Data / Signal Analysis using Blind Source Separation (BSS) for Beam Analysis

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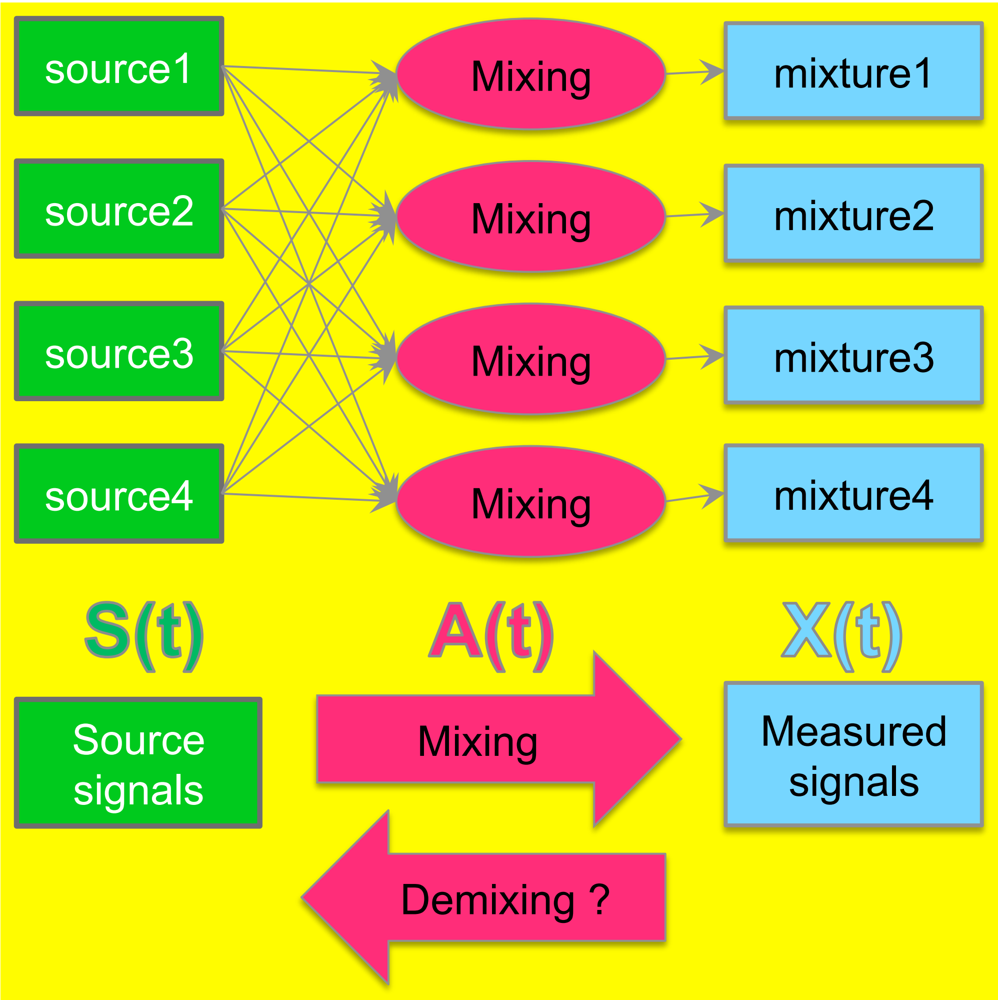
Outline

- **What is and why blind source separation (BSS)**
  - Principal Component Analysis (PCA)
  - Independent Component Analysis (ICA)
- **Applications:**
  - Betatron motion
  - Nonlinear motion and sextupole strength
  - Slices along the bunch

Motivation

- **We record an overwhelming amount of data, most of which goes unused. Can we analyze the data to uncover underlying structure, which we can then relate to some physical process?**
- **Blind source separation (BSS)**
  - Is a class of statistical methods from the field of data / signal analysis.
  - **Blind** – Only uses initial assumptions about the data and statistical algorithms. BSS analysis does not use a model and has no knowledge of the measurement.
  - **Source** – Data's underlying structure. Component. Must be identified.
  - **Separation** – The ability to distinguish sources.
- **Applications of BSS**
  - Image feature extraction, audio separation, brain imaging, telecommunications, econometrics …

Cocktail Party Problem



Blind Source Separation for Beams Analysis

- **Use BSS for time series data.**
- **Arrange the recorded data from *M* monitors and *N* measurements**

Temporal Variation →

$$x(t) = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(N) \\ x_2(1) & x_2(2) & \cdots & x_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(1) & x_M(2) & \cdots & x_M(N) \end{pmatrix}$$

↓ Spatial Variation

- **Data Matrix:**
  - Rows: Contain all measurements from one monitor. (Temporal variation)
  - Columns: Contain data from all monitors for one measurement. (Spatial variation)

Principal Component Analysis (PCA)

- **Simplest BSS method**
  - Many other BSS methods use PCA for preprocessing and noise reduction.
  - Originally used in the well known Model Independent Analysis (MIA) [Irwin].
- **Expresses high-dimensional data matrix by highlighting the underlying structures represented as uncorrelated principle components (PCs)**
  - Minimizes the redundancy measured by covariance;
  - Maximizes the signal measured by variance.
- **The core of PCA is Singular Value Decomposition (SVD).**

$$x(t) = U\Lambda V^T$$

where Λ ($M \times N$) is diagonal matrix of singular values (SVs).

- **The columns of U ($M \times M$) span column-space**
  - M-dimensional space of monitor number. Spatial patterns.
- **The columns of V ($N \times N$) span row-space**
  - N-dimensional space of measurement number. Temporal patterns.

Independent Component Analysis

- **Separates independent sources s (*L* × *N*) given x(t) (*M* × *N*), but the mixing matrix A (*M* × *L*) is unknown**

$$x(t) = As$$

- **For time series data, source independence is related to diagonality of covariance matrices.**
  - The auto-covariance of a signal is where τ is called a time lag $\operatorname{cov}\left(\vec{y}_i(t), \vec{y}_i(t-\tau)\right), \quad \tau = 0,\ 1,\ 2,\ \ldots,\ K$
  - The covariance between two signals is $\operatorname{cov}\left(\vec{y}_i(t), \vec{y}_j(t-\tau)\right), \quad i \neq \mathrm{j}$

- **Combining these results for mean zero source signals**

$$C_s(\tau) = \left\langle s_i(t) s_j(t-\tau)^T \right\rangle = 0, \quad i \neq j, \quad \tau = 0,\ 1,\ 2,\ \ldots,\ K$$

- **So A^{-1}x(t) must also possess diagonal time-lagged covariance matrices.**
- **The BSS problem is solved by obtaining a demixing matrix (A^{-1}) that diagonalizes the time-lagged covariance matrices of x(t).**

Independent Component Analysis (cont.)

- **$C_x(\tau = 0)$ does not contain enough information to obtain A.**
  - Utilize the additional information contained $C_x(\tau)$.
- **Including multiple time lags improves ICA's performance by resolving degenerate SVs.**
  - Introduces additional complication of simultaneously diagonalizing many $C_x(\tau)$.
- **A numerical technique for simultaneously diagonalizing several matrices with Jacobi angles is discussed in Ref. [Cardoso].**
- **We use the ICA algorithm Second Order Blind Identification (SOBI) [Belouchrani], which accommodates multiple time lags in our analysis.**
- **The SOBI algorithm**
  - Preprocessing
  - Joint diagonalization

SOBI: Preprocessing

- **Mean-zero data simplifies the covariance matrix calculation**
  - Calculated by subtracting the average over the temporal variation of **x(t)**.
- **Use PCA to reduce redundancy and remove some noise.**

$$C_{\bar{x}}(\tau = 0) = \left\langle \bar{x}(t)\bar{x}(t)^T \right\rangle = (U_1, U_2)\begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

Λ_1 and Λ_2 are diagonal matrices of SVs separated by a cutoff threshold λ_c.

  - λ_c is calculated from the number of SVs *L* in the analysis.
  - U_1 and U_2 are eigenvectors corresponding to Λ_1 and Λ_2 respectively.
- **Mean-zero whitened data**

$$z = Y\bar{x}, \quad \text{where} \quad Y = \Lambda_1^{-1/2} U_1^T \quad \text{and} \quad \left\langle zz^T \right\rangle = \mathrm{I}$$

  - $\Lambda^{-1/2}$ indicates the inverse square root of the diagonal elements individually.

SOBI: Joint diagonalization

- **For a set of time-lagged covariance matrices of z**

$$C_z(\tau_k) = \left\langle z(t) z(t-\tau)^T \right\rangle, \quad \tau = 0,\ 1,\ 2,\ \ldots\ ,\ K$$

Construct modified time-lagged covariance matrices

$$\overline{C}_z(\tau_k) = \left(C_z(\tau_k) + C_z(\tau_k)^T \right) \Big/ 2$$

- **SVD is well defined, since $C_z(\tau_k)$ is real and symmetric**

$$\overline{C}_z(\tau_k) = W D_k W$$

where **W** is the unitary demixing matrix and $\mathbf{D}_k$ is a diagonal matrix

- **The Jacobi angle technique is used to find W, which is a joint diagonalizer for all modified time-lagged covariance matrices.**

SOBI: Joint diagonalization (cont.)

- **The source signals s and the mixing matrix A are calculated**

$$s = W^T Y \bar{x} \quad \text{and} \quad A = Y^{-1} W$$

- **The columns of A span column-space**
  - *M*-dimensional space of monitor number. Spatial modes.
- **The rows of s span row-space**
  - *N*-dimensional space of measurement number. Temporal modes.
- **The mean-zero signal represented by the i^{th} independent component (IC) is constructed by multiplying the i^{th} column of A and the i^{th} row of s**

$$z_{IC_i} = \vec{A}_i \cdot \vec{s}_i$$

BSS Application to BPM Data

- **Applying BSS to beam analysis**
  - Monitor -> Beam position monitor (BPM) (Spatial variation)
  - Measurement -> Turn (Temporal variation)

Turn Number →

$$x(t) = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(N) \\ x_2(1) & x_2(2) & \cdots & x_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(1) & x_M(2) & \cdots & x_M(N) \end{pmatrix}$$

↓ BPM Number

- **Consider betatron motion for *M* BPMs and *N* turns**
  - The (m,n) element of the turn-by-turn BPM data matrix **x**(t) is

$$x_{m,n} = \sqrt{2\beta_m J}\,\sin\left(2\pi\nu(n-1) + \nu\phi_m\right)$$

where m = 1, 2, …, M and n = 1, 2, …, N.

PCA Result for Betatron Motion

- **SVD:** $x(t) = U\Lambda V^T$

Spatial patterns

$$U = \begin{pmatrix} P\sqrt{\frac{2\beta_1}{M}}\sin(\nu\phi_1) & P\sqrt{\frac{2\beta_1}{M}}\cos(\nu\phi_1) & 0 & \cdots \\ P\sqrt{\frac{2\beta_2}{M}}\sin(\nu\phi_2) & P\sqrt{\frac{2\beta_2}{M}}\cos(\nu\phi_2) & 0 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} \frac{\sqrt{2JMN}}{2P} & 0 & 0 & \cdots \\ 0 & \frac{\sqrt{2JMN}}{2P} & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

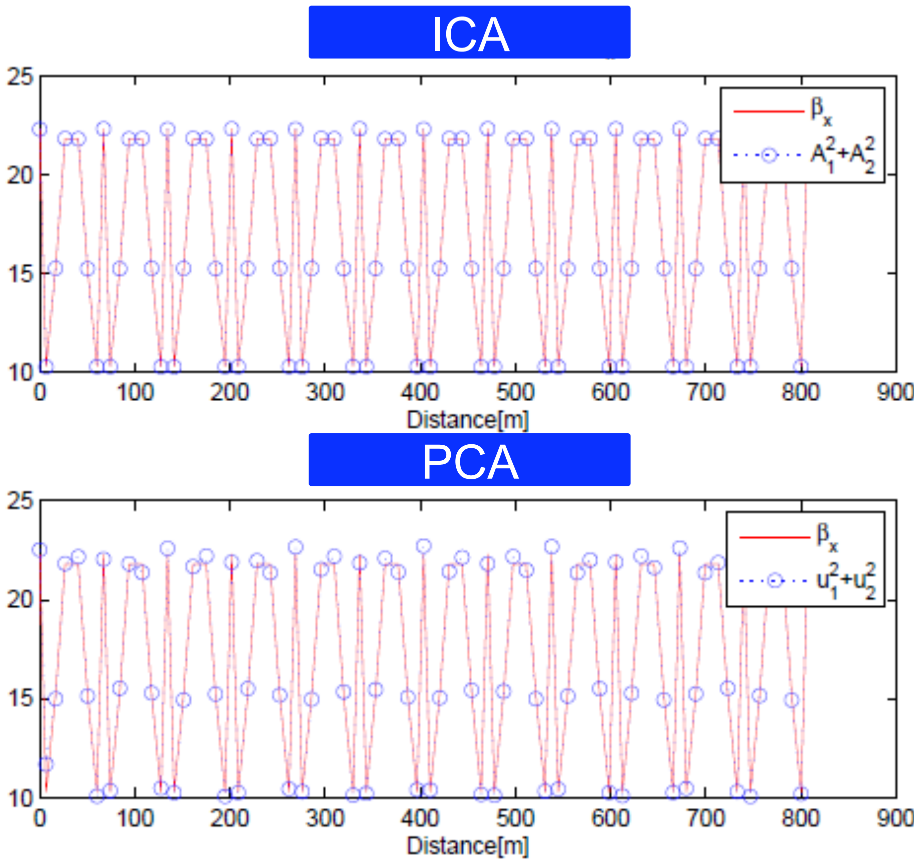
Temporal patterns

$$V^T = \begin{pmatrix} \sqrt{\frac{2}{N}}\cos(2\pi\nu\cdot 0) & \sqrt{\frac{2}{N}}\cos(2\pi\nu\cdot 1) & \cdots & \sqrt{\frac{2}{N}}\cos(2\pi\nu\cdot(N-1)) \\ \sqrt{\frac{2}{N}}\sin(2\pi\nu\cdot 0) & \sqrt{\frac{2}{N}}\sin(2\pi\nu\cdot 1) & \cdots & \sqrt{\frac{2}{N}}\sin(2\pi\nu\cdot(N-1)) \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \end{pmatrix}$$

