BEAM DYNAMICS STUDY CONCERNING SIS-100 PROTON OPERATION INCLUDING SPACE CHARGE EFFECTS

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SIS-100 Proton Cycle

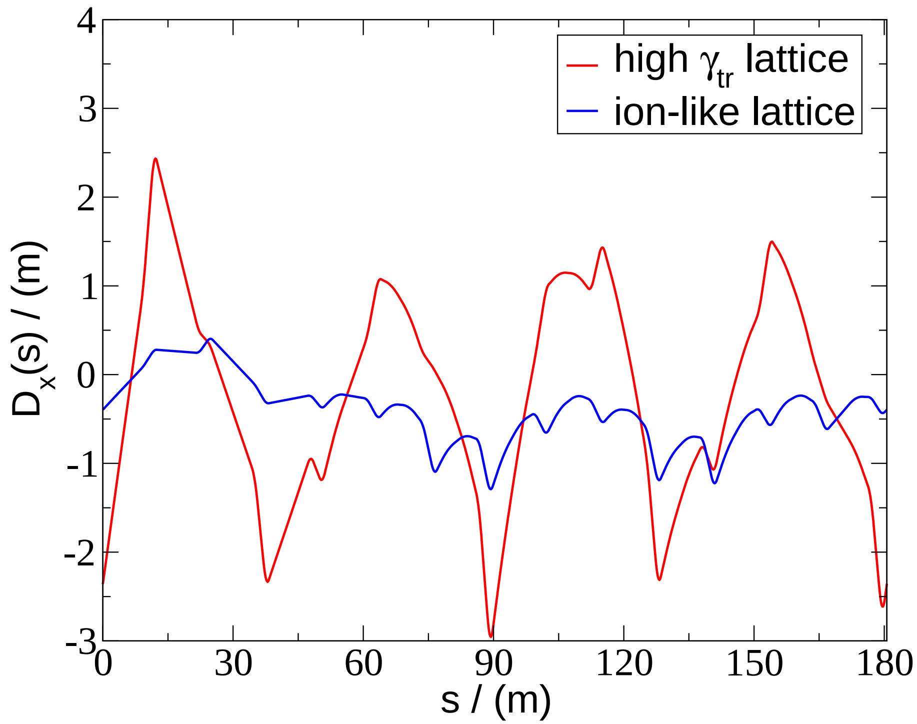
- Basically, SIS-100 is heavy ion synchrotron within FAIR project.
- In addition, there will be proton operation.
  - Four bunches injected and merged to single bunch.
  - Number of protons per cycle: $N_p = 2 \times 10^{13}$.
  - Injection energy: $E_{inj} = 4$ GeV.
  - 2σ emittances at injection: $(\epsilon_x, \epsilon_y) = (13, 4)$ mm mrad
  - Maximum energy at extraction: $E_{ext} = 29$ GeV $\rightarrow \gamma = 31.9$.
  - On the other hand, operation is planned to occur below transition energy. Therefore, transition energy corresponding to $\gamma_{tr} = 45.5$.

Dispersion Function

Dispersion function functions of the unperturbed lattice in one sector, high γ_{tr} lattice vs "ion-like" lattice, WP: $(21.8,\ 17.7)$.

A strongly oscillating dispersion function $D_x(s)$ is required to reach $\gamma_{tr} = 45.5$, i.e. to reach small momentum compaction factor

$$\alpha_c = \frac{1}{\gamma_{tr}^2} = \frac{1}{C}\oint \frac{D_x(s)\mathrm{d}s}{\rho(s)}.$$



