varphi C_U(\varphi') G_{ds,U}(\varphi - \varphi'|\varphi') \delta_{ds,U}(\varphi - \varphi'|\varphi') d\varphi' + \Psi_D(\varphi)$$

- Volterra integral equations of the second

# Appendix: Formulas for Non-Stationary Theory III

- Impact rate (electrons/radian) in plate  $U/D$  at phase  $\varphi$ :

$$I_U(\varphi) = \int_0^\varphi C_D(\varphi') G_{ds,D}(\varphi - \varphi'|\varphi') d\varphi' \\ + \int_0^\varphi C_U(\varphi') G_{ss,U}(\varphi - \varphi'|\varphi') d\varphi'$$

$$I_D(\varphi) = \int_0^\varphi C_D(\varphi') G_{ss,D}(\varphi - \varphi'|\varphi') d\varphi' \\ + \int_0^\varphi C_U(\varphi') G_{ds,U}(\varphi - \varphi'|\varphi') d\varphi'$$

- Number of particles:

$$N(\varphi) = \int_0^\varphi (C_U(\varphi') + C_D(\varphi') - I_U(\varphi') - I_D(\varphi')) d\varphi'$$