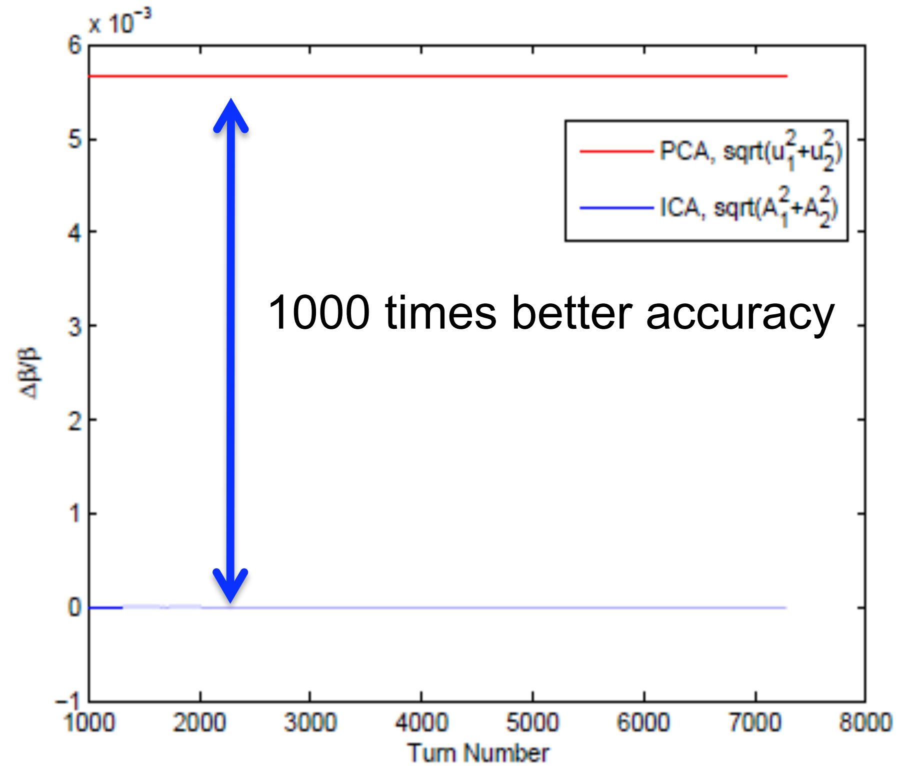
$$U_1^2 + U_2^2 = \frac{2\beta(s)P^2}{M} \propto \beta(s)$$

$$\phi(s) = \tan^{-1}\left(\frac{U_1}{U_2}\right)$$

BSS Application to Betatron Motion



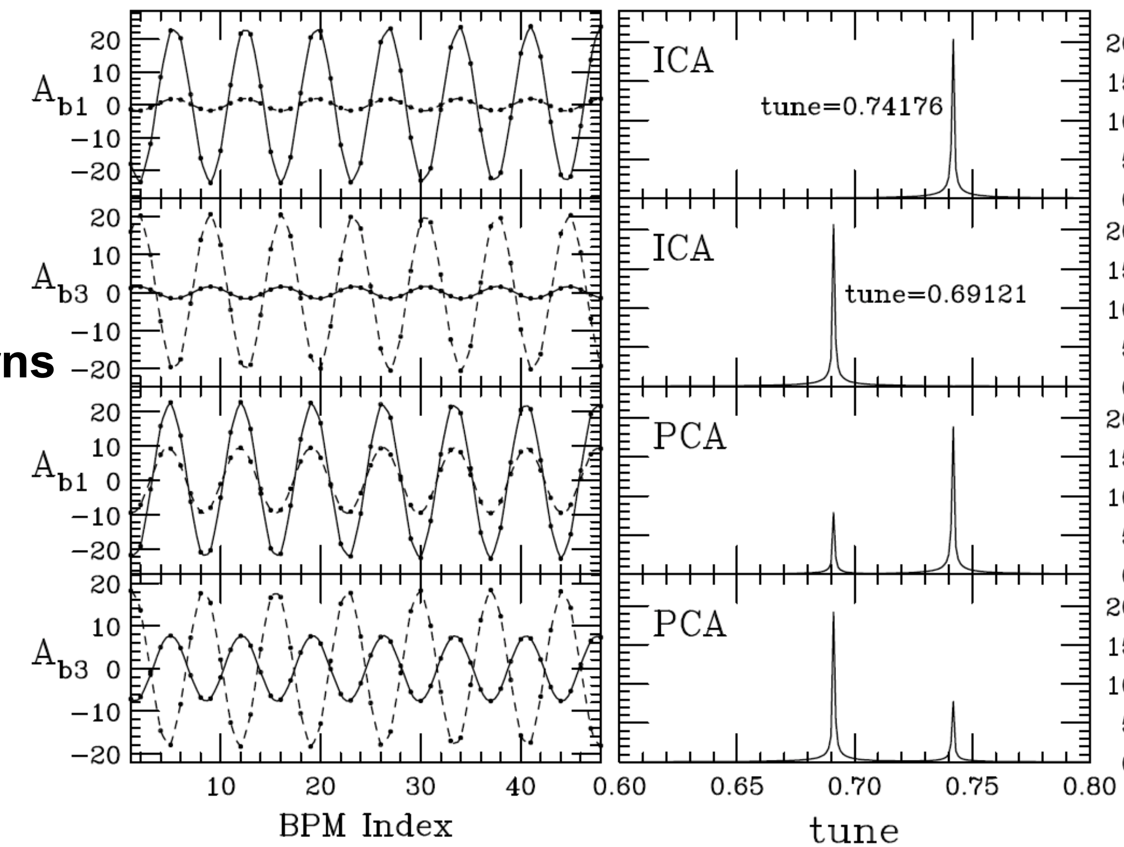
Single particle tracking of Brookhaven National Lab's AGS Booster with MAD.



X. Pang, Ph.D. thesis, unpublished (Indiana University 2009).

BSS Application to Linear Coupling

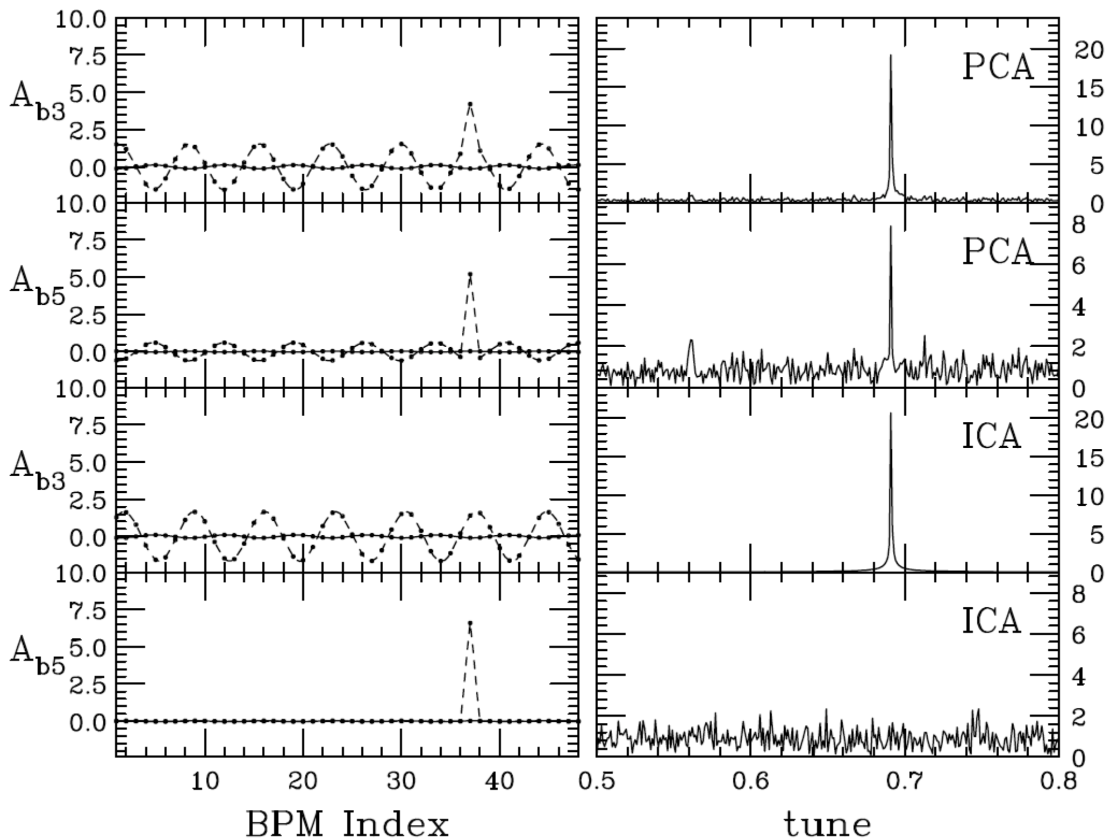
- **Single particle tracking simulations using the Fermilab Booster lattice.**
- **ν_x = 6.7415, ν_y = 6.6915, C = 0.05, x_0 = y_0 = 1.0.**
- **Left: spatial modes/patterns**
  - Horizontal (solid)
  - Vertical (dash)
- **Right: fft spectra of temporal modes/patterns**



X. Huang, et. al., PRST-AB **8**, 064001 (2005).

BSS Application to Noisy BPM

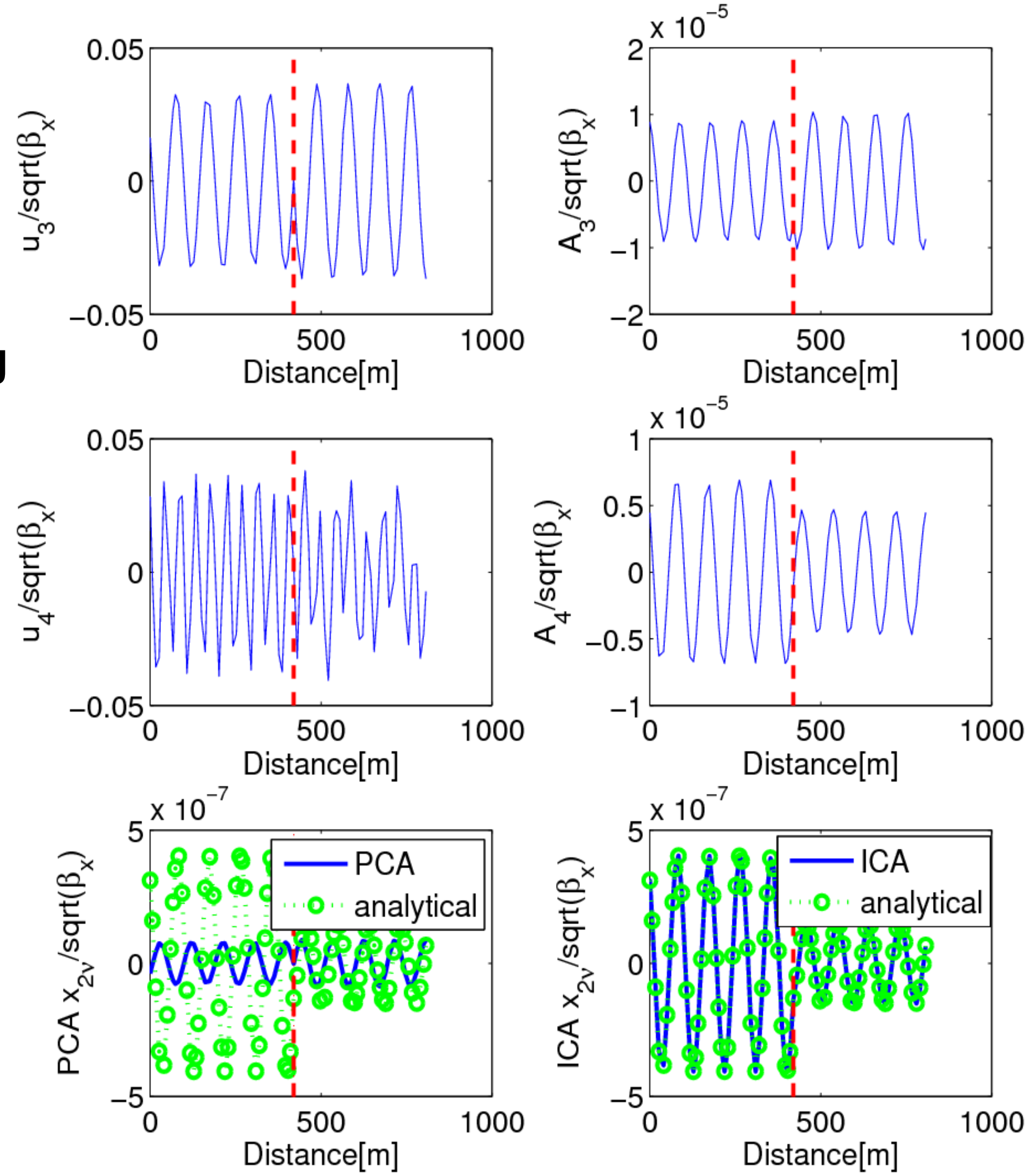
- **Linear coupling and localized bad BPM with Gaussian noise: BPM 37y.**
- **$\nu_x = 6.7415$, $\nu_y = 6.6915$, $C = 0.05$, $x_0 = 1.0$, $y_0 = 0.0$.**



X. Huang, et. al., PRST-AB **8**, 064001 (2005).

BSS Application to Nonlinear Motion, 2ν

- **Single particle tracking of Brookhaven National Lab's AGS Booster with one sextupole using MAD.**
- **Compare the spatial modes/ patterns obtained by BSS**
  - After ICA, the spatial modes of the 3rd and 4th IC have simple linear betatron motion outside the sextupole.