General Lattice Properties

Unperturbed lattice with working point $(21.8, 17.7)$

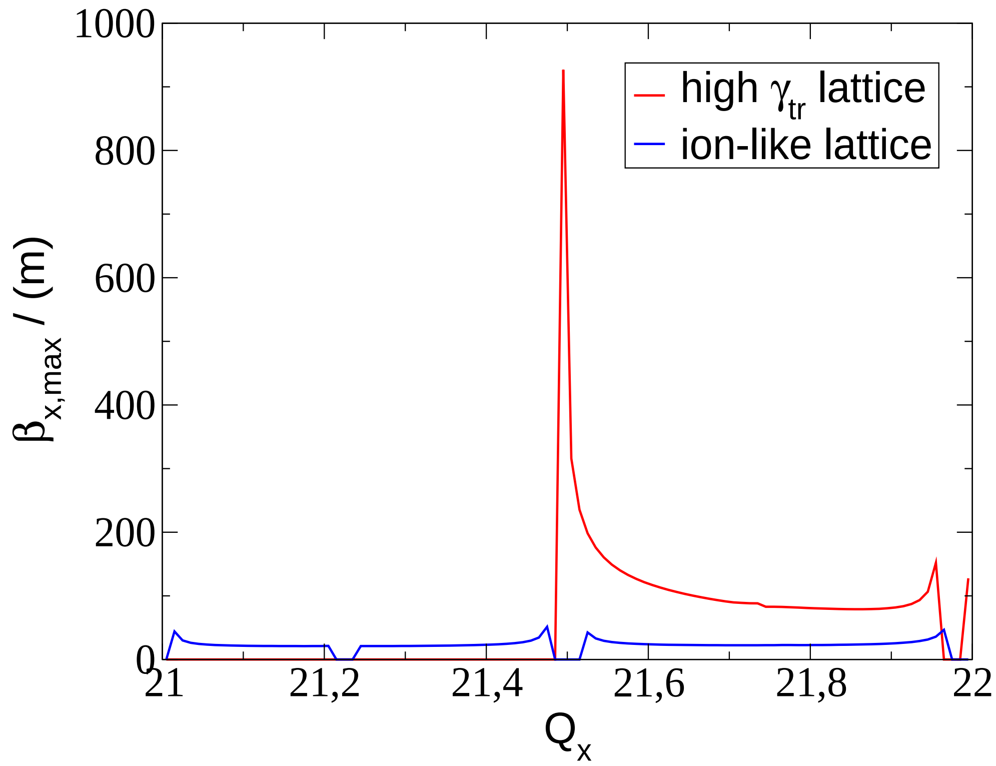
	high γ_{tr} lattice	"ion-like" lattice
Number of quadrupole families	3	2
γ_{tr}	45.5	18.4
max dispersion function $D_{x,max}/(\mathrm{m})$	2.9	1.3
max beta functions, $(\beta_{x,max}, \beta_{y,max})/(\mathrm{m})$	$(72,\ 29)$	$(19,\ 21)$
max 2σ beam width for $\delta = 0$ and		
$(\epsilon_{x,2\sigma} \times \epsilon_{y,2\sigma}) = (13 \times 4)$ mm mrad	(31×11) mm	(16×9) mm
natural chromaticities, $(\xi_{nat,x}, \xi_{nat,y})$	$(-2.4, -1.4)$	$(-0.9, -1.1)$

Chromaticities defined by $\Delta Q = \xi \delta Q$

Maximum Horizontal Beta Function

1. **High-γ_{tr} lattice:** in general, no lattice functions found for $Q_x < 21.5$.
2. Lattice perturbed by magnet errors.
   - Linear lattice functions perturbed by **random gradient errors** in the main quadrupoles.
   - Assume Gaussian distribution truncated at 2σ with $\sigma_{rel} \approx 3.0 \cdot 10^{-3}$
   - At $Q_x = 21.8$:
     - ideal lattice: $\beta_{x,max} = 72$ m
     - pert lattice: $\beta_{x,max} = 99$ m

Maximum horizontal beta function for $Q_y = 17.7$.

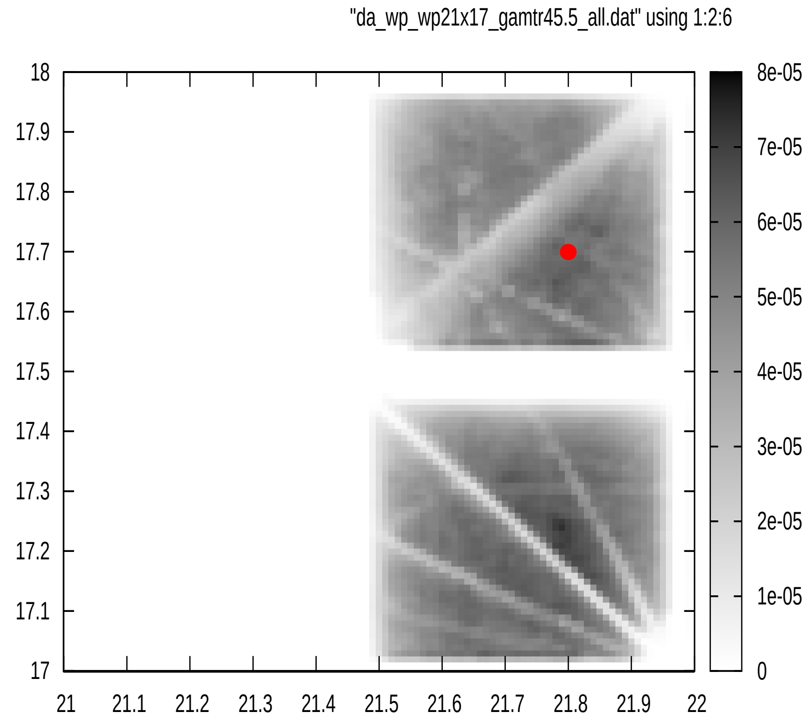


Resonance Diagram

- In addition, non-linear random multipole errors in magnets drive resonances which reduce Dynamic Aperture (DA).
- DA scan using MAD-X to see them.
- Look for WP which fits well in resonance diagram, confirms WP $(21.8, 17.7)$.
- For $Q_x < 21.5$ no DA calculated because lattice functions not determined.