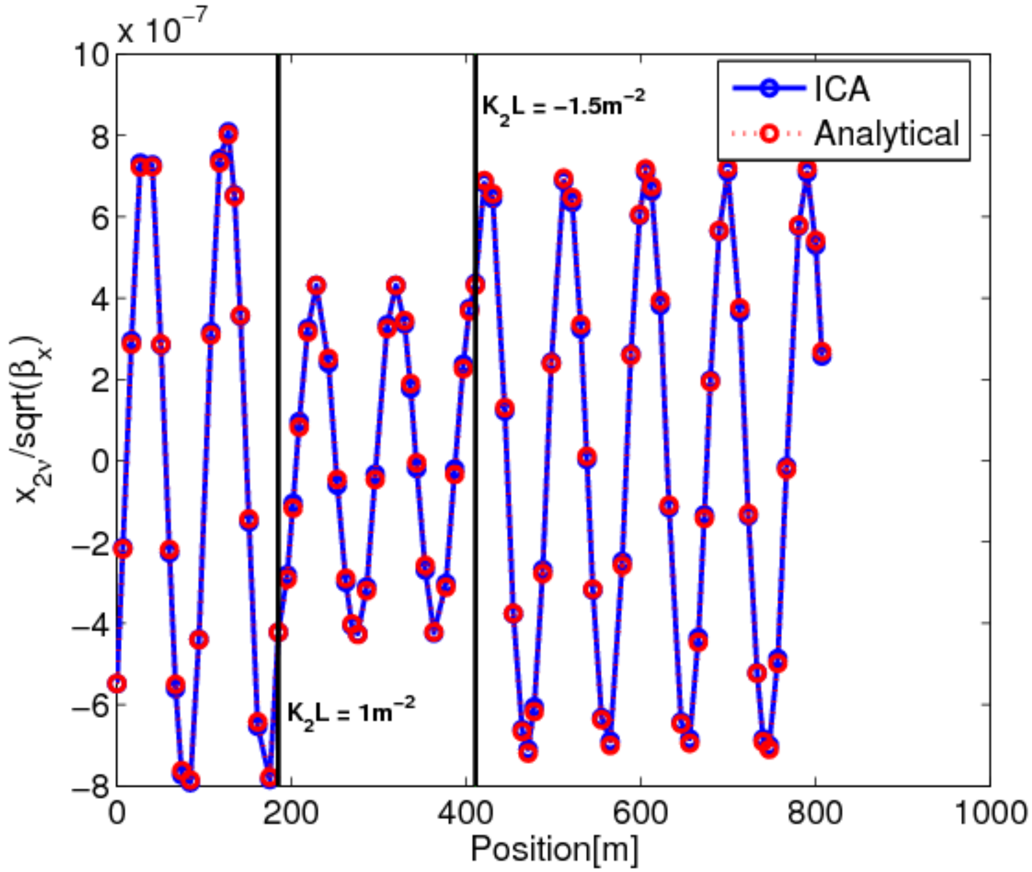


X. Pang and S.Y. Lee, Journal of Applied Physics, **106**, 074902 (2009).

BSS Application to Nonlinear Motion (cont.)

- **AGS lattice with two sextuples located at 185m and 420.37m, with strength $K_2L = 1m^{-2}$ and $-1.5m^{-2}$.**
- **Black lines indicate the locations of two sextupoles.**
- **At the sextupole, the change of particle motion action is related to sextupole strength by**

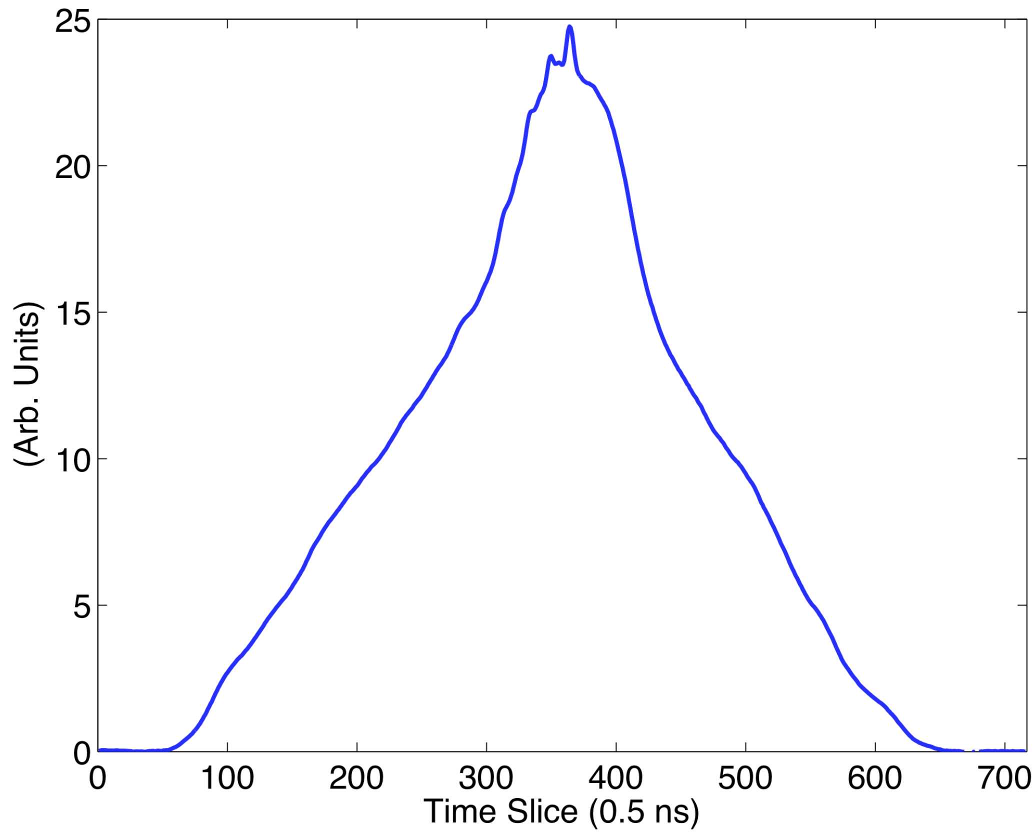
$$\Delta J = (\alpha x + \beta x')\Delta x' = \frac{1}{2}K_2 L x^2(\alpha x + \beta x')$$



X. Pang and S.Y. Lee, Journal of Applied Physics, **106**, 074902 (2009).

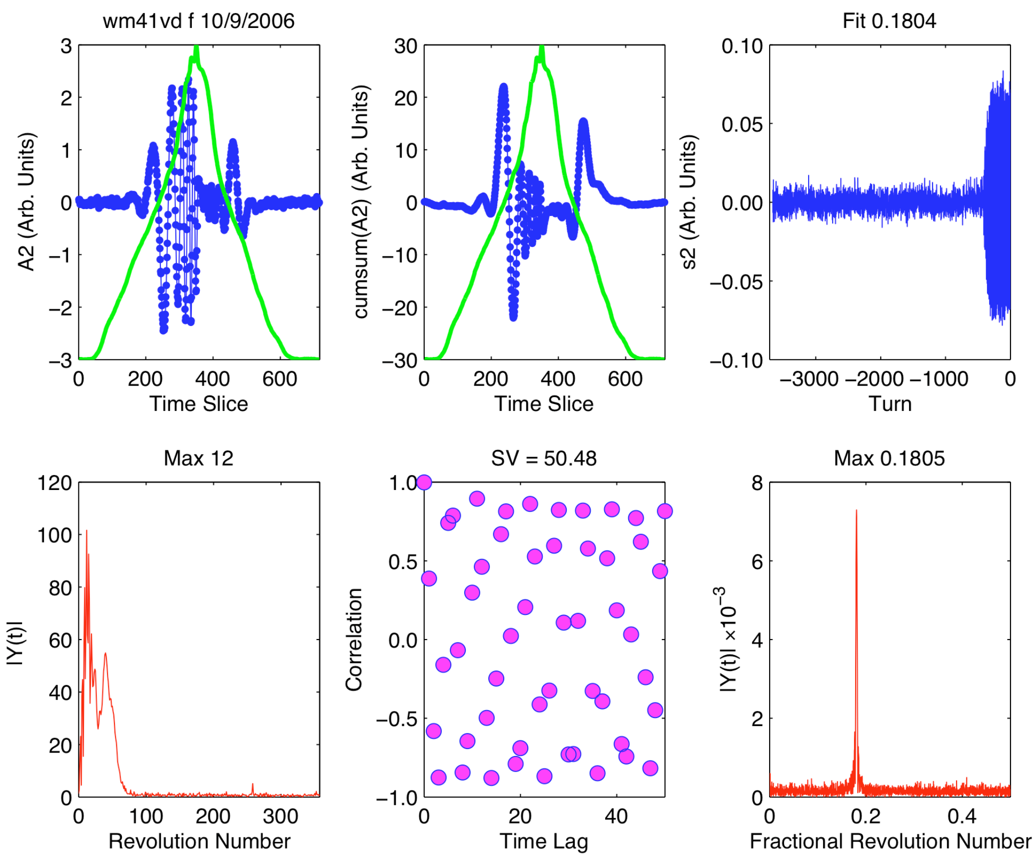
BSS of Slices Along the Bunch
Los Alamos Proton Storage Ring (PSR)

- **Apply BSS in a new way to slices along the bunch.**
  - Digitize beam signals for injection-extraction cycle (≥1800 turns).
  - We divide the digitized signal into time slices of equal length using the 0.5 ns digitization bin length.
  - The long digitized signal vector is stacked turn-by-turn to form **x**(t), such that each row of **x**(t) is beam signal from a time slice.
- **Applying BSS to slices along the bunch**
  - Monitor -> Slice (Spatial variation)
  - Measurement -> Turn (Temporal variation)



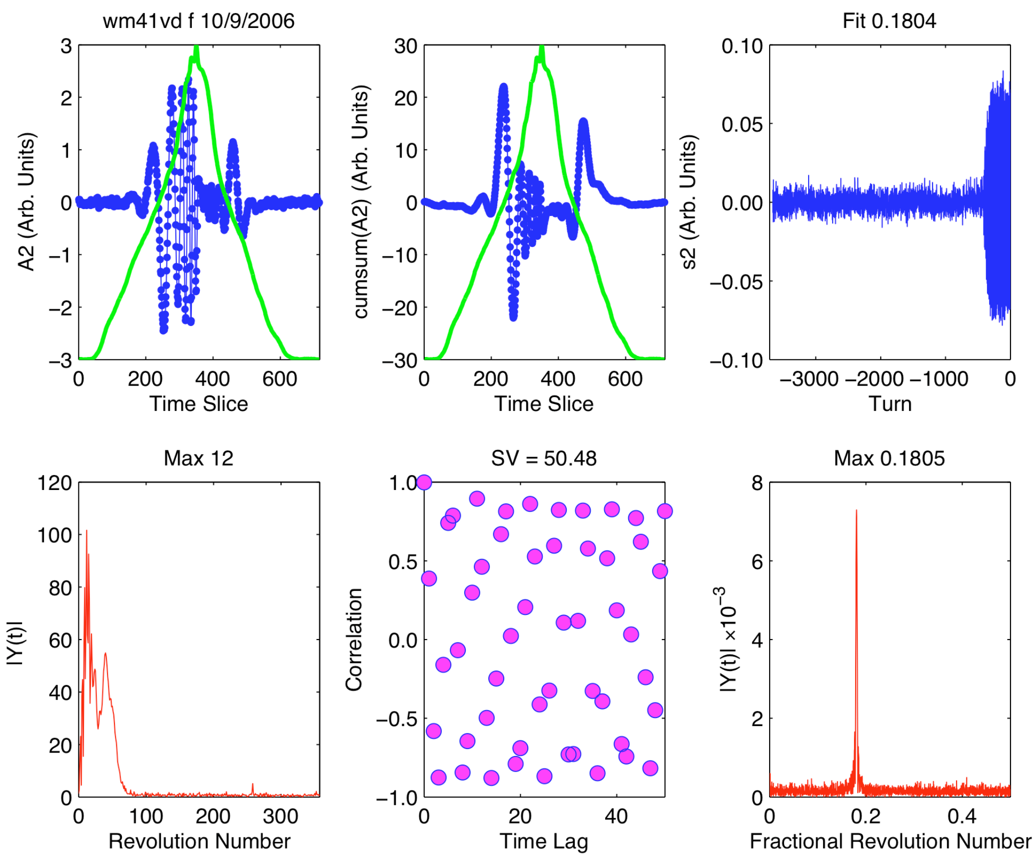
The last turn beam profile plotted is an example of the last column of x(t).

Visual Representation of ICs



- **Top left**
  - Spatial mode (blue)
  - Last turn beam profile (green)
- **Bottom left**
  - Spatial mode fft
  - Integer revolution number (quoted)
- **Top center**
  - Integrated spatial mode (blue)
  - Last turn beam profile (green)

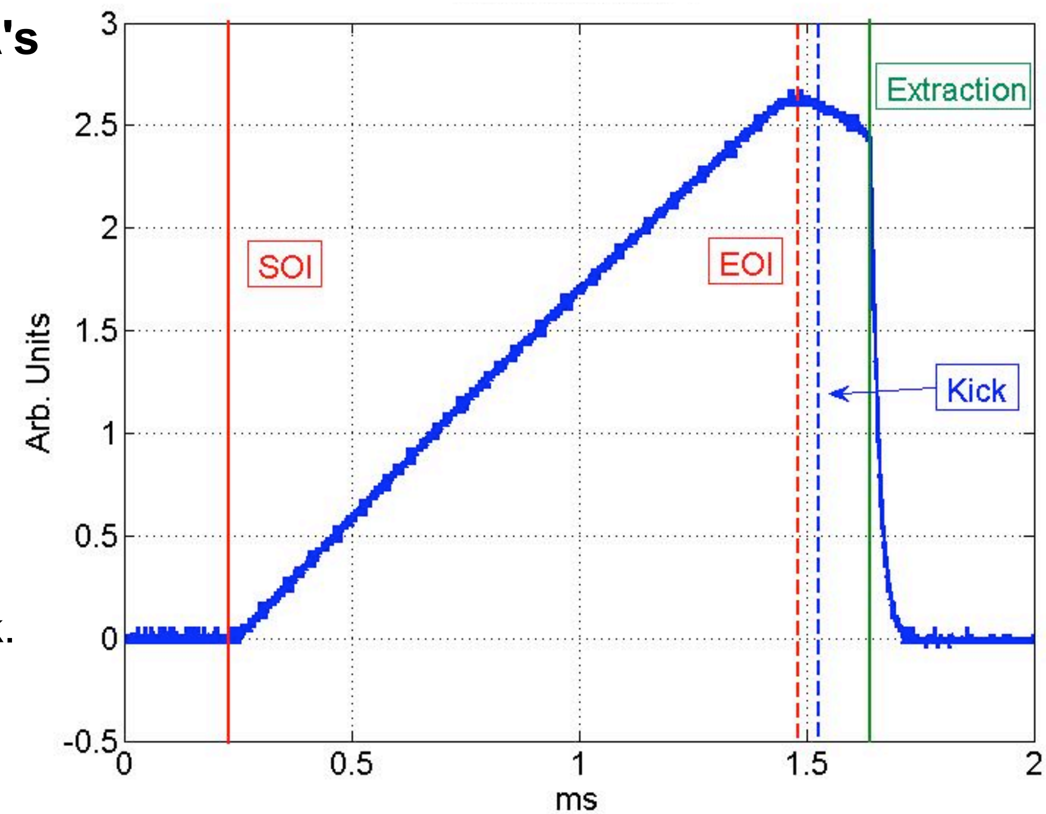
Visual Representation of ICs (cont.)



- **Bottom center**
  - Correlation between the constructed signal and **z**(t)
  - SV (quoted)
- **Top right**
  - Temporal Mode
  - Turn -1 is last turn
  - Fractional revolution number result of sinusoid fit (quoted)
- **Bottom right**
  - Temporal mode fft
  - Fractional revolution number

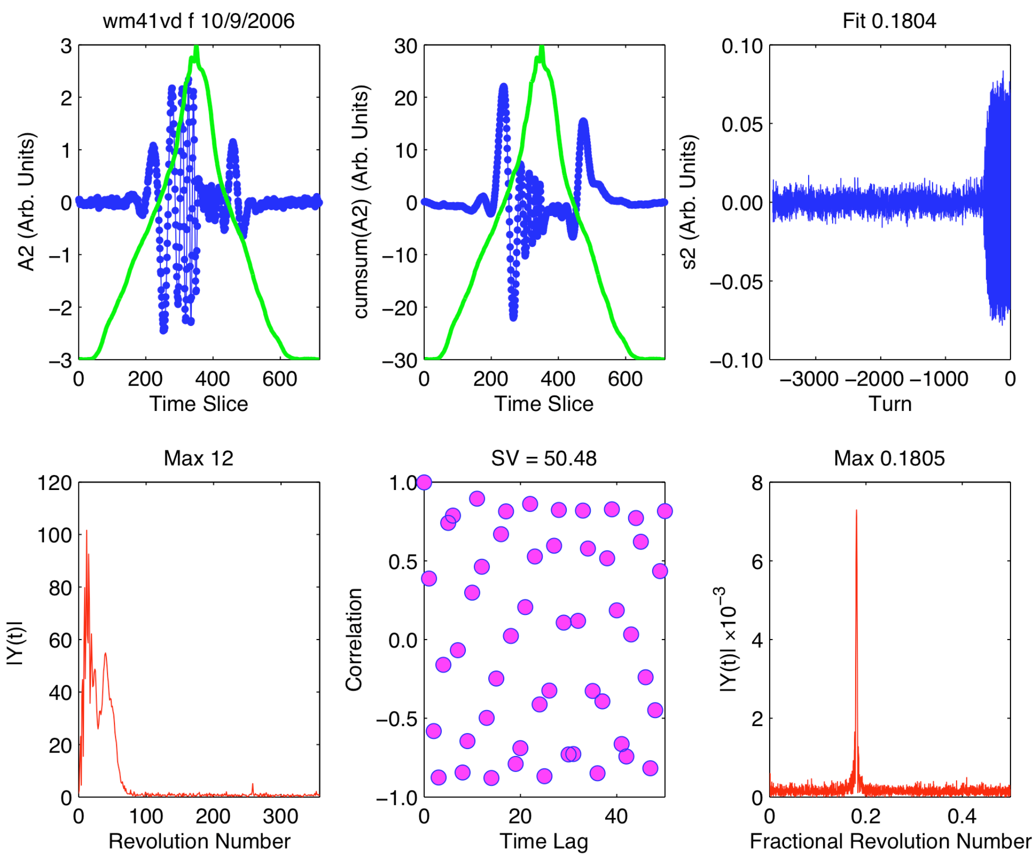
Single-Turn Kick Experiment

- **It is of interest to examine ICA's results concerning betatron motion when applied to slices along the bunch.**
- **The beam must undergo coherent betatron oscillation**
  - Induced by a vertical single-turn kick during a store time after accumulation.
  - The beam is stored for 420 turns (150 μs) after the single-turn kick.



Data taken by R. Macek in Oct. 2006; 1225 μs accumulation, 200 μs store.

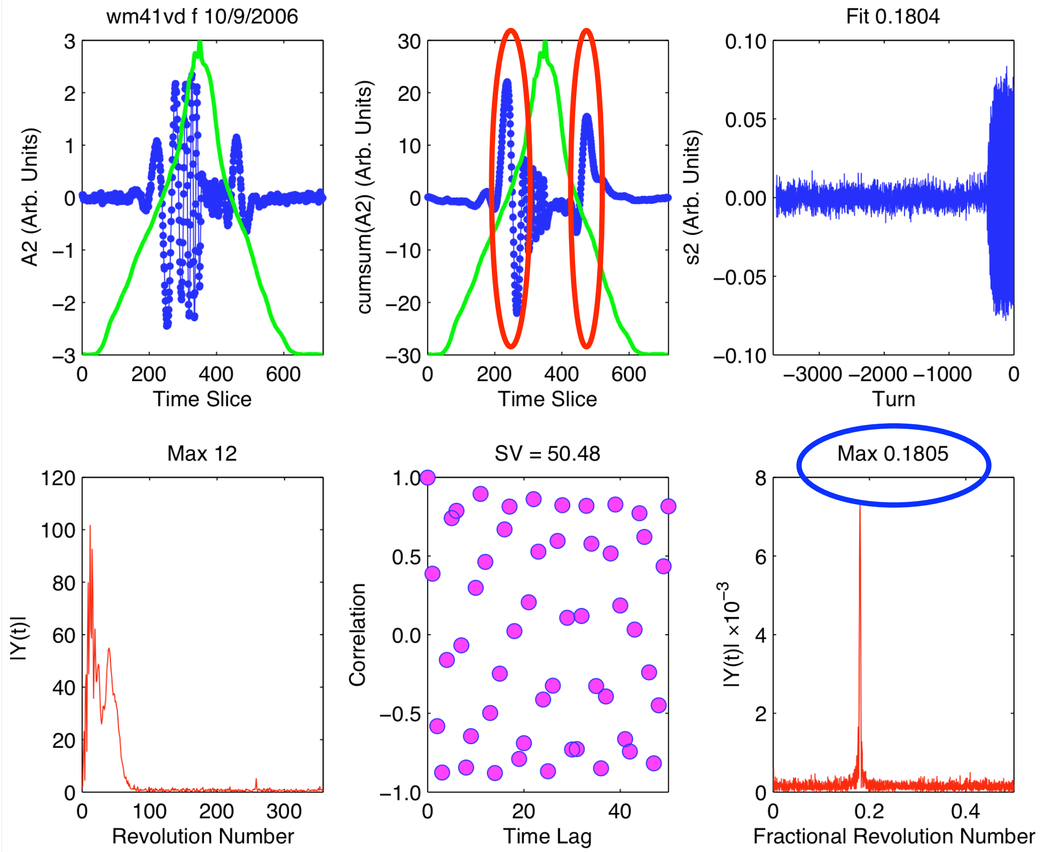
Betatron ICs



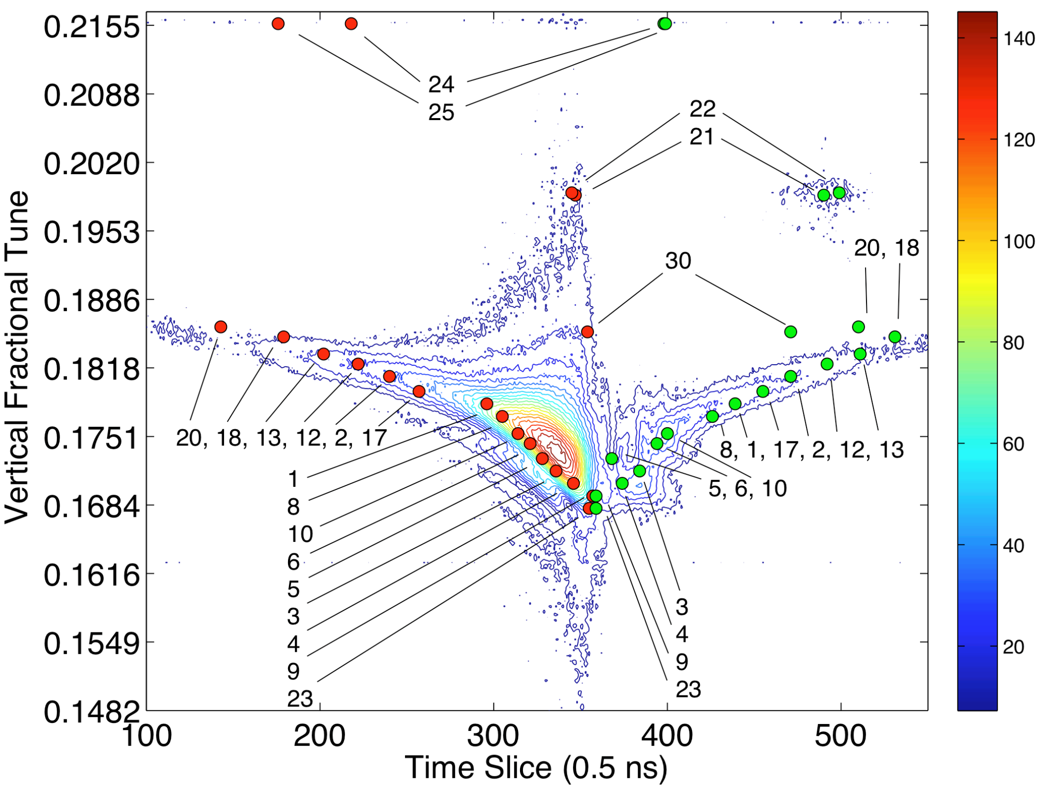
- **From vertical difference signal**
- **Identified by**
  - Temporal mode fft (bottom right) peak (fractional revolution harmonic) is close to the operating fractional betatron tune.
  - Temporal mode (top right) is only non-zero after the single-turn kick.
- **Spatial mode**
  - Describes the strength of the 0.1805 fractional betatron oscillation along the bunch.
  - Units [~ current derivative].
- **Integrated spatial mode**
  - Units [approx. ~ current].

Greatest Strength Locations

- **Want to compare the betatron ICs with the data matrix fft.**
- **Greatest strength location:**
  - Fractional revolution number and location along the bunch where an IC is strongest.
  - Use the integrated spatial mode (top center) maximum.



Betatron ICs Viewed in Concert

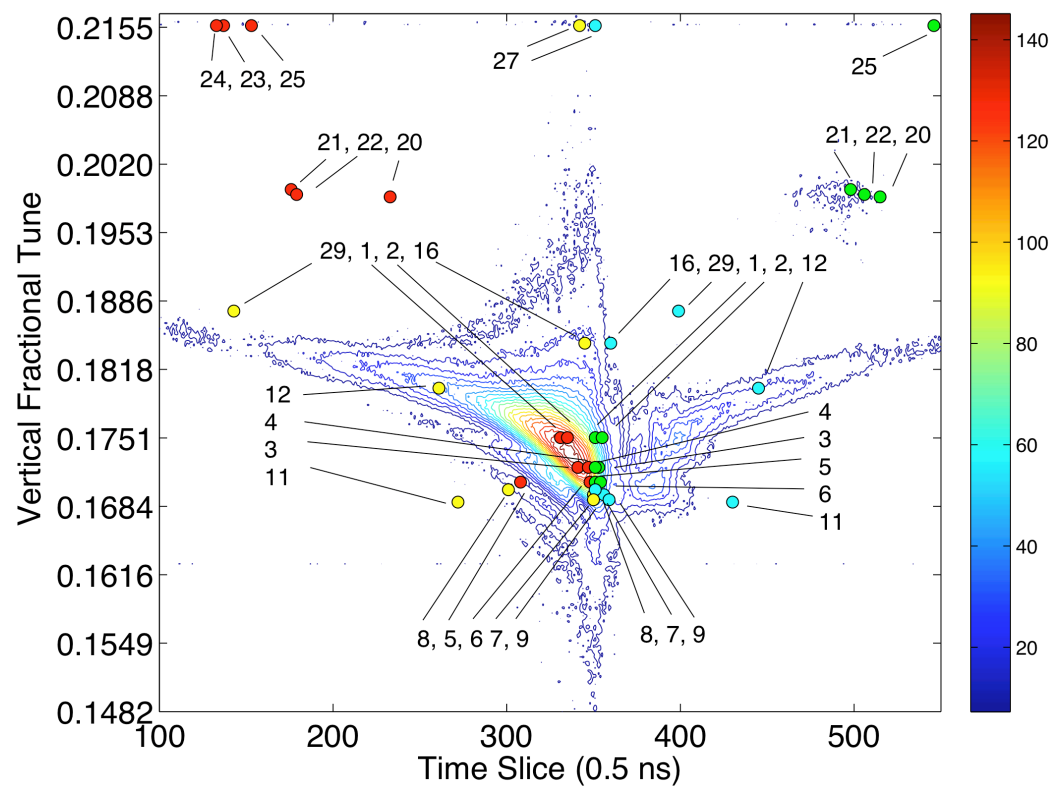


Red and green circles are the leading and trailing edge greatest strength locations respectively.

- **15 betatron ICs**
  - The greatest strength locations reproduce tune distribution given by the data matrix fft along turn for each time slice.