$Q_x \in [21, 22], Q_y \in [17, 18]$

10000 WP's



Resonance Diagram and Tune Spread

In addition:

- High γ_{tr} lattice with

  large natural chromaticities

  $\xi_{nat,x} = -2.4, \; \xi_{nat,y} = -1.4,$

  and momentum spread up to

  $\delta = \pm 0.005$ (at $E = 7$ GeV).

- Resulting tune spread $\Delta Q = \xi Q \delta$:

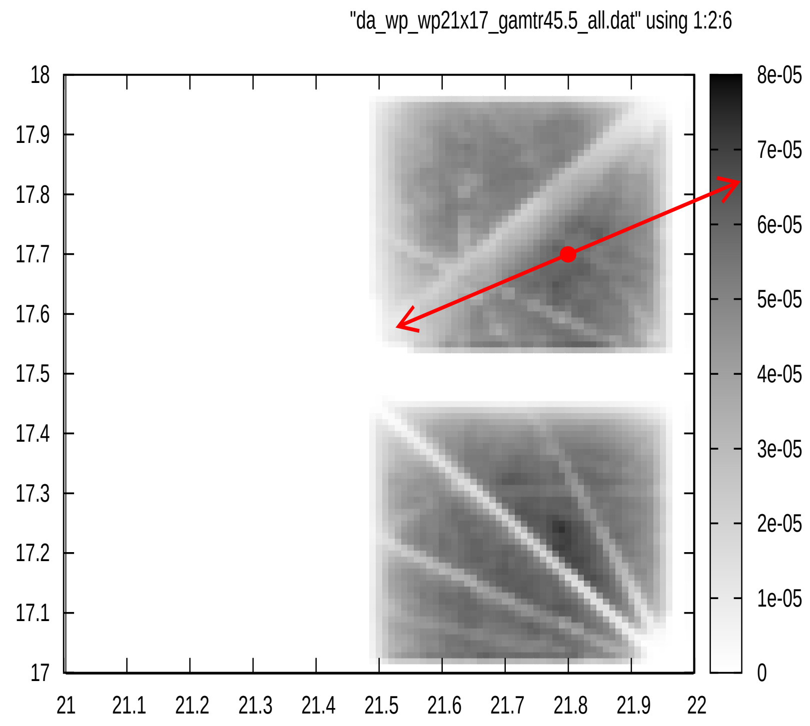
  $\Delta Q_x = \pm 0.27, \; \Delta Q_y = \pm 0.12$

- Loss of particles with large δ within

  one synchrotron period.

$Q_x \in [21, 22], Q_y \in [17, 18]$

10000 WP's



Cures:

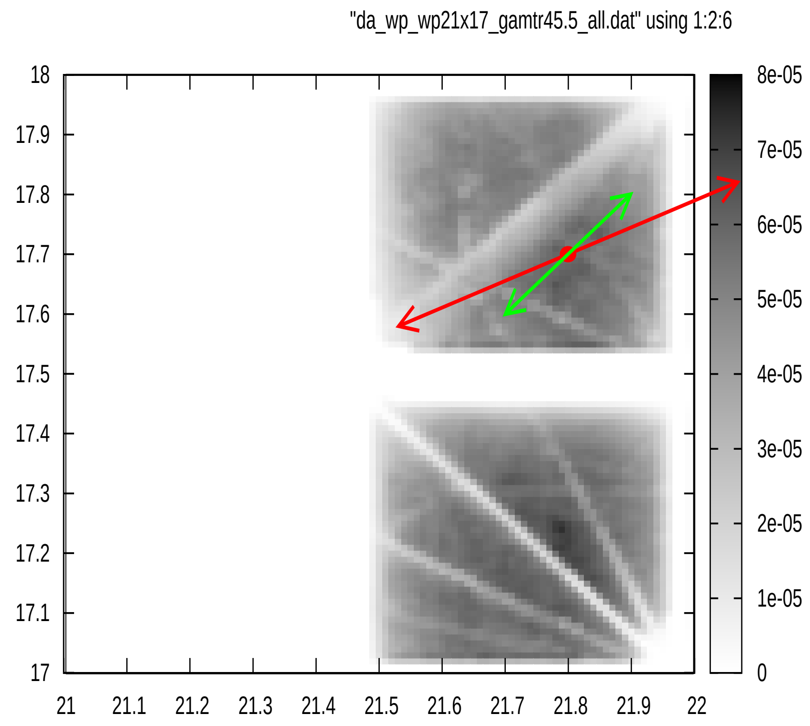
1. Correct chromaticity to reduce tune spread to $\Delta Q_{x,y} = \pm 0.1$:
    - Correction with sextupoles, reduce DA.
    - Use 52 sextupoles magnets in SIS-100 to correct two variables, ξ_x, ξ_y.
    - Provides freedom to apply additional condition

$$\sum_{n=1}^{52} (k_2 L)_n^2 \rightarrow \text{minimum}$$

to reduce influence of sextupoles.