Betatron PCs Viewed in Concert

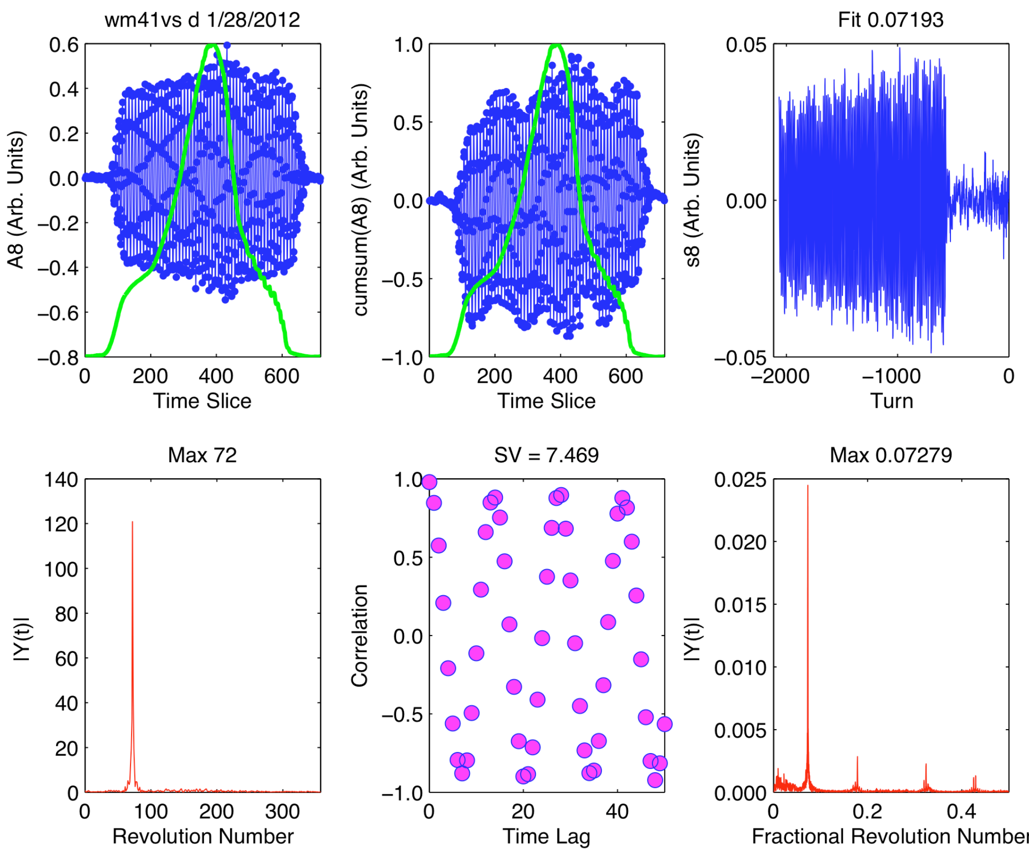
- **All betatron PCs have peak strengths located near the bunch center where the vertical difference signal is largest because PCA is unable to diagonalize the frequency continuum beyond its peak strength and average location.**
- **It is clear that PCA is unable to recover the coherent space charge tune shift along the bunch.**



Red (green) circles are the leading (trailing) edge greatest strength location for PCs with dominate betatron motion.

Yellow (cyan) circles are the leading (trailing) edge greatest strength location for PCs with dominate source signal other than betatron motion.

201.25 MHz ICs

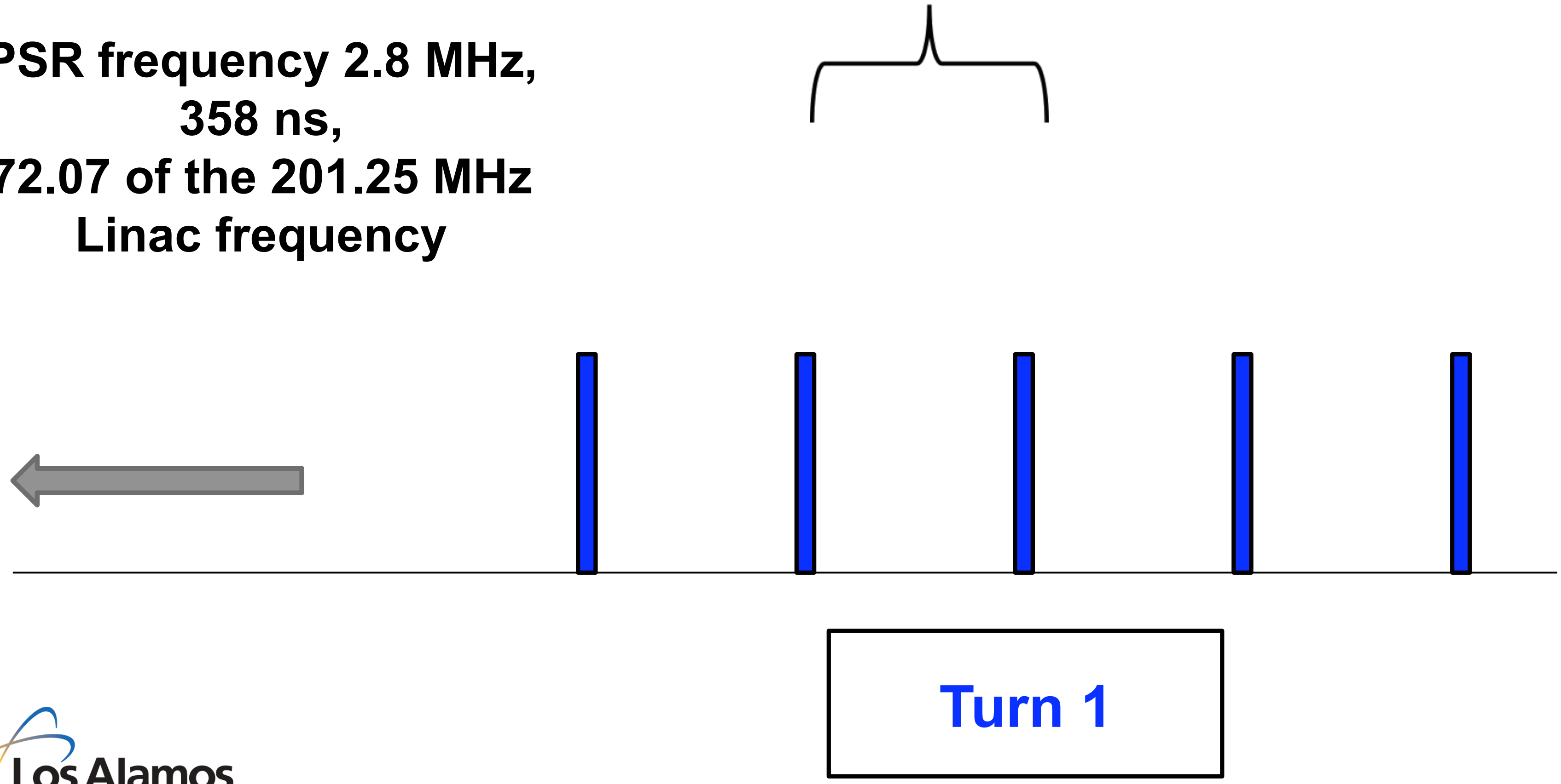


- **From vertical sum signal**
- **Identified by**
  - Temporal mode (top right) has constant amplitude during accumulation and reduces to noise during storage.
- **Represents the longitudinal structure of injected beam.**
- **The total revolution number is 72.07279 as expected because the PSR revolution frequency is set to the 72.07 subharmonic of 201.25 MHz.**
- **Frequency exactly 201.25 MHz.**

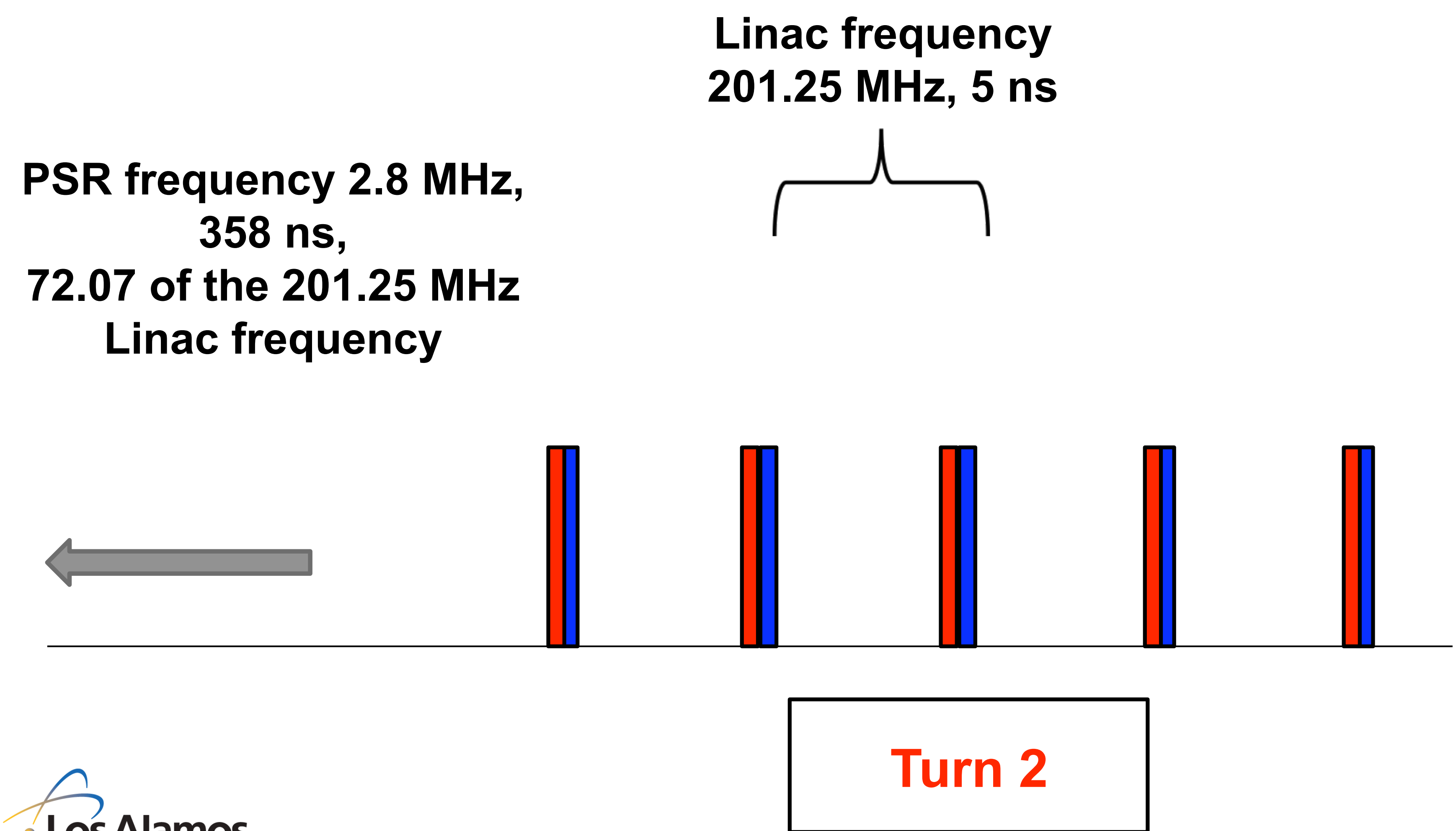
Longitudinal Injection Scheme

Linac frequency
201.25 MHz, 5 ns

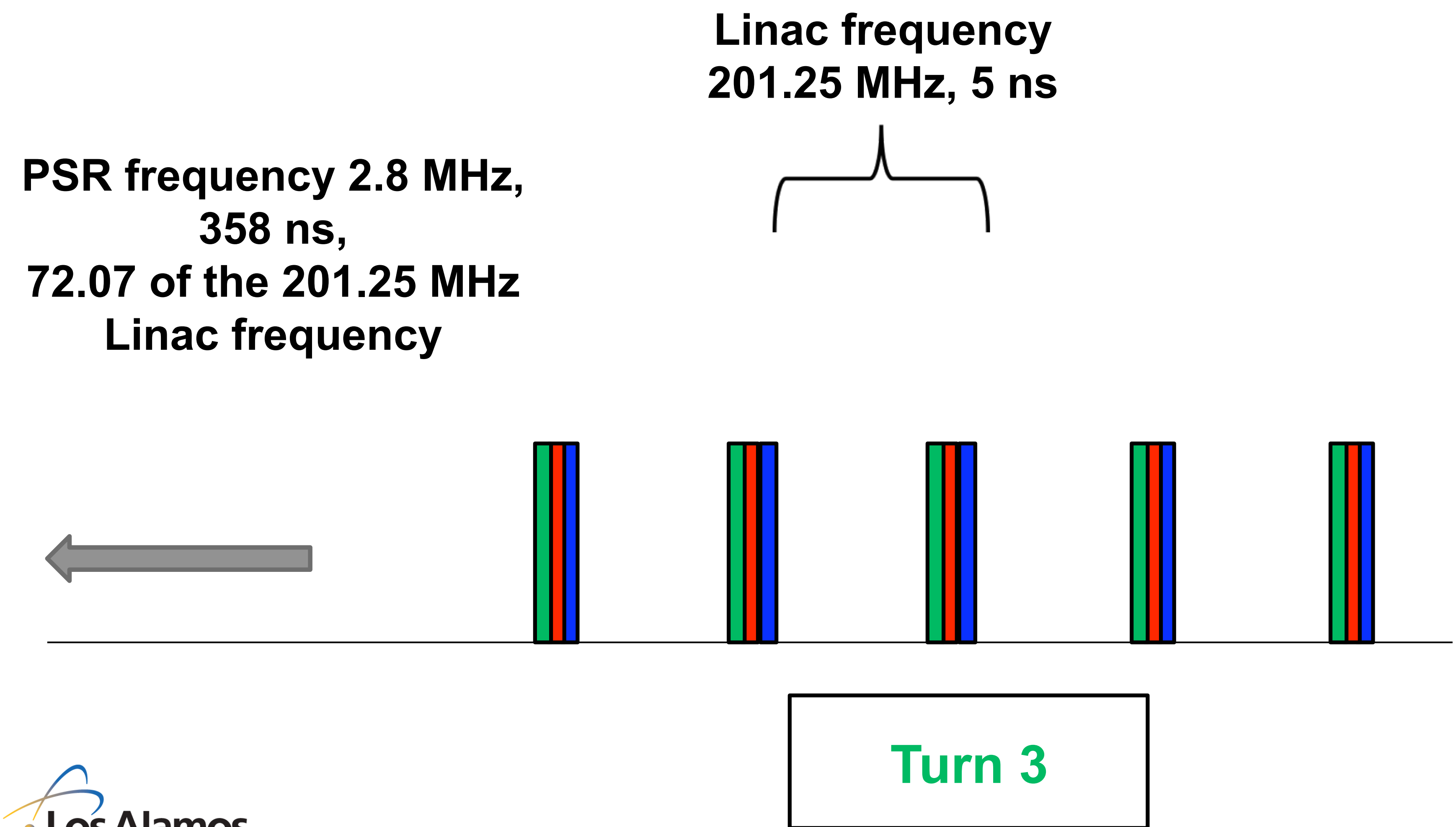
PSR frequency 2.8 MHz,
358 ns,
72.07 of the 201.25 MHz
Linac frequency