$Q_x \in [21, 22], Q_y \in [17, 18]$

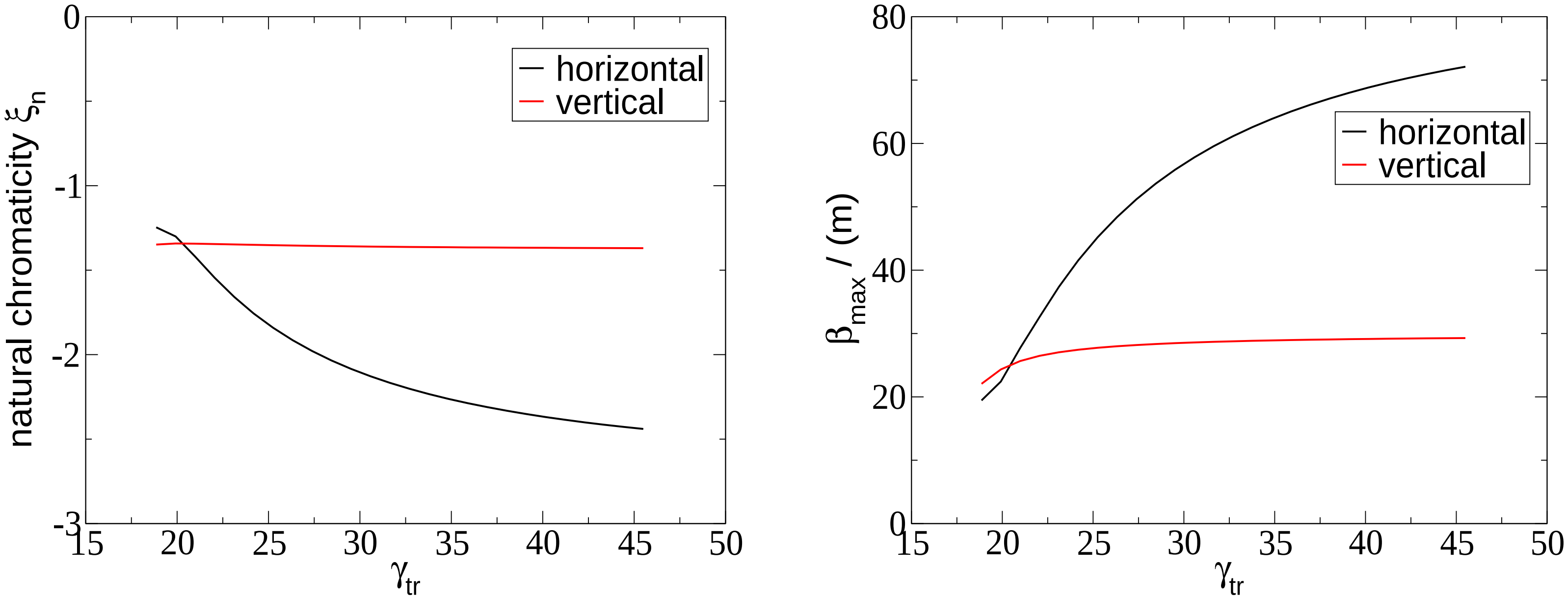
10000 WP's



Tune Spread and Changed Optics

2. Change optics during ramp to use the high γ_{tr} lattice only at high energy.

- At low energies, usage of ion-like lattice with $\gamma_{tr} = 18.4$.



- Smaller chromaticity (Fig left) yields reduced tune spread.

- Smaller maximum beta function (Fig right) provides larger DA.

Diagonal Dynamic Aperture

"Diagonal" DA: $\epsilon_{x,lim}(\phi) = \epsilon_{lim}(\phi)\cos^2\phi,\ \epsilon_{y,lim}(\phi) = \epsilon_{lim}(\phi)\sin^2\phi$.

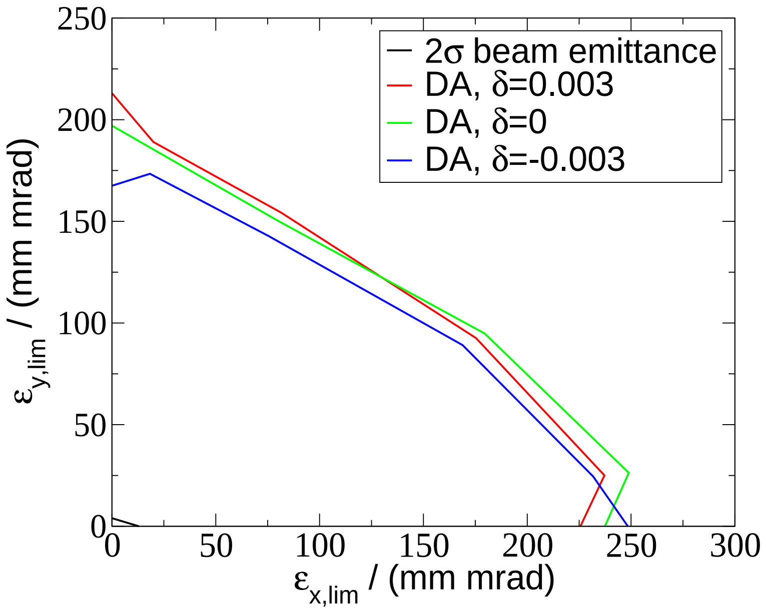
Procedure:

- Choose $\phi = (0,\ 0.1\pi,\ 0.2\pi,\ 0.3\pi,\ 0.4\pi,\ 0.5\pi)$
- Track a single particle with initial coordinates according to $\epsilon_{lim}(\phi)$ relative to the closed orbit deviation created by a momentum deviation.
  Apply $\delta = (-\delta_{max},\ 0,\ \delta_{max})$.
- Short term: 500 turns
- Vary $\epsilon_{lim}(\phi)$ for each ϕ, δ until the maximum for stable particle motion is found. Apply nested interval procedure.
- Use MAD-X code.

Diagonal Dynamic Aperture

$$E = 4\ \text{GeV},\ \delta = 0,\ \pm 0.003$$

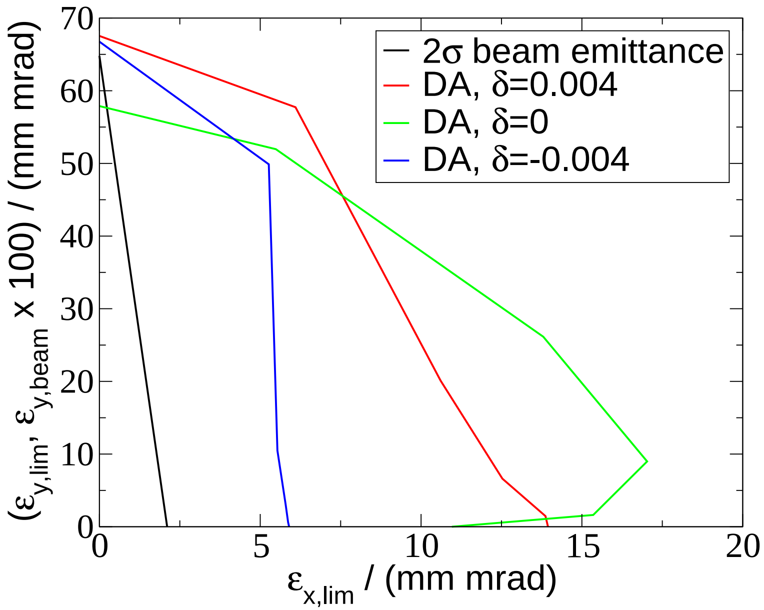
$$\epsilon_{beam} = (13 \times 4)\ \text{mm mrad}$$