Longitudinal Injection Scheme



Longitudinal Injection Scheme



Longitudinal Injection Scheme

Linac frequency
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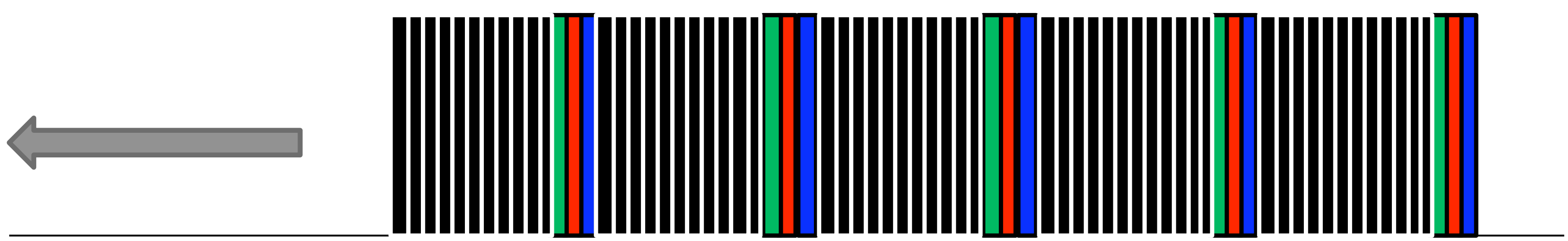
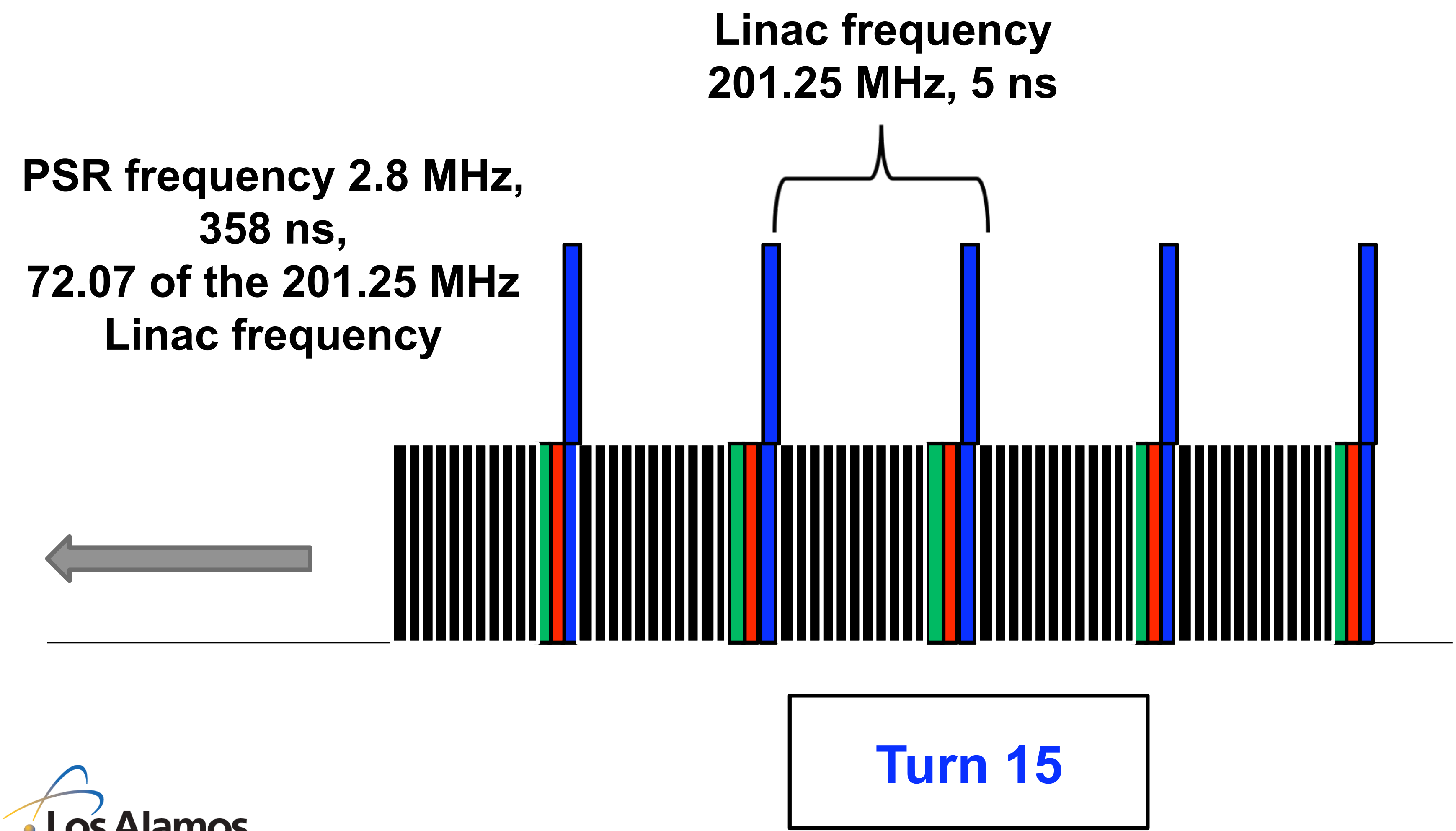
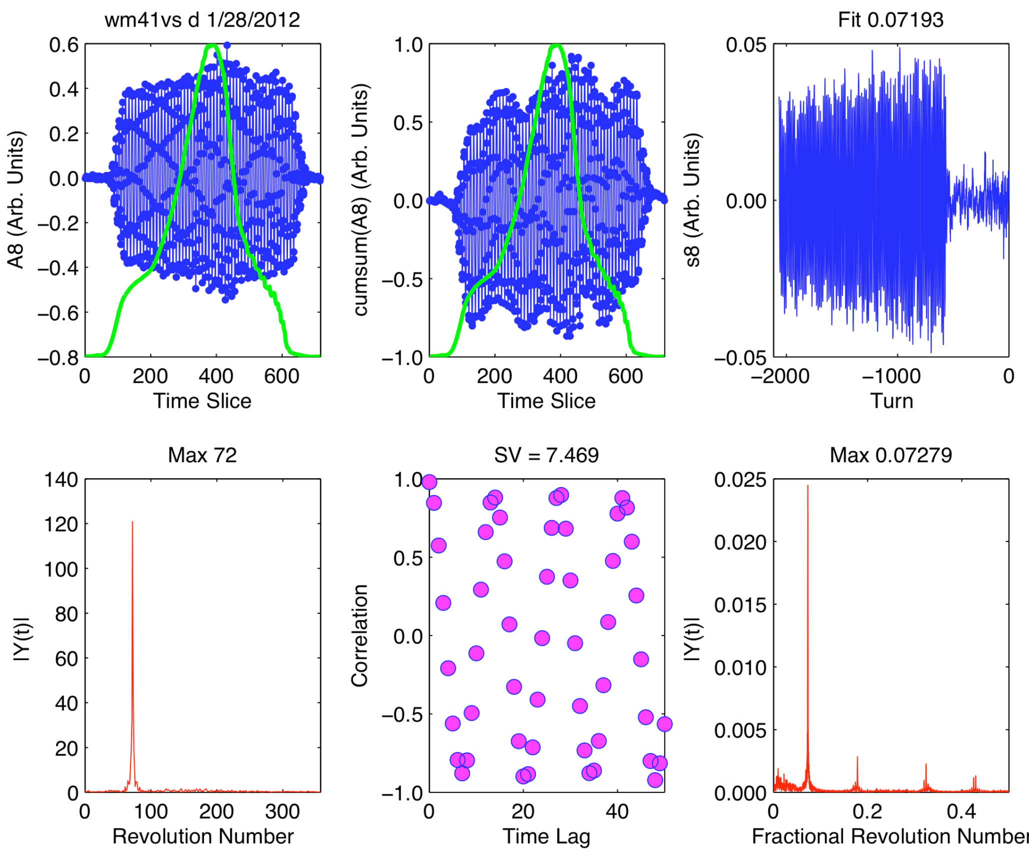


Turn 14

Longitudinal Injection Scheme



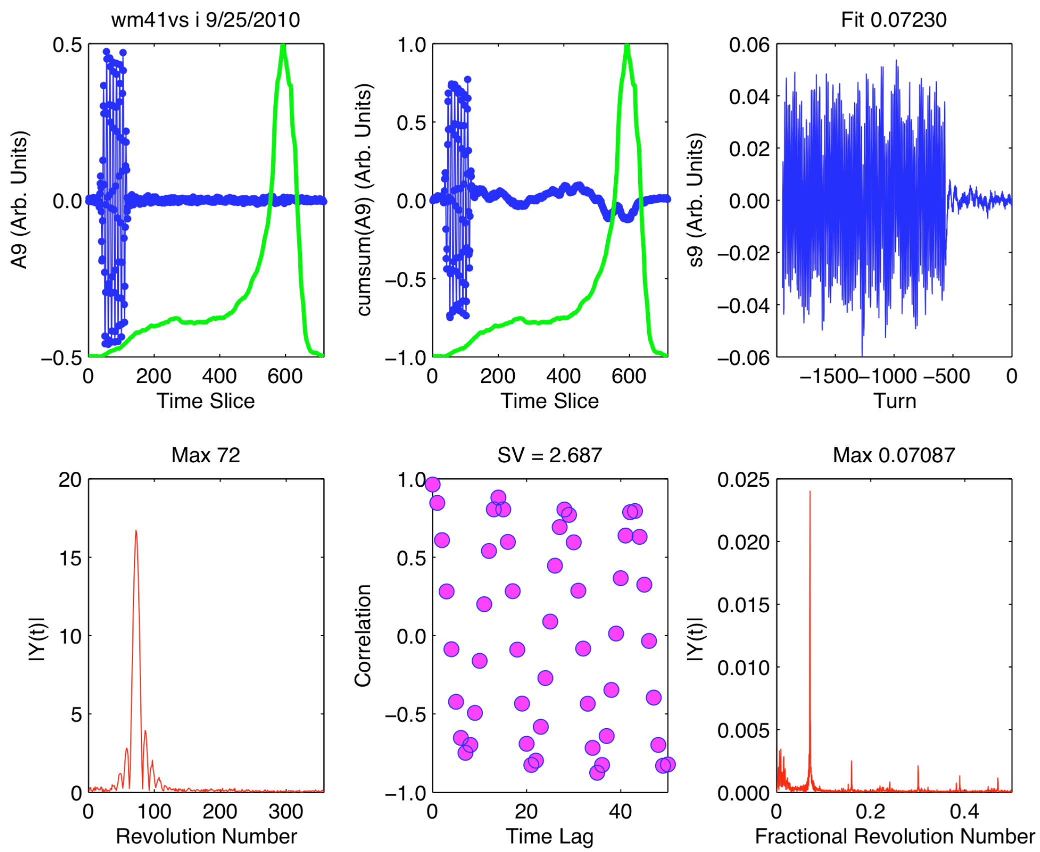
201.25 MHz ICs (cont.)



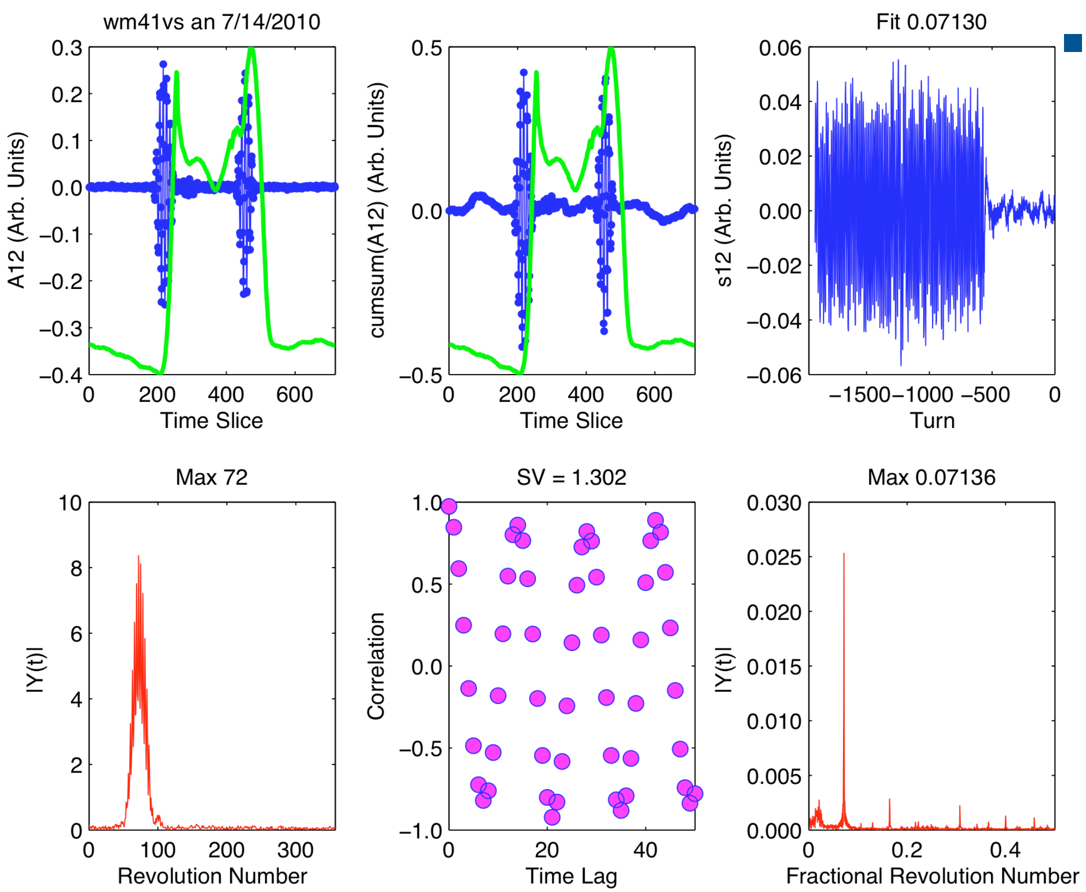
- **The spatial mode describes**
  - Longitudinal phase of injection.
  - The injection length each turn (pattern width).
- **58 peaks**
  - 5 ns period, a 201.25 MHz period.
  - represents a single linac pulse (linac RF bucket) injected into the PSR.