$$\epsilon_y = 0:\ \epsilon_{x,lim} \approx 17 \times \epsilon_{x,beam}$$

$$E = 29\ \text{GeV},\ \delta = 0,\ \pm 0.004$$

$$\epsilon_{beam} = (2.1 \times 0.65)\ \text{mm mrad}$$



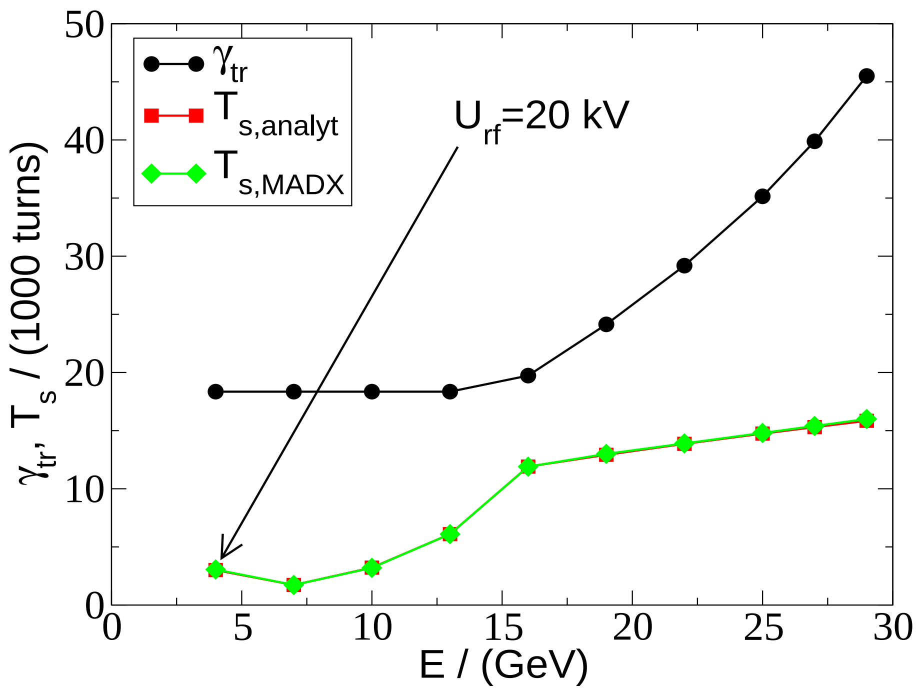
$$\epsilon_y = 0:\ \epsilon_{x,lim} \approx 3 \times \epsilon_{x,beam}$$

Maximum energy: Short term horizontal DA $\approx 2\times$horizontal spatial beam width.

Multi Particle Simulation

Simulation using MAD-X:

- Constant energy.
- Rf cavity introduces synchrotron motion $\longrightarrow$ oscillation of tune due to δ spread.
- 100 particles during 16000 turns to cover at least one synchrotron period.
- γ_{tr} at high energies: keep $\eta = \text{const}$.
- Compare synchrotron periods T_s, simulation vs analytic formula, where $U_{rf} = 300\ \text{kV},\ h = 5,\ \phi_s = 0$.

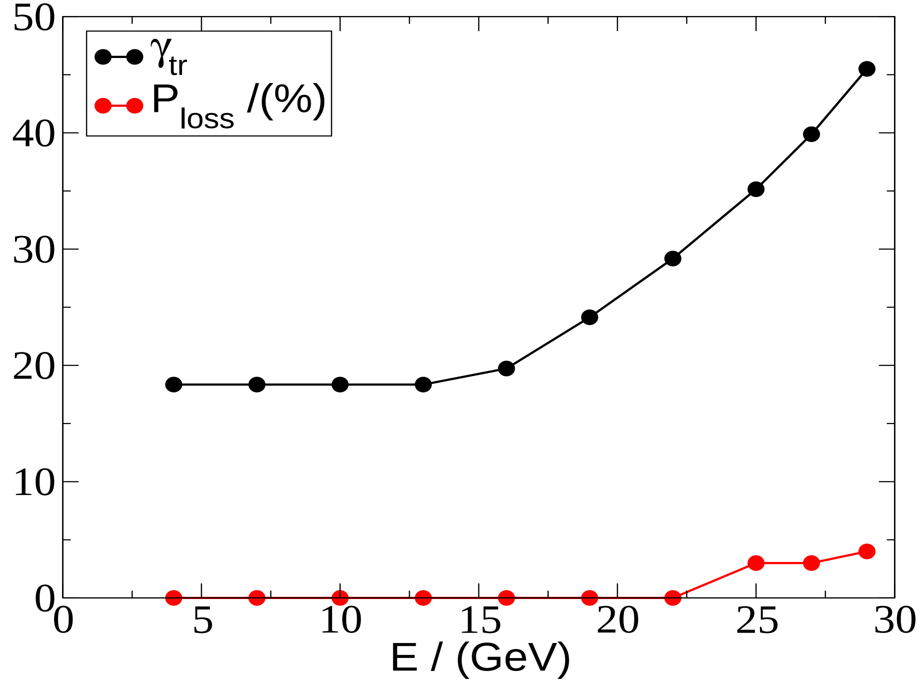


$$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$$

$$T_s = \sqrt{\frac{2\pi\beta^2 E}{heU_{rf}|\eta\cos\phi_s|}}$$

Multi Particle Simulation, Beam Loss

- Thin lens tracking to keep chance to include later space charge.
- Particle loss found without space charge:
  - P_{loss} up to $4\ \%$
  - only for high γ_{tr}



Inclusion of Space Charge

- Regard for incoherent space charge fields.
- Frozen space charge, no oscillation of betatron tune by synchrotron motion.
- Space charge introduced by thin `beambeam` elements with truncated Gaussian profile characterised by rms width $z_{rms}(s) = \sqrt{\beta_z(s)\epsilon_{z,rms}}, \ z = x, y.$
- Algorithm[1] consists of four steps:

1. Convert lattice to thin lens lattice (MAD-X).
2. Insert marker elements equidistantly around the ring (extern).
3. Determine beta functions at marker positions $\longrightarrow z_{rms}(s)$ (MAD-X).
4. Replace markers with `beambeam` elements (extern).

[1] Method received from V Kapin

Space Charge Parameters during Ramp

Laslett tune shift: $\Delta Q_z = -\dfrac{N_p r_c}{2\pi\beta^2\gamma^3 B_f \sqrt{\epsilon_z}\left(\sqrt{\epsilon_x}+\sqrt{\epsilon_y}\right)}, \quad z = x, y$

for a Gaussian beam with 2×10^{13} protons

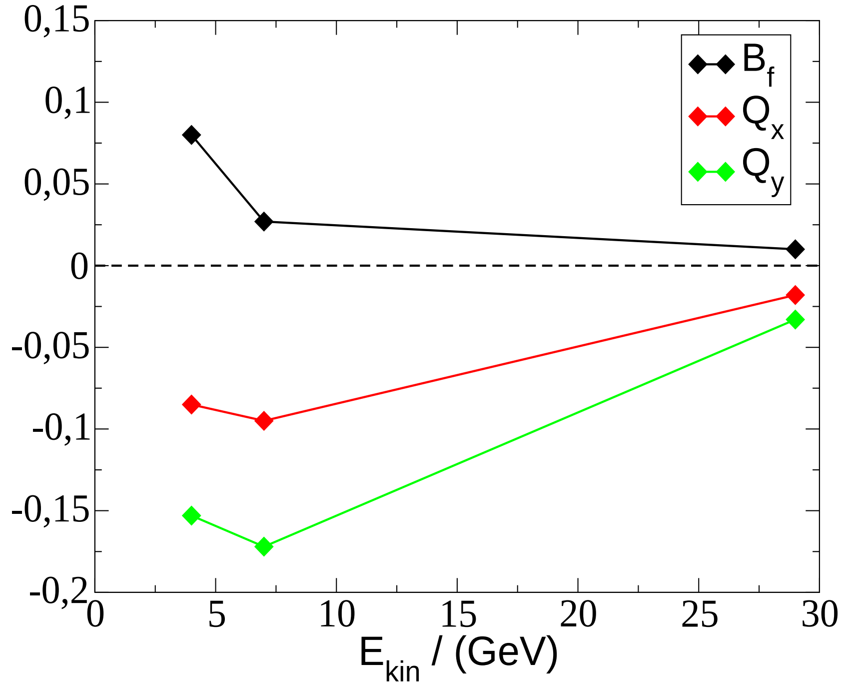
Rf cycle (O Chorniy): Maximum rf voltage reached at $E = 7$ GeV. In addition, assume bunching factor B_f according to rf voltage.

- B_f changes $0.08 \rightarrow 0.027$.
- Reach largest space charge tune shift here.

$\Downarrow$

Expect strongest influence of space charge at

$E = 7$ GeV.