201.25 MHz ICs and Injection

- Inject a 50 ns pattern width beam 120° early in the PSR's RF bucket.



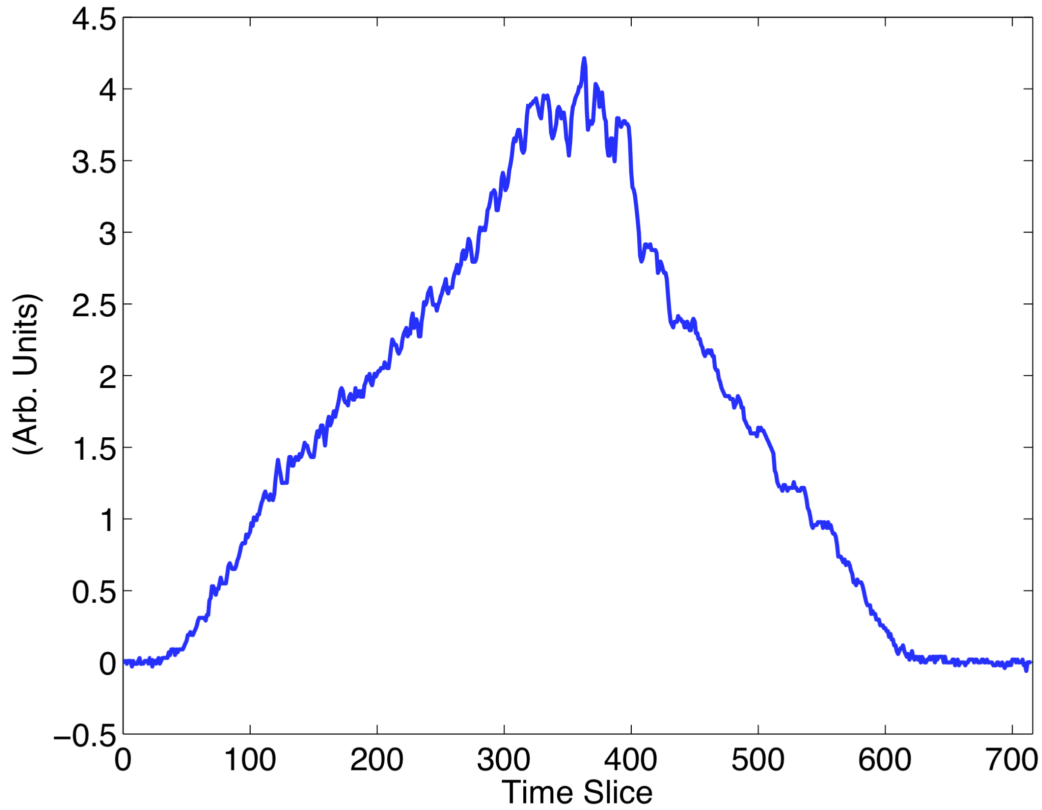
201.25 MHz ICs and Injection (cont.)



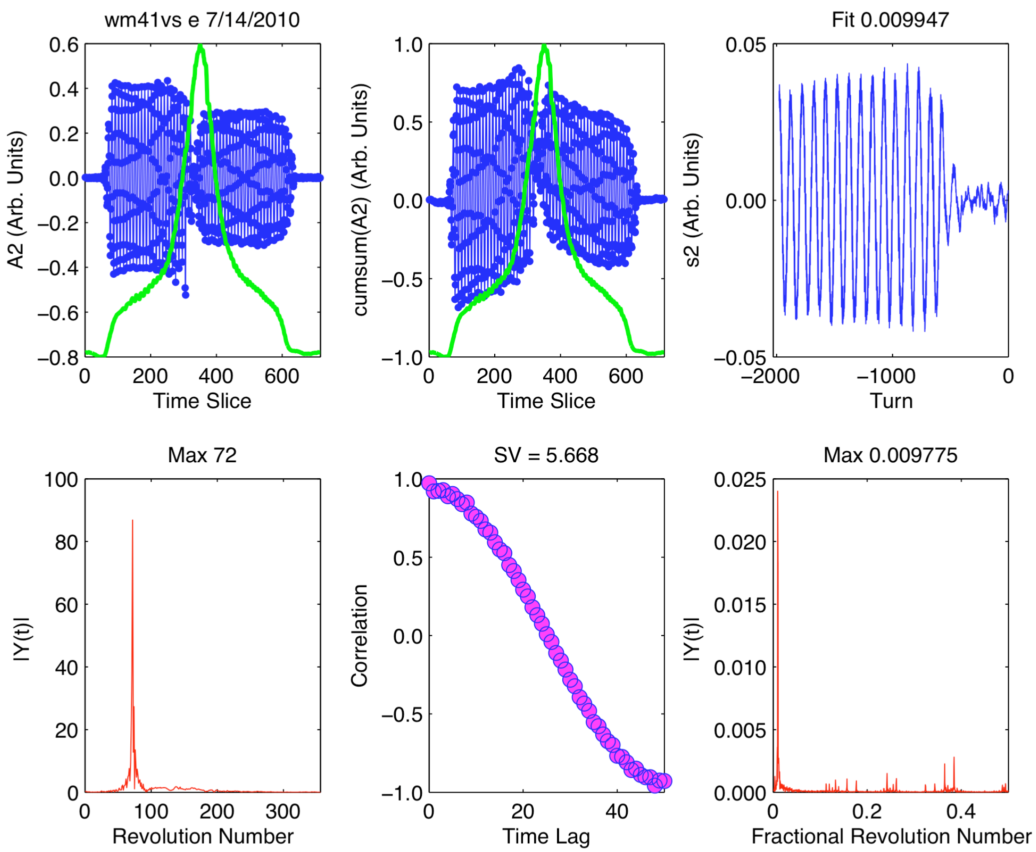
- **Inject with a ``notch" in the beam, illustrated by the spatial mode (top left).**

Beginning of the 2010 LANSCE Production Run Cycle

- **We observed an unusual amount of ``hash" noise-like structure on the PSR beam profile**
- **The hashy beam profile is an operational concern because it could excite longitudinal microwave instabilities and other longitudinal space charge effects.**



Beginning of the 2010 LANSCE Production Run Cycle (cont.)



- **Fractional revolution number 0.009 describes an oscillation that repeats every 111 turns.**
- **The slow injection rastering causes the hashy beam profile by longitudinally stacking the beam upon itself more than in nominal operations.**
- **The revolution frequency differs from design by 2.4 kHz.**
- **The number inputted into the frequency generator computer was 2.7948 MHz; exactly the number predicted by ICA.**

Conclusions

- **Introduced blind source separation (BSS) methods**
  - Principal component analysis (PCA)
  - Independent component analysis (ICA)
- **BSS application to BPM data**
  - Betatron motion -> beta functions
  - Linear coupling -> PCA mixes the betatron components
  - Bad BPM with Gaussian noise -> IC identifies the bad BPM
  - Nonlinear motion, 2ν -> ICA determines sextupole strength
- **BSS application to slices along the bunch**
  - Betatron tune along the bunch -> Space charge tune shift
  - 201.25 MHz IC describing the injection phase and pattern width -> Measured the revolution frequency

Bibliography

[Belouchrani] A. Belouchrani, K. Abed-Merain, J.F. Cardoso, and E. Moulines, ``A Blind Source Separation Technique using Second Order Statistics'', IEEE Trans. Signal Processing, 45, 434-444, (1997).

[Cardoso] J.F. Cardoso and A. Souloumiac, ``Jacobi Angles for Simultaneous Diagonalization'', SIAM J. Mat. Anal. Appl., 17, 161, (1996).

[Irwin] J. Irwin, C.X. Wang, Y.T. Yan, K.L.F. Bane, Y. Cai, F.-J. Decker, M.G. Minty, G.V. Stupakov, and F. Zimmerman, ``Model-Independent Beam Dynamics Analysis'', Phys. Rev. Lett. 82, 1684 (1999); C.X. Wang, Ph.D. thesis, unpublished (Standford University 1999).

Thank you for you attention!

Backup Slides

Uncorrelated vs. Independent

- **Two random variable vectors y_1 and y_2 are uncorrelated if their covariance is zero,**

$$\mathrm{cov}\left(\vec{y}_1,\vec{y}_2\right)=\left\langle \vec{y}_1,\vec{y}_2\right\rangle-\left\langle \vec{y}_1\right\rangle\left\langle \vec{y}_2\right\rangle=0$$

where <…> is the expectation operator.

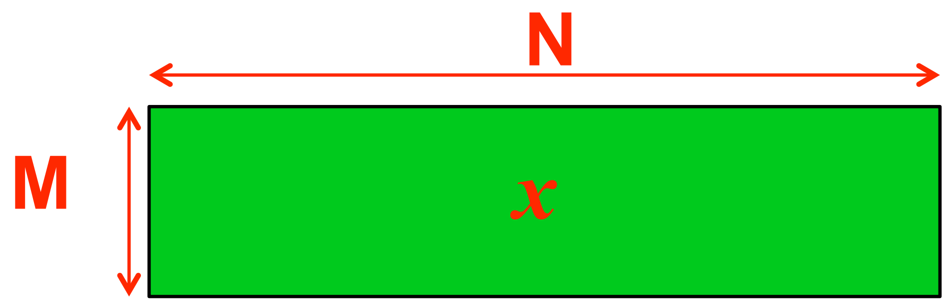
- **Two random variable vectors y_1 and y_2 are independent if the covariance of any function of y_1 and any function of y_2 is zero**

$$\mathrm{cov}\left(f\left(\vec{y}_1\right),g\left(\vec{y}_2\right)\right)=\left\langle f\left(\vec{y}_1\right),g\left(\vec{y}_2\right)\right\rangle-\left\langle f\left(\vec{y}_1\right)\right\rangle\left\langle g\left(\vec{y}_2\right)\right\rangle=0$$

- **Uncorrelatedness is a special case of independence when $f(y_1) = y_1$ and $g(y_2) = y_2$.**

Time Lags

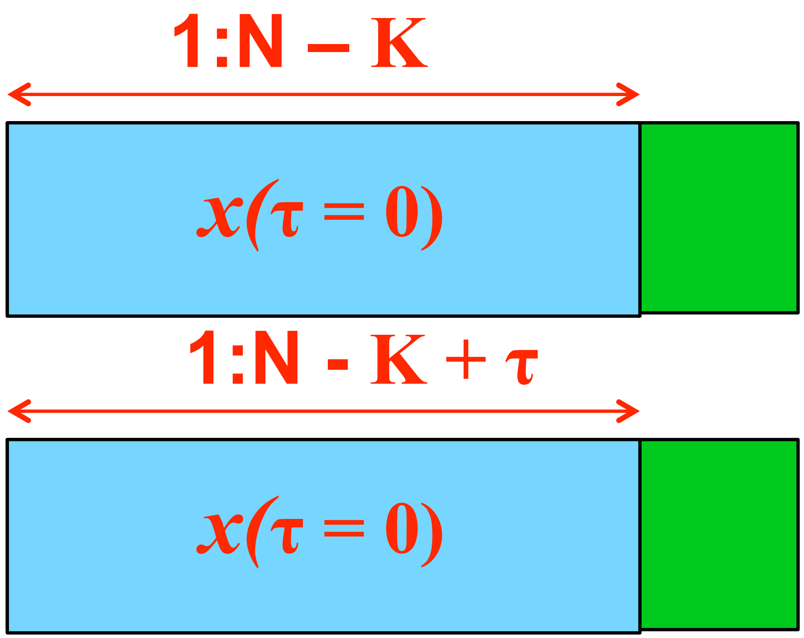
The Data Matrix



Time Lags

Zero Time-Lag Correlation Matrix

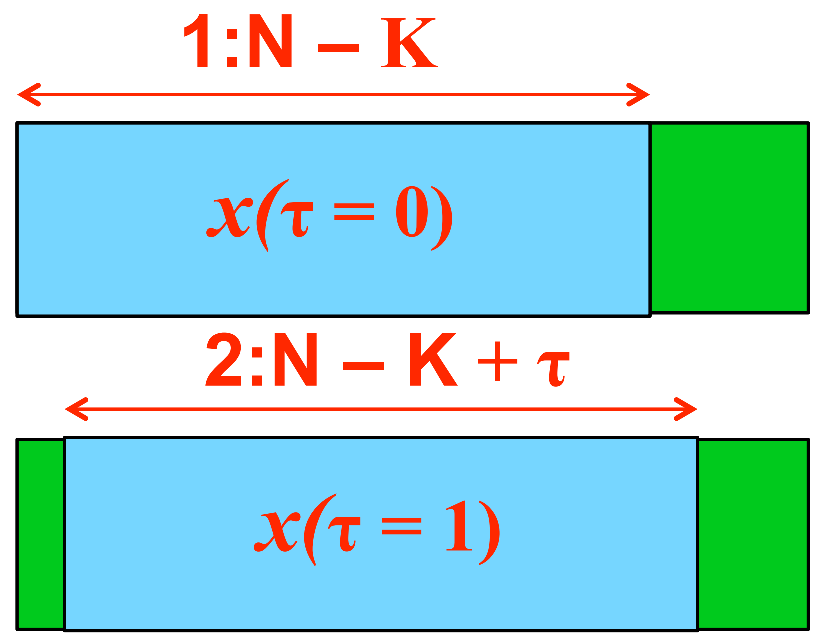
$$C_x(\tau = 0) = x(\tau = 0)\,x(\tau = 0)^T$$