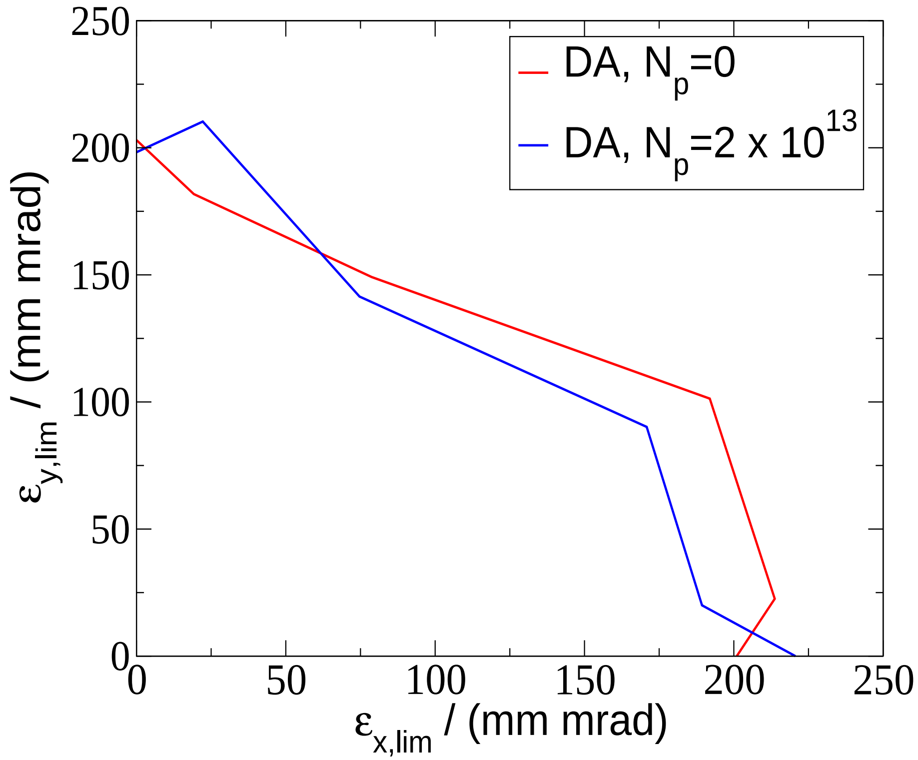


Data from O Chorniy, priv comm

Dynamic Aperture with Space Charge

- **In fact,** attempt to determine lattice function at $E = 7$ GeV including space charge as well as systematic and random magnet errors failed for some random magnet error samples.

- **On the other hand,** if lattice functions were found, very similar dynamic apertures with and without space were found.

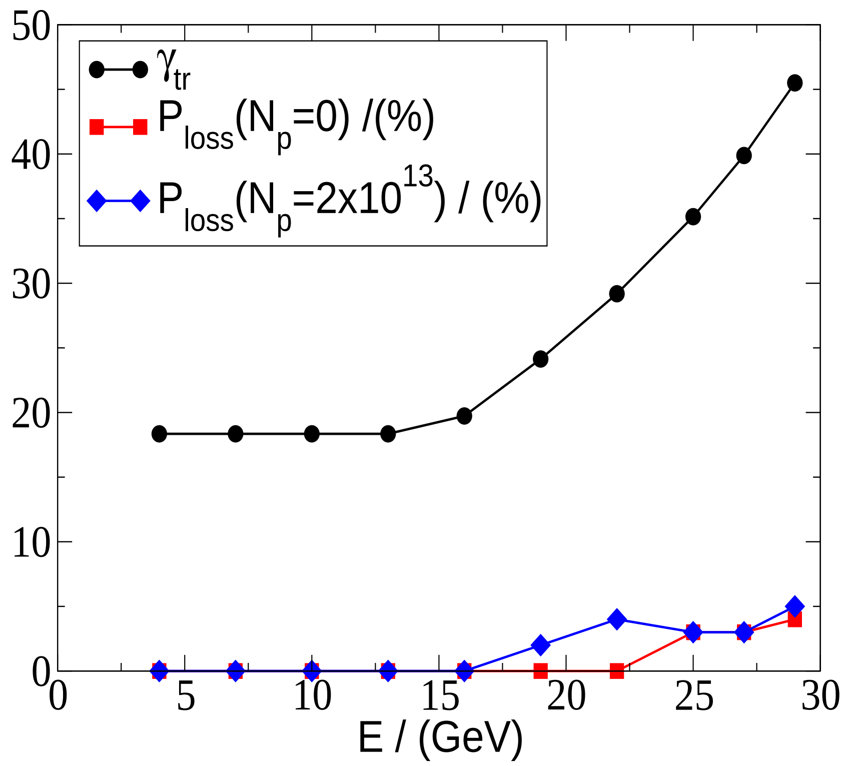


Result: no particle loss at $E = 7$ GeV, instead ...

Particle Loss with Space Charge

... instead, particle loss found at high energies

- Close to maximum energy, major beam loss due to lattice, is only slightly modified by space charge.
- Around $E = 20$ GeV significant beam loss only if space charge is present.
  $\longrightarrow$ Space charge stronger at lower energy.
- Minimum energy for beam loss:
  Beam loss due to interconnection of space charge and high γ_{tr} lattice.



Summary

- Numerical study on the proton cycle of SIS-100 to estimate beam loss.
- Simulation with independent particles, thin lens tracking tool of MAD-X.
  - **Crucial point:** High γ_{tr} optics required $\longrightarrow$ complicated lattice functions.
  - Restrict usage of high γ_{tr} optics to high energies.
  - Simulations without and with space charge $\longrightarrow$ frozen space charge.
  - Influence of synchrotron oscillations regarded with respect to chromatic tune shift, neglected with respect to space charge tune shift.
    $\longrightarrow$ Important mechanism for space charge induced beam loss neglected.
- **Results:**
  - No beam loss at low energies with ion-like optics.
  - Close to maximum energy, beam loss dominated by high γ_{tr} optics.
  - At medium energies, beam loss due to interconnection of space charge and lattice properties.

Open Points

Quantitative results are preliminary because of small particle numbers and usage of only one sample of random magnet in simulations, and because of the strongly simplified space charge treatment. Therefore:

- Next step: repeat simulations with larger particle numbers and different random error samples to consolidate results.
- It would be desirable to use a tracking code which can include the influence of synchrotron motion on the space charge because it is an important ingredient to the description of space charge induced beam loss.
  $\longrightarrow$ PTC-ORBIT installed, (special thanks to F. Schmidt).