Time Lags

Covariance Matrix $\tau = 1$

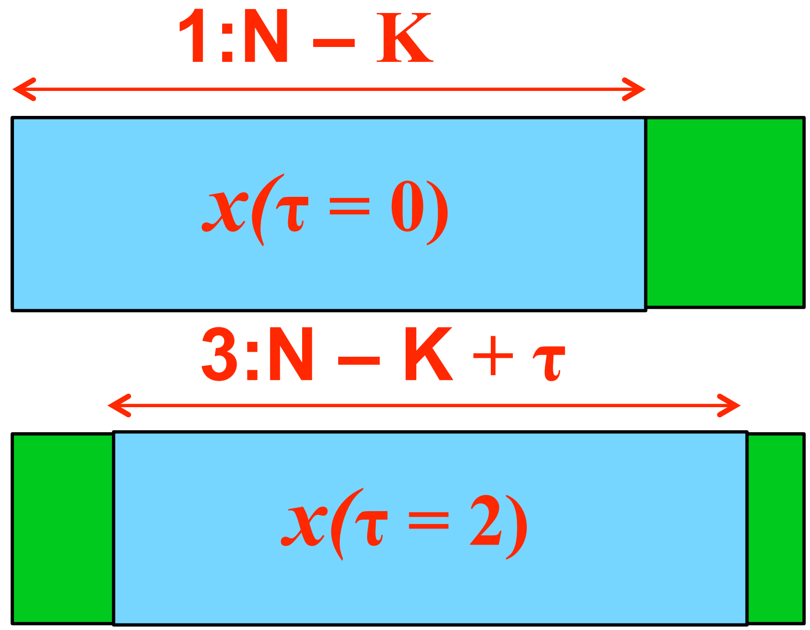
$$C_x(\tau = 1) = x(\tau = 0)\,x(\tau = 1)^T$$



Time Lags

Covariance Matrix τ = 2

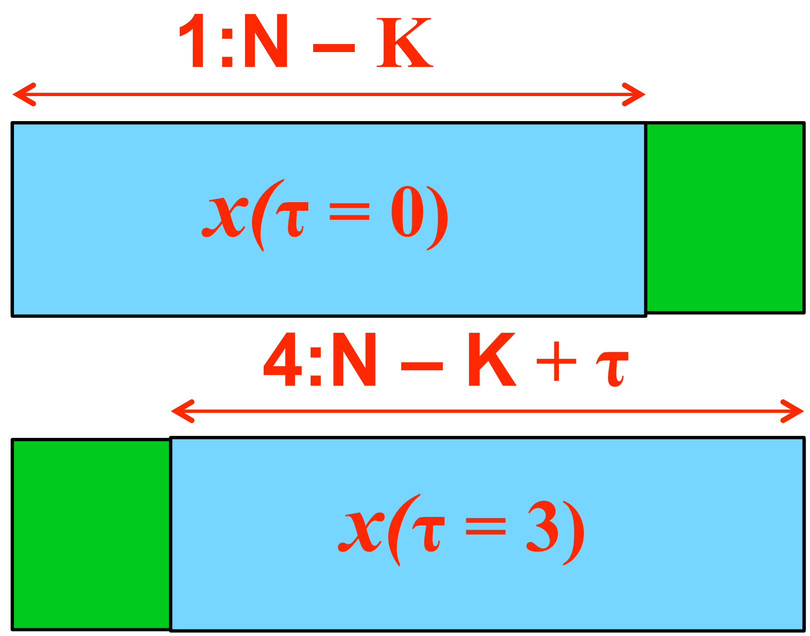
$$C_x(\tau = 0) = x(\tau = 0)x(\tau = 2)^T$$



Time Lags

Covariance Matrix τ = 3

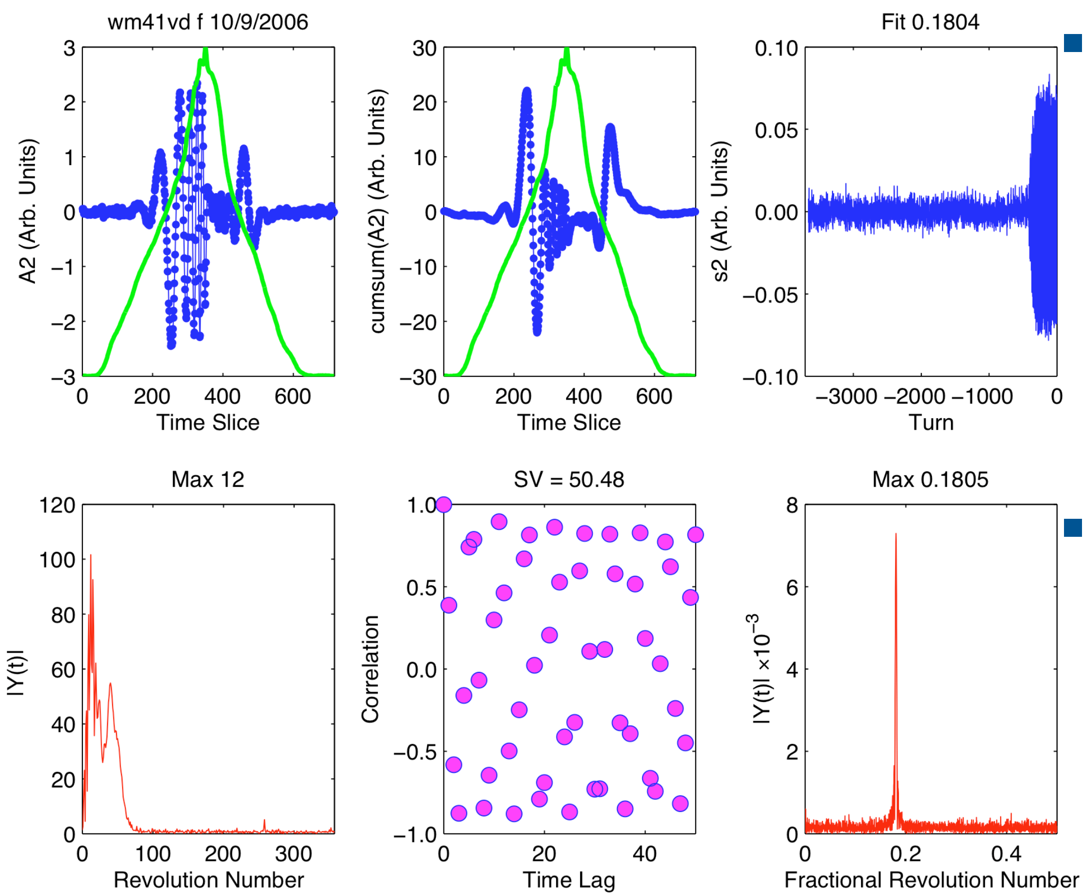
$$C_x(\tau = 0) = x(\tau = 0)x(\tau = 3)^T$$



BSS of Slices Along the Bunch (cont.)

- **The ICs produced by ICA of slices along the bunch describe longitudinal motion, longitudinal beam structure, and motion that varies along the bunch.**
- **We digitize beam signals for ICA**
  - a fast current monitor, SRWC41 [units: ~ current]
  - a short stripline BPM with 400 MHz peak frequency response, SRWM41.
    - Horizontal and vertical sum and difference signals
      - Sum signal units [~ current derivative]
      - Difference signal unit [~ position × current derivative]
    - The integral of the sum and difference signals are calculated offline for ICA
      - Integrated sum signal units [~ current]
      - Integrated difference signal units [approx. ~ position × current]
- **We typically ran our analysis for *L* = 30 SVs, *K* = 50 time lags, and the maximum number of turns possible.**

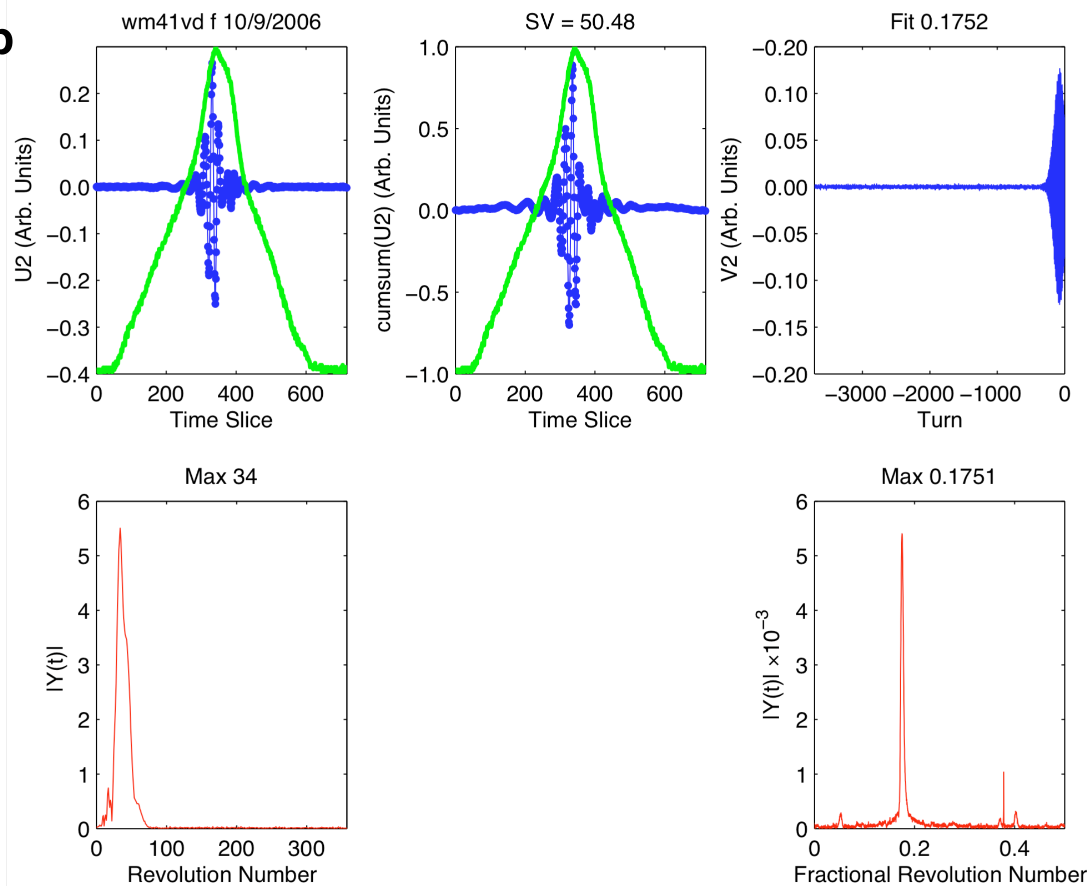
Betatron ICs (cont.)



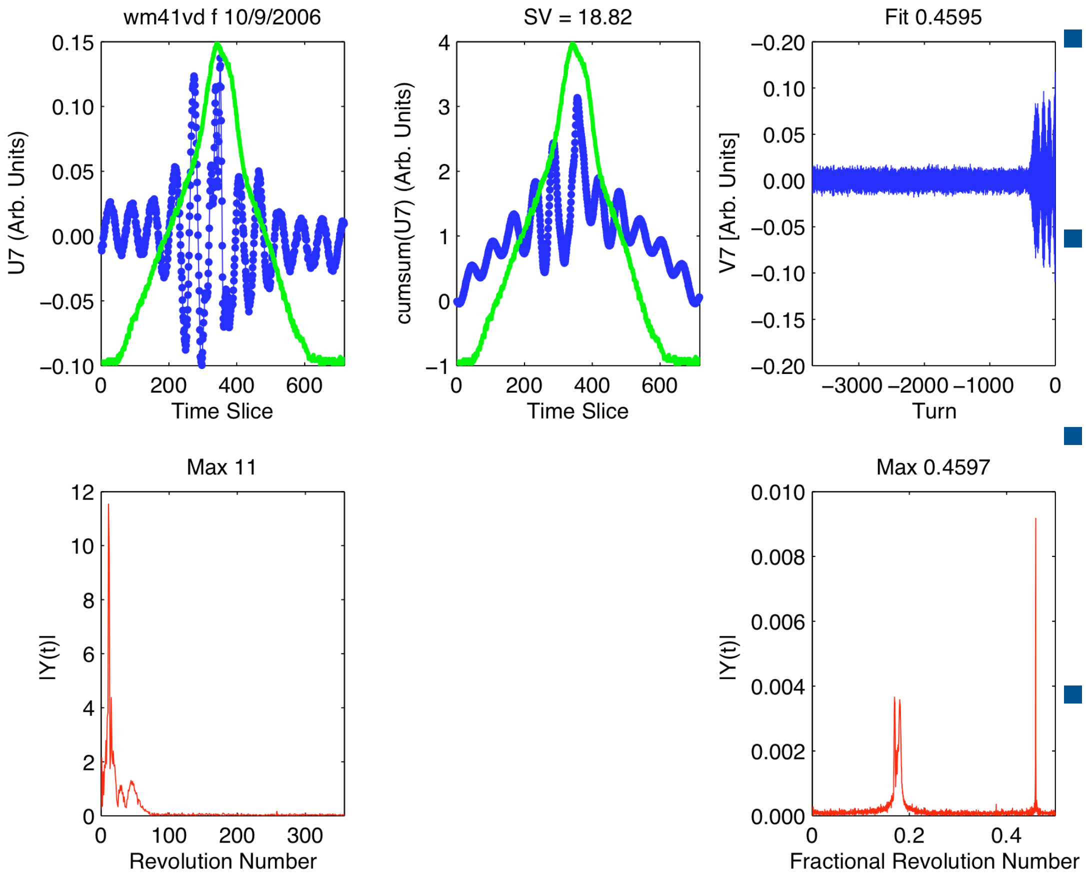
- The majority of particles undergoing coherent betatron motion with a fractional tune of 0.1805 are located symmetric about the bunch center at time slices 237 and 474 and represent the coherent space charge tune shifted beam.
- The fast oscillation slightly forward of the bunch center describes mixing of betatron tunes for the central time slices.

Betatron PCs

- **The integral spatial pattern (top center) indicates the greatest strength location of the 0.1751 fractional tune oscillation is at the bunch center.**
- **The strength of the PC decays symmetrically about the peak current and does not indicate asymmetry in the coherent space charge tune shift.**
- **The temporal pattern fft (bottom right) peak's full width half maximum is twice as large as for the betatron IC.**



Betatron PCs (cont.)



- **The ICA and PCA result differs as the betatron PCs are mixed with other source signals.**
- **The dominate source signal has fractional revolution number (bottom right) of 0.46.**
- **The board betatron peak in the temporal pattern fft indicates that this PC mixes with much of the tune continuum.**
- **The spatial pattern (top left) is a mixture of betatron motion and the other source signal.**
- **The interpretation of this PC is corrupted by PCA's incomplete source separation